Application

In this course, you will learn that there are many ways to approach a problem. Example 9 illustrates three common approaches.

A numerical approach: Construct and use a table.

A graphical approach: Draw and use a graph.

An algebraic approach: Use the rules of algebra.

REMARK You should develop the habit of using at least two approaches to solve every problem. This helps build your intuition and helps you check that your answers are reasonable.

EXAMPLE 9 Maximum Weight

The maximum weight y (in pounds) for a man in the United States Marine Corps can be approximated by the mathematical model

$$y = 0.040x^2 - 0.11x + 3.9, \quad 58 \le x \le 80$$

where x is the man's height (in inches). *(Source: U.S. Department of Defense)*

a. Construct a table of values that shows the maximum weights for men with heights of 62, 64, 66, 68, 70, 72, 74, and 76 inches.

b. Use the table of values to sketch a graph of the model. Then use the graph to estimate *graphically* the maximum weight for a man whose height is 71 inches.

c. Use the model to confirm *algebraically* the estimate you found in part (b).

Solution

a. Use a calculator to construct a table, as shown at the left.

DATA

Height, x	Weight, y
62	150.8
64	160.7
66	170.9
68	181.4
70	192.2
72	203.3
74	214.8
76	226.6

Spreadsheet at LarsonPrecalculus.com

b. Use the table of values to sketch the graph of the equation, as shown in Figure 1.19. From the graph, you can estimate that a height of 71 inches corresponds to a weight of about 198 pounds.

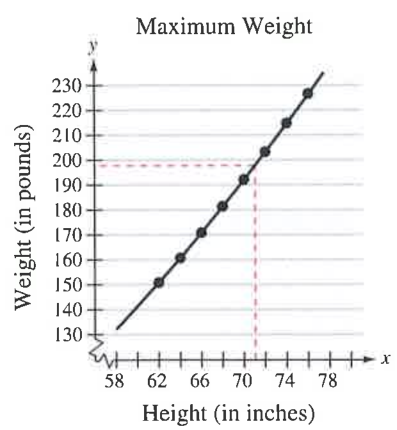


Figure 1.19

c. To confirm algebraically the estimate you found in part (b), substitute 71 for x in the model.

$$y = 0.040(71)^2 - 0.11(71) + 3.9$$
$$\approx 197.7$$

So, the graphical estimate of 198 pounds is fairly good.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Use Figure 1.19 to estimate *graphically* the maximum weight for a man whose height is 75 inches. Then confirm the estimate *algebraically*.

Summarize (Section 1.2)

1. Explain how to sketch the graph of an equation *(page 11)*. For examples of sketching graphs of equations, see Examples 2 and 3.
2. Explain how to find the x- and y-intercepts of a graph *(page 14)*. For an example of finding x- and y-intercepts, see Example 4.
3. Explain how to use symmetry to graph an equation *(page 15)*. For an example of using symmetry to graph an equation, see Example 6.
4. State the standard form of the equation of a circle *(page 17)*. For an example of writing the standard form of the equation of a circle, see Example 8.
5. Describe an example of how to use the graph of an equation to solve a real-life problem *(page 18, Example 9)*.

1.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. An ordered pair (a, b) is a ________ of an equation in x and y when the substitutions $x = a$ and $y = b$ result in a true statement.
2. The set of all solution points of an equation is the ________ of the equation.
3. The points at which a graph intersects or touches an axis are the ________ of the graph.
4. A graph is symmetric with respect to the ________ if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
5. The equation $(x - h)^2 + (y - k)^2 = r^2$ is the standard form of the equation of a ________ with center ________ and radius ________.
6. When you construct and use a table to solve a problem, you are using a ________ approach.

Skills and Applications

Determining Solution Points In Exercises 7–14, determine whether each point lies on the graph of the equation.

Equation	Points	
7. $y = \sqrt{x + 4}$	(a) $(0, 2)$	(b) $(5, 3)$
8. $y = \sqrt{5 - x}$	(a) $(1, 2)$	(b) $(5, 0)$
9. $y = x^2 - 3x + 2$	(a) $(2, 0)$	(b) $(-2, 8)$
10. $y = 3 - 2x^2$	(a) $(-1, 1)$	(b) $(-2, 11)$
11. $y = 4 - \lvert x - 2\rvert$	(a) $(1, 5)$	(b) $(6, 0)$
12. $y = \lvert x - 1\rvert + 2$	(a) $(2, 3)$	(b) $(-1, 0)$
13. $x^2 + y^2 = 20$	(a) $(3, -2)$	(b) $(-4, 2)$
14. $2x^2 + 5y^2 = 8$	(a) $(6, 0)$	(b) $(0, 4)$

Sketching the Graph of an Equation In Exercises 15–18, complete the table. Use the resulting solution points to sketch the graph of the equation.

15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y					
(x, y)					

16. $y + 1 = \frac{3}{4}x$

x	-2	0	1	$\frac{4}{3}$	2
y					
(x, y)					

17. $y + 3x = x^2$

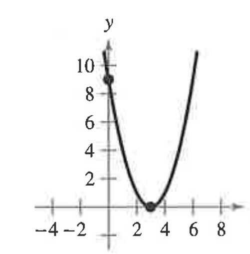
x	-1	0	1	2	3
y					
(x, y)					

18. $y = 5 - x^2$

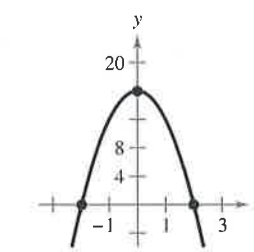
x	-2	-1	0	1	2
y					
(x, y)					

Identifying x- and y-Intercepts In Exercises 19–22, identify the x- and y-intercepts of the graph. Verify your results algebraically.

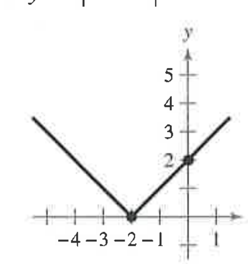
19. $y = (x - 3)^2$



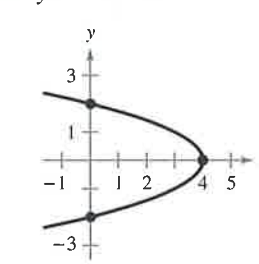
20. $y = 16 - 4x^2$



21. $y = \lvert x + 2\rvert$



22. $y^2 = 4 - x$



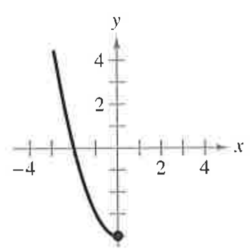
Finding x- and y-Intercepts In Exercises 23–32, find the x- and y-intercepts of the graph of the equation.

23. $y = 5x - 6$
24. $y = 8 - 3x$
25. $y = \sqrt{x + 4}$
26. $y = \sqrt{2x - 1}$
27. $y = \lvert 3x - 7\rvert$
28. $y = -\lvert x + 10\rvert$
29. $y = 2x^3 - 4x^2$
30. $y = x^4 - 25$
31. $y^2 = 6 - x$
32. $y^2 = x + 1$

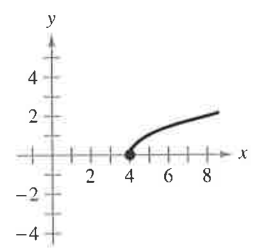
Testing for Symmetry In Exercises 33–40, use the algebraic tests to check for symmetry with respect to both axes and the origin.

33. $x^2 - y = 0$
34. $x - y^2 = 0$
35. $y = x^3$
36. $y = x^4 - x^2 + 3$
37. $y = \dfrac{x}{x^2 + 1}$
38. $y = \dfrac{1}{x^2 + 1}$
39. $xy^2 + 10 = 0$
40. $xy = 4$

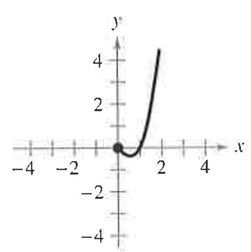
Using Symmetry as a Sketching Aid In Exercises 41–44, assume that the graph has the given type of symmetry. Complete the graph of the equation. To print an enlarged copy of the graph, go to *MathGraphs.com*.

41.


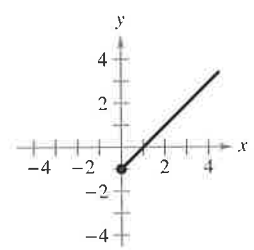
y-Axis symmetry

42.


x-Axis symmetry

43.


Origin symmetry

44.


y-Axis symmetry

Sketching the Graph of an Equation In Exercises 45–56, find any intercepts and test for symmetry. Then sketch the graph of the equation.

45. $y = -3x + 1$
46. $y = 2x - 3$
47. $y = x^2 - 2x$
48. $y = -x^2 - 2x$
49. $y = x^3 + 3$
50. $y = x^3 - 1$
51. $y = \sqrt{x - 3}$
52. $y = \sqrt{1 - x}$
53. $y = |x - 6|$
54. $y = 1 - |x|$
55. $x = y^2 - 1$
56. $x = y^2 - 5$

Using Technology In Exercises 57–66, use a graphing utility to graph the equation. Use a standard setting. Approximate any intercepts.

57. $y = 3 - \frac{1}{2}x$
58. $y = \frac{2}{3}x - 1$
59. $y = x^2 - 4x + 3$
60. $y = x^2 + x - 2$
61. $y = \dfrac{2x}{x - 1}$
62. $y = \dfrac{4}{x^2 + 1}$
63. $y = \sqrt[3]{x + 1}$
64. $y = x\sqrt{x + 6}$
65. $y = |x + 3|$
66. $y = 2 - |x|$

Writing the Equation of a Circle In Exercises 67–74, write the standard form of the equation of the circle with the given characteristics.

67. Center: $(0, 0)$; Radius: 3
68. Center: $(0, 0)$; Radius: 7
69. Center: $(-4, 5)$; Radius: 2
70. Center: $(1, -3)$; Radius: $\sqrt{11}$
71. Center: $(3, 8)$; Solution point: $(-9, 13)$
72. Center: $(-2, -6)$; Solution point: $(1, -10)$
73. Endpoints of a diameter: $(3, 2)$, $(-9, -8)$
74. Endpoints of a diameter: $(11, -5)$, $(3, 15)$

Sketching a Circle In Exercises 75–80, find the center and radius of the circle with the given equation. Then sketch the circle.

75. $x^2 + y^2 = 25$
76. $x^2 + y^2 = 16$
77. $(x - 1)^2 + (y + 3)^2 = 9$
78. $x^2 + (y - 1)^2 = 1$
79. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$
80. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

81. **Depreciation** A hospital purchases a new magnetic resonance imaging (MRI) machine for \$1.2 million. The depreciated value y (reduced value) after t years is given by $y = 1{,}200{,}000 - 80{,}000t$, $0 \le t \le 10$. Sketch the graph of the equation.

82. **Depreciation** You purchase an all-terrain vehicle (ATV) for \$9500. The depreciated value y (reduced value) after t years is given by $y = 9500 - 1000t$, $0 \le t \le 6$. Sketch the graph of the equation.

83. **Geometry** A regulation NFL playing field of length x and width y has a perimeter of $346\frac{2}{3}$ or $\frac{1040}{3}$ yards.
    (a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.
    (b) Show that the width of the rectangle is $y = \frac{520}{3} - x$ and its area is $A = x\left(\frac{520}{3} - x\right)$.
    (c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.
    (d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.
    (e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation NFL playing field and compare your findings with the results of part (d).

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

84. **Architecture** The arch support of a bridge is modeled by $y = -0.0012x^2 + 300$, where x and y are measured in feet and the x-axis represents the ground.
    (a) Use a graphing utility to graph the equation.
    (b) Find one x-intercept of the graph. Explain how to use the intercept and the symmetry of the graph to find the width of the arch support.

85. **Population Statistics** The table shows the life expectancies of a child (at birth) in the United States for selected years from 1940 through 2010. *(Source: U.S. National Center for Health Statistics)*

DATA

Spreadsheet at LarsonPrecalculus.com

Year	Life Expectancy, y
1940	62.9
1950	68.2
1960	69.7
1970	70.8
1980	73.7
1990	75.4
2000	76.8
2010	78.7

A model for the life expectancy during this period is

$$y = \frac{63.6 + 0.97t}{1 + 0.01t}, \quad 0 \le t \le 70$$

where y represents the life expectancy and t is the time in years, with $t = 0$ corresponding to 1940.

(a) Use a graphing utility to graph the data from the table and the model in the same viewing window. How well does the model fit the data? Explain.

(b) Determine the life expectancy in 1990 both graphically and algebraically.
(c) Use the graph to determine the year when life expectancy was approximately 70.1. Verify your answer algebraically.
(d) Find the y-intercept of the graph of the model. What does it represent in the context of the problem?
(e) Do you think this model can be used to predict the life expectancy of a child 50 years from now? Explain.

86. **Electronics** The resistance y (in ohms) of 1000 feet of solid copper wire at 68 degrees Fahrenheit is

$$y = \frac{10{,}370}{x^2}$$

where x is the diameter of the wire in mils (0.001 inch).
    (a) Complete the table.

x	5	10	20	30	40	50
y						

x	60	70	80	90	100
y					

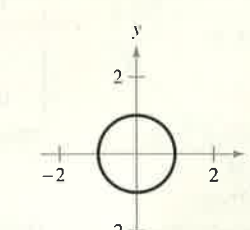
    (b) Use the table of values in part (a) to sketch a graph of the model. Then use your graph to estimate the resistance when $x = 85.5$.
    (c) Use the model to confirm algebraically the estimate you found in part (b).
    (d) What can you conclude about the relationship between the diameter of the copper wire and the resistance?

Exploration

True or False? In Exercises 87–89, determine whether the statement is true or false. Justify your answer.

87. The graph of a linear equation cannot be symmetric with respect to the origin.
88. The graph of a linear equation can have either no x-intercepts or only one x-intercept.
89. A circle can have a total of zero, one, two, three, or four x- and y-intercepts.

90. **HOW DO YOU SEE IT?** The graph shows the circle with the equation $x^2 + y^2 = 1$. Describe the types of symmetry that you observe.



91. **Think About It** Find a and b when the graph of $y = ax^2 + bx^3$ is symmetric with respect to (a) the y-axis and (b) the origin. (There are many correct answers.)