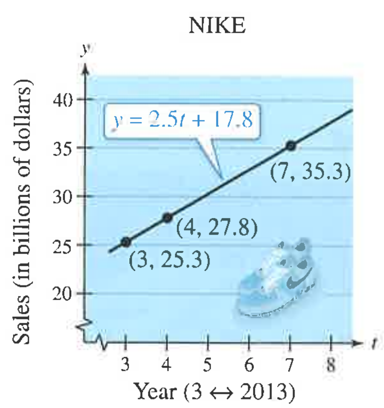


Figure 1.29

EXAMPLE 8 Predicting Sales

The sales for NIKE were approximately \$25.3 billion in 2013 and \$27.8 billion in 2014. Using only this information, write a linear equation that gives the sales in terms of the year. Then predict the sales in 2017. *(Source: NIKE Inc.)*

Solution Let $t = 3$ represent 2013. Then the two given values are represented by the data points (3, 25.3) and (4, 27.8) The slope of the line through these points is

$$m = \frac{27.8 - 25.3}{4 - 3} = 2.5.$$

Use the point-slope form to write an equation that relates the sales y and the year t.

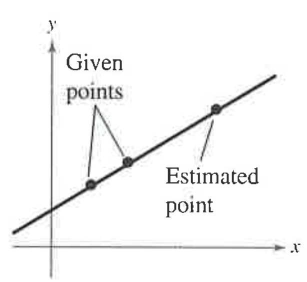
$$y - 25.3 = 2.5(t - 3) \qquad \text{Write in point-slope form.}$$
$$y = 2.5t + 17.8 \qquad \text{Write in slope-intercept form.}$$

According to this equation, the sales in 2017 will be

$y = 2.5(7) + 17.8 = 17.5 + 17.8 = \$35.3$ billion. (See Figure 1.29.)

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

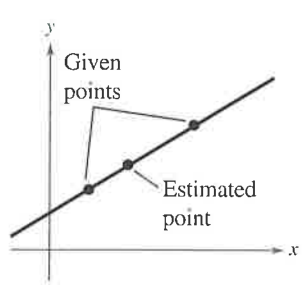
The sales for Foot Locker were approximately \$6.5 billion in 2013 and \$7.2 billion in 2014. Repeat Example 8 using this information. *(Source: Foot Locker)*



Linear extrapolation
Figure 1.30

The prediction method illustrated in Example 8 is called **linear extrapolation.** Note in Figure 1.30 that an extrapolated point does not lie between the given points. When the estimated point lies between two given points, as shown in Figure 1.31, the procedure is called **linear interpolation.**

The slope of a vertical line is undefined, so its equation cannot be written in slope-intercept form. However, every line has an equation that can be written in the **general form** $Ax + By + C = 0$, where A and B are not both zero.



Linear interpolation
Figure 1.31

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$
6. Two-point form: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Summarize (Section 1.3)

1. Explain how to use slope to graph a linear equation in two variables *(page 22)* and how to find the slope of a line passing through two points *(page 24)*. For examples of using and finding slopes, see Examples 1 and 2.
2. State the point-slope form of the equation of a line *(page 26)*. For an example of using point-slope form, see Example 3.
3. Explain how to use slope to identify parallel and perpendicular lines *(page 27)*. For an example of finding parallel and perpendicular lines, see Example 4.
4. Describe examples of how to use slope and linear equations in two variables to model and solve real-life problems *(pages 28–30, Examples 5–8)*.

1.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

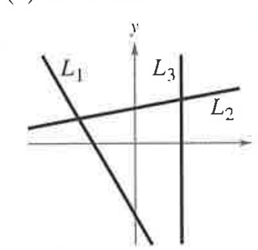
Vocabulary: Fill in the blanks.

1. The simplest mathematical model for relating two variables is the ________ equation in two variables $y = mx + b$.
2. For a line, the ratio of the change in y to the change in x is the ________ of the line.
3. The ________-________ form of the equation of a line with slope m passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.
4. Two distinct nonvertical lines are ________ if and only if their slopes are equal.
5. Two nonvertical lines are ________ if and only if their slopes are negative reciprocals of each other.
6. When the x-axis and y-axis have different units of measure, the slope can be interpreted as a ________.
7. ________ ________ is the prediction method used to estimate a point on a line when the point does not lie between the given points.
8. Every line has an equation that can be written in ________ form.

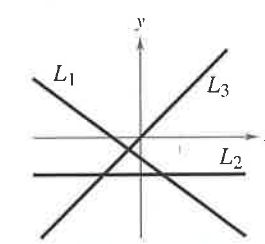
Skills and Applications

Identifying Lines In Exercises 9 and 10, identify the line that has each slope.

9. (a) $m = \frac{2}{3}$
(b) m is undefined.
(c) $m = -2$



10. (a) $m = 0$
(b) $m = -\frac{3}{4}$
(c) $m = 1$

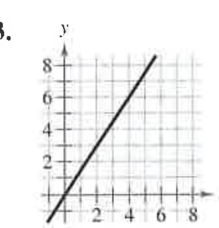


Sketching Lines In Exercises 11 and 12, sketch the lines through the point with the given slopes on the same set of coordinate axes.

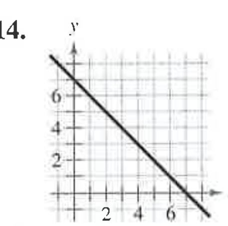
Point	Slopes	
11. (2, 3)	(a) 0	(b) 1
	(c) 2	(d) -3
12. $(-4, 1)$	(a) 3	(b) -3
	(c) $\frac{1}{2}$	(d) Undefined

Estimating the Slope of a Line In Exercises 13 and 14, estimate the slope of the line.

13.



14.



Graphing a Linear Equation In Exercises 15–24, find the slope and y-intercept (if possible) of the line. Sketch the line.

15. $y = 5x + 3$
16. $y = -x - 10$
17. $y = -\frac{3}{4}x - 1$
18. $y = \frac{2}{3}x + 2$
19. $y - 5 = 0$
20. $x + 4 = 0$
21. $5x - 2 = 0$
22. $3y + 5 = 0$
23. $7x - 6y = 30$
24. $2x + 3y = 9$

Finding the Slope of a Line Through Two Points In Exercises 25–34, find the slope of the line passing through the pair of points.

25. (0, 9), (6, 0)
26. (10, 0), (0, −5)
27. (−3, −2), (1, 6)
28. (2, −1), (−2, 1)
29. (5, −7), (8, −7)
30. (−2, 1), (−4, −5)
31. (−6, −1), (−6, 4)
32. (0, −10), (−4, 0)
33. (4.8, 3.1), (−5.2, 1.6)
34. $\left(\frac{11}{2}, -\frac{4}{3}\right), \left(-\frac{3}{2}, -\frac{1}{3}\right)$

Using the Slope and a Point In Exercises 35–42, use the slope of the line and the point on the line to find three additional points through which the line passes. (There are many correct answers.)

35. $m = 0$, (5, 7)
36. $m = 0$, (3, −2)
37. $m = 2$, (−5, 4)
38. $m = -2$, (0, −9)
39. $m = -\frac{1}{3}$, (4, 5)
40. $m = \frac{1}{4}$, (3, −4)
41. m is undefined, (−4, 3)
42. m is undefined, (2, 14)

Using the Point-Slope Form In Exercises 43–54, find the slope-intercept form of the equation of the line that has the given slope and passes through the given point. Sketch the line.

43. $m = 3$, $(0, -2)$
44. $m = -1$, $(0, 10)$
45. $m = -2$, $(-3, 6)$
46. $m = 4$, $(0, 0)$
47. $m = -\frac{1}{3}$, $(4, 0)$
48. $m = \frac{1}{4}$, $(8, 2)$
49. $m = -\frac{1}{2}$, $(2, -3)$
50. $m = \frac{3}{4}$, $(-2, -5)$
51. $m = 0$, $\left(4, \frac{5}{2}\right)$
52. $m = 6$, $\left(2, \frac{3}{2}\right)$
53. $m = 5$, $(-5.1, 1.8)$
54. $m = 0$, $(-2.5, 3.25)$

Finding an Equation of a Line In Exercises 55–64, find an equation of the line passing through the pair of points. Sketch the line.

55. $(5, -1), (-5, 5)$
56. $(4, 3), (-4, -4)$
57. $(-7, 2), (-7, 5)$
58. $(-6, -3), (2, -3)$
59. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$
60. $(1, 1), \left(6, -\frac{2}{3}\right)$
61. $(1, 0.6), (-2, -0.6)$
62. $(-8, 0.6), (2, -2.4)$
63. $(2, -1), \left(\frac{1}{3}, -1\right)$
64. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

Parallel and Perpendicular Lines In Exercises 65–68, determine whether the lines are parallel, perpendicular, or neither.

65. L_1: $y = -\frac{2}{3}x - 3$
L_2: $y = -\frac{2}{3}x + 4$
66. L_1: $y = \frac{1}{4}x - 1$
L_2: $y = 4x + 7$
67. L_1: $y = \frac{1}{2}x - 3$
L_2: $y = -\frac{1}{2}x + 1$
68. L_1: $y = -\frac{4}{5}x - 5$
L_2: $y = \frac{5}{4}x + 1$

Parallel and Perpendicular Lines In Exercises 69–72, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.

69. L_1: $(0, -1), (5, 9)$
L_2: $(0, 3), (4, 1)$
70. L_1: $(-2, -1), (1, 5)$
L_2: $(1, 3), (5, -5)$
71. L_1: $(-6, -3), (2, -3)$
L_2: $\left(3, -\frac{1}{2}\right), \left(6, -\frac{1}{2}\right)$
72. L_1: $(4, 8), (-4, 2)$
L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

Finding Parallel and Perpendicular Lines In Exercises 73–80, find equations of the lines that pass through the given point and are (a) parallel to and (b) perpendicular to the given line.

73. $4x - 2y = 3$, $(2, 1)$
74. $x + y = 7$, $(-3, 2)$
75. $3x + 4y = 7$, $\left(-\frac{2}{3}, \frac{7}{8}\right)$
76. $5x + 3y = 0$, $\left(\frac{7}{8}, \frac{3}{4}\right)$
77. $y + 5 = 0$, $(-2, 4)$
78. $x - 4 = 0$, $(3, -2)$
79. $x - y = 4$, $(2.5, 6.8)$
80. $6x + 2y = 9$, $(-3.9, -1.4)$

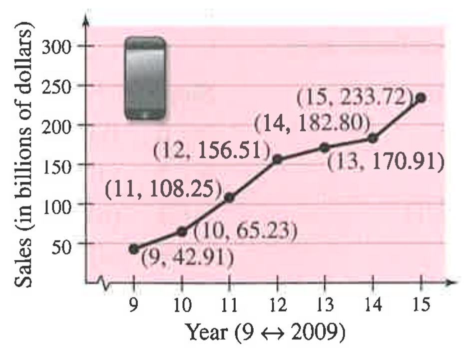
Using Intercept Form In Exercises 81–86, use the *intercept form* to find the general form of the equation of the line with the given intercepts. The intercept form of the equation of a line with intercepts $(a, 0)$ and $(0, b)$ is

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, \quad b \neq 0.$$

81. x-intercept: $(3, 0)$
y-intercept: $(0, 5)$
82. x-intercept: $(-3, 0)$
y-intercept: $(0, 4)$
83. x-intercept: $\left(-\frac{1}{6}, 0\right)$
y-intercept: $\left(0, -\frac{2}{3}\right)$
84. x-intercept: $\left(\frac{2}{3}, 0\right)$
y-intercept: $(0, -2)$
85. Point on line: $(1, 2)$
x-intercept: $(c, 0)$, $c \neq 0$
y-intercept: $(0, c)$, $c \neq 0$
86. Point on line: $(-3, 4)$
x-intercept: $(d, 0)$, $d \neq 0$
y-intercept: $(0, d)$, $d \neq 0$

87. **Sales** The slopes of lines representing annual sales y in terms of time x in years are given below. Use the slopes to interpret any change in annual sales for a one-year increase in time.
(a) The line has a slope of $m = 135$.
(b) The line has a slope of $m = 0$.
(c) The line has a slope of $m = -40$.

88. **Sales** The graph shows the sales (in billions of dollars) for Apple Inc. in the years 2009 through 2015. *(Source: Apple Inc.)*



(a) Use the slopes of the line segments to determine the years in which the sales showed the greatest increase and the least increase.
(b) Find the slope of the line segment connecting the points for the years 2009 and 2015.
(c) Interpret the meaning of the slope in part (b) in the context of the problem.

89. **Road Grade** You are driving on a road that has a 6% uphill grade. This means that the slope of the road is $\frac{6}{100}$. Approximate the amount of vertical change in your position when you drive 200 feet.

90. **Road Grade**
From the top of a mountain road, a surveyor takes several horizontal measurements x and several vertical measurements y, as shown in the table (x and y are measured in feet).

x	300	600	900	1200
y	-25	-50	-75	-100

x	1500	1800	2100
y	-125	-150	-175

(a) Sketch a scatter plot of the data.
(b) Use a straightedge to sketch the line that you think best fits the data.
(c) Find an equation for the line you sketched in part (b).
(d) Interpret the meaning of the slope of the line in part (c) in the context of the problem.
(e) The surveyor needs to put up a road sign that indicates the steepness of the road. For example, a surveyor would put up a sign that states "8% grade" on a road with a downhill grade that has a slope of $-\frac{8}{100}$. What should the sign state for the road in this problem?

Rate of Change In Exercises 91 and 92, you are given the dollar value of a product in 2016 and the rate at which the value of the product is expected to change during the next 5 years. Use this information to write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 16$ represent 2016.)

	2016 Value	Rate
91.	$3000	$150 decrease per year
92.	$200	$6.50 increase per year

93. **Cost** The cost C of producing n computer laptop bags is given by

$$C = 1.25n + 15{,}750, \quad n > 0.$$

Explain what the C-intercept and the slope represent.

94. **Monthly Salary** A pharmaceutical salesperson receives a monthly salary of \$5000 plus a commission of 7% of sales. Write a linear equation for the salesperson's monthly wage W in terms of monthly sales S.

95. **Depreciation** A sandwich shop purchases a used pizza oven for \$875. After 5 years, the oven will have to be discarded and replaced. Write a linear equation giving the value V of the equipment during the 5 years it will be in use.

96. **Depreciation** A school district purchases a high-volume printer, copier, and scanner for \$24,000. After 10 years, the equipment will have to be replaced. Its value at that time is expected to be \$2000. Write a linear equation giving the value V of the equipment during the 10 years it will be in use.

97. **Temperature Conversion** Write a linear equation that expresses the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F. Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F).

98. **Neurology** The average weight of a male child's brain is 970 grams at age 1 and 1270 grams at age 3. *(Source: American Neurological Association)*
(a) Assuming that the relationship between brain weight y and age t is linear, write a linear model for the data.
(b) What is the slope and what does it tell you about brain weight?
(c) Use your model to estimate the average brain weight at age 2.
(d) Use your school's library, the Internet, or some other reference source to find the actual average brain weight at age 2. How close was your estimate?
(e) Do you think your model could be used to determine the average brain weight of an adult? Explain.

99. **Cost, Revenue, and Profit** A roofing contractor purchases a shingle delivery truck with a shingle elevator for \$42,000. The vehicle requires an average expenditure of \$9.50 per hour for fuel and maintenance, and the operator is paid \$11.50 per hour.
(a) Write a linear equation giving the total cost C of operating this equipment for t hours. (Include the purchase cost of the equipment.)
(b) Assuming that customers are charged \$45 per hour of machine use, write an equation for the revenue R obtained from t hours of use.
(c) Use the formula for profit $P = R - C$ to write an equation for the profit obtained from t hours of use.
(d) Use the result of part (c) to find the break-even point—that is, the number of hours this equipment must be used to yield a profit of 0 dollars.

100. Geometry The length and width of a rectangular garden are 15 meters and 10 meters, respectively. A walkway of width x surrounds the garden.

(a) Draw a diagram that gives a visual representation of the problem.

(b) Write the equation for the perimeter y of the walkway in terms of x.

(c) Use a graphing utility to graph the equation for the perimeter.

(d) Determine the slope of the graph in part (c). For each additional one-meter increase in the width of the walkway, determine the increase in its perimeter.

Exploration

True or False? **In Exercises 101 and 102, determine whether the statement is true or false. Justify your answer.**

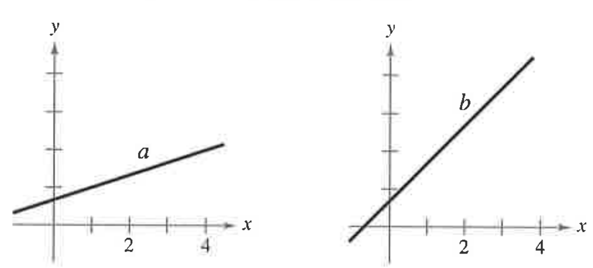
101. A line with a slope of $-\frac{5}{7}$ is steeper than a line with a slope of $-\frac{6}{7}$.

102. The line through $(-8, 2)$ and $(-1, 4)$ and the line through $(0, -4)$ and $(-7, 7)$ are parallel.

103. Right Triangle Explain how you can use slope to show that the points $A(-1, 5)$, $B(3, 7)$, and $C(5, 3)$ are the vertices of a right triangle.

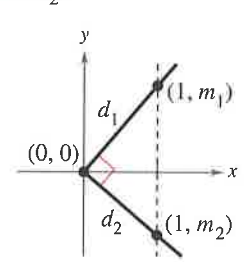
104. Vertical Line Explain why the slope of a vertical line is undefined.

105. Error Analysis Describe the error.



Line b has a greater slope than line a. X

106. Perpendicular Segments Find d_1 and d_2 in terms of m_1 and m_2, respectively (see figure). Then use the Pythagorean Theorem to find a relationship between m_1 and m_2.

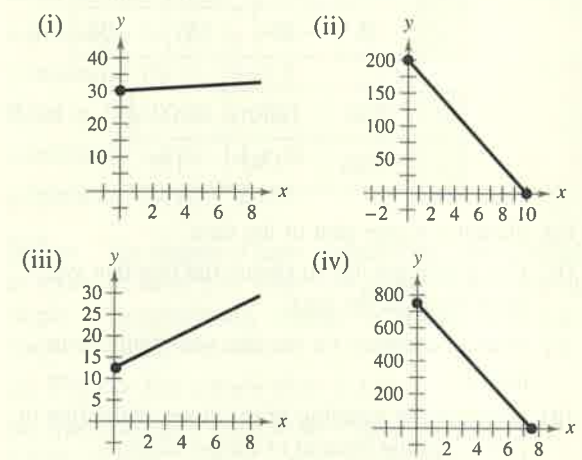


107. Think About It Is it possible for two lines with positive slopes to be perpendicular? Explain.

108. Slope and Steepness The slopes of two lines are -4 and $\frac{5}{2}$. Which is steeper? Explain.

109. Comparing Slopes Use a graphing utility to compare the slopes of the lines $y = mx$, where $m = 0.5$, 1, 2, and 4. Which line rises most quickly? Now, let $m = -0.5$, -1, -2, and -4. Which line falls most quickly? Use a square setting to obtain a true geometric perspective. What can you conclude about the slope and the "rate" at which the line rises or falls?

110. HOW DO YOU SEE IT? Match the description of the situation with its graph. Also determine the slope and y-intercept of each graph and interpret the slope and y-intercept in the context of the situation. [The graphs are labeled (i), (ii), (iii), and (iv).]



(a) A person is paying \$20 per week to a friend to repay a \$200 loan.

(b) An employee receives \$12.50 per hour plus \$2 for each unit produced per hour.

(c) A sales representative receives \$30 per day for food plus \$0.32 for each mile traveled.

(d) A computer that was purchased for \$750 depreciates \$100 per year.

Finding a Relationship for Equidistance **In Exercises 111–114, find a relationship between x and y such that (x, y) is equidistant (the same distance) from the two points.**

111. $(4, -1), (-2, 3)$ **112.** $(6, 5), (1, -8)$

113. $(3, \frac{5}{2}), (-7, 1)$ **114.** $(-\frac{1}{2}, -4), (\frac{7}{2}, \frac{5}{4})$

Project: Bachelor's Degrees To work an extended application analyzing the numbers of bachelor's degrees earned by women in the United States from 2002 through 2013, visit this text's website at *LarsonPrecalculus.com*. *(Source: National Center for Education Statistics)*

1.4 Functions

Functions are used to model and solve real-life problems. For example, in Exercise 70 on page 47, you will use a function that models the force of water against the face of a dam.

- **Determine whether relations between two variables are functions, and use function notation.**
- **Find the domains of functions.**
- **Use functions to model and solve real-life problems.**
- **Evaluate difference quotients.**

Introduction to Functions and Function Notation

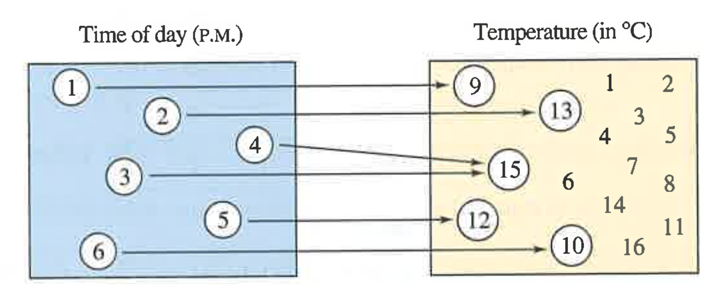
Many everyday phenomena involve two quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation.** In mathematics, equations and formulas often represent relations. For example, the simple interest I earned on \$1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.

The formula $I = 1000r$ represents a special kind of relation that matches each item from one set with *exactly one* item from a different set. Such a relation is a **function.**

Definition of Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

To help understand this definition, look at the function below, which relates the time of day to the temperature.



Set A is the domain.
Inputs: 1, 2, 3, 4, 5, 6

Set B contains the range.
Outputs: 9, 10, 12, 13, 15

The ordered pairs below can represent this function. The first coordinate (x-value) is the input and the second coordinate (y-value) is the output.

$$\{(1, 9), (2, 13), (3, 15), (4, 15), (5, 12), (6, 10)\}$$

Characteristics of a Function from Set A to Set B

1. Each element in A must be matched with an element in B.
2. Some elements in B may not be matched with any element in A.
3. Two or more elements in A may be matched with the same element in B.
4. An element in A (the domain) cannot be matched with two different elements in B.