## 1.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

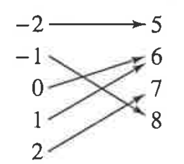
**Vocabulary:** Fill in the blanks.

1. A relation that assigns to each element $x$ from a set of inputs, or ________, exactly one element $y$ in a set of outputs, or ________, is a ________.
2. For an equation that represents $y$ as a function of $x$, the set of all values taken on by the ________ variable $x$ is the domain, and the set of all values taken on by the ________ variable $y$ is the range.
3. If the domain of the function $f$ is not given, then the set of values of the independent variable for which the expression is defined is the ________ ________.
4. One of the basic definitions in calculus uses the ratio $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$. This ratio is a ________ ________.

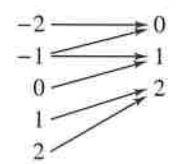
### Skills and Applications

**Testing for Functions** In Exercises 5–8, determine whether the relation represents $y$ as a function of $x$.

5. *Domain, x* *Range, y*




6. *Domain, x* *Range, y*




7.

| Input, $x$ | 10 | 7 | 4 | 7 | 10 |
|---|---|---|---|---|---|
| Output, $y$ | 3 | 6 | 9 | 12 | 15 |

8.

| Input, $x$ | −2 | 0 | 2 | 4 | 6 |
|---|---|---|---|---|---|
| Output, $y$ | 1 | 1 | 1 | 1 | 1 |

**Testing for Functions** In Exercises 9 and 10, which sets of ordered pairs represent functions from $A$ to $B$? Explain.

9. $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2\}$
   (a) $\{(0, 1), (1, -2), (2, 0), (3, 2)\}$
   (b) $\{(0, -1), (2, 2), (1, -2), (3, 0), (1, 1)\}$
   (c) $\{(0, 0), (1, 0), (2, 0), (3, 0)\}$
   (d) $\{(0, 2), (3, 0), (1, 1)\}$
10. $A = \{a, b, c\}$ and $B = \{0, 1, 2, 3\}$
   (a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$
   (b) $\{(a, 1), (b, 2), (c, 3)\}$
   (c) $\{(1, a), (0, a), (2, c), (3, b)\}$
   (d) $\{(c, 0), (b, 0), (a, 3)\}$

**Testing for Functions Represented Algebraically** In Exercises 11–18, determine whether the equation represents $y$ as a function of $x$.

11. $x^2 + y^2 = 4$
12. $x^2 - y = 9$
13. $y = \sqrt{16 - x^2}$
14. $y = \sqrt{x + 5}$
15. $y = 4 - |x|$
16. $|y| = 4 - x$
17. $y = -75$
18. $x - 1 = 0$

**Evaluating a Function** In Exercises 19–30, find each function value, if possible.

19. $f(x) = 3x - 5$
   (a) $f(1)$ (b) $f(-3)$ (c) $f(x + 2)$
20. $V(r) = \frac{4}{3}\pi r^3$
   (a) $V(3)$ (b) $V(\frac{3}{2})$ (c) $V(2r)$
21. $g(t) = 4t^2 - 3t + 5$
   (a) $g(2)$ (b) $g(t - 2)$ (c) $g(t) - g(2)$
22. $h(t) = -t^2 + t + 1$
   (a) $h(2)$ (b) $h(-1)$ (c) $h(x + 1)$
23. $f(y) = 3 - \sqrt{y}$
   (a) $f(4)$ (b) $f(0.25)$ (c) $f(4x^2)$
24. $f(x) = \sqrt{x + 8} + 2$
   (a) $f(-8)$ (b) $f(1)$ (c) $f(x - 8)$
25. $q(x) = 1/(x^2 - 9)$
   (a) $q(0)$ (b) $q(3)$ (c) $q(y + 3)$
26. $q(t) = (2t^2 + 3)/t^2$
   (a) $q(2)$ (b) $q(0)$ (c) $q(-x)$
27. $f(x) = |x|/x$
   (a) $f(2)$ (b) $f(-2)$ (c) $f(x - 1)$
28. $f(x) = |x| + 4$
   (a) $f(2)$ (b) $f(-2)$ (c) $f(x^2)$
29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$
   (a) $f(-1)$ (b) $f(0)$ (c) $f(2)$
30. $f(x) = \begin{cases} -3x - 3, & x < -1 \\ x^2 + 2x - 1, & x \geq -1 \end{cases}$
   (a) $f(-2)$ (b) $f(-1)$ (c) $f(1)$



**Evaluating a Function** In Exercises 31–34, complete the table.

31. $f(x) = -x^2 + 5$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

32. $h(t) = \frac{1}{2}|t + 3|$

| $t$ | −5 | −4 | −3 | −2 | −1 |
|---|---|---|---|---|---|
| $h(t)$ | | | | | |

33. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \leq 0 \\ (x - 2)^2, & x > 0 \end{cases}$

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

34. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

**Finding Values for Which $f(x) = 0$** In Exercises 35–42, find all real values of $x$ for which $f(x) = 0$.

35. $f(x) = 15 - 3x$
36. $f(x) = 4x + 6$
37. $f(x) = \frac{3x - 4}{5}$
38. $f(x) = \frac{12 - x^2}{8}$
39. $f(x) = x^2 - 81$
40. $f(x) = x^2 - 6x - 16$
41. $f(x) = x^3 - x$
42. $f(x) = x^3 - x^2 - 3x + 3$

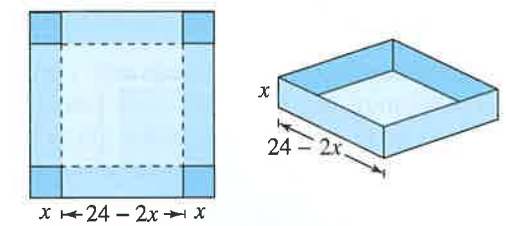
**Finding Values for Which $f(x) = g(x)$** In Exercises 43–46, find the value(s) of $x$ for which $f(x) = g(x)$.

43. $f(x) = x^2$, $g(x) = x + 2$
44. $f(x) = x^2 + 2x + 1$, $g(x) = 5x + 19$
45. $f(x) = x^4 - 2x^2$, $g(x) = 2x^2$
46. $f(x) = \sqrt{x} - 4$, $g(x) = 2 - x$

**Finding the Domain of a Function** In Exercises 47–56, find the domain of the function.

47. $f(x) = 5x^2 + 2x - 1$
48. $g(x) = 1 - 2x^2$
49. $g(y) = \sqrt{y + 6}$
50. $f(t) = \sqrt[3]{t + 4}$
51. $g(x) = \frac{1}{x} - \frac{3}{x + 2}$
52. $h(x) = \frac{6}{x^2 - 4x}$
53. $f(s) = \frac{\sqrt{s - 1}}{s - 4}$
54. $f(x) = \frac{\sqrt{x + 6}}{6 + x}$
55. $f(x) = \frac{x - 4}{\sqrt{x}}$
56. $f(x) = \frac{x + 2}{\sqrt{x - 10}}$

57. **Maximum Volume** An open box of maximum volume is made from a square piece of material 24 centimeters on a side by cutting equal squares from the corners and turning up the sides (see figure).




   (a) The table shows the volumes $V$ (in cubic centimeters) of the box for various heights $x$ (in centimeters). Use the table to estimate the maximum volume.

| Height, $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Volume, $V$ | 484 | 800 | 972 | 1024 | 980 | 864 |

   (b) Plot the points $(x, V)$ from the table in part (a). Does the relation defined by the ordered pairs represent $V$ as a function of $x$?
   (c) Given that $V$ is a function of $x$, write the function and determine its domain.

58. **Maximum Profit** The cost per unit in the production of an MP3 player is \$60. The manufacturer charges \$90 per unit for orders of 100 or less. To encourage large orders, the manufacturer reduces the charge by \$0.15 per MP3 player for each unit ordered in excess of 100 (for example, the charge is reduced to \$87 per MP3 player for an order size of 120).
   (a) The table shows the profits $P$ (in dollars) for various numbers of units ordered, $x$. Use the table to estimate the maximum profit.

| Units, $x$ | 130 | 140 | 150 | 160 | 170 |
|---|---|---|---|---|---|
| Profit, $P$ | 3315 | 3360 | 3375 | 3360 | 3315 |

   (b) Plot the points $(x, P)$ from the table in part (a). Does the relation defined by the ordered pairs represent $P$ as a function of $x$?
   (c) Given that $P$ is a function of $x$, write the function and determine its domain. (*Note:* $P = R - C$, where $R$ is revenue and $C$ is cost.)

**59. Geometry** Write the area $A$ of a square as a function of its perimeter $P$.

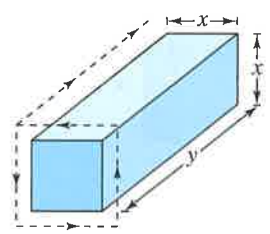
**60. Geometry** Write the area $A$ of a circle as a function of its circumference $C$.

**61. Path of a Ball** You throw a baseball to a child 25 feet away. The height $y$ (in feet) of the baseball is given by

$$y = -\frac{1}{10}x^2 + 3x + 6$$

where $x$ is the horizontal distance (in feet) from where you threw the ball. Can the child catch the baseball while holding a baseball glove at a height of 5 feet?

**62. Postal Regulations** A rectangular package has a combined length and girth (perimeter of a cross section) of 108 inches (see figure).

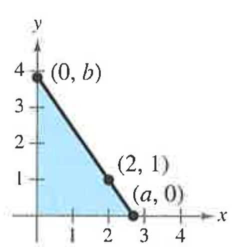



(a) Write the volume $V$ of the package as a function of $x$. What is the domain of the function?

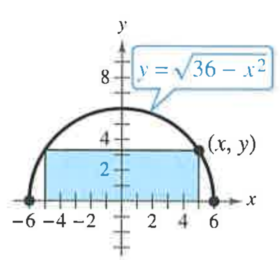
(b) Use a graphing utility to graph the function. Be sure to use an appropriate window setting.

(c) What dimensions will maximize the volume of the package? Explain.

**63. Geometry** A right triangle is formed in the first quadrant by the $x$- and $y$-axes and a line through the point $(2, 1)$ (see figure). Write the area $A$ of the triangle as a function of $x$, and determine the domain of the function.



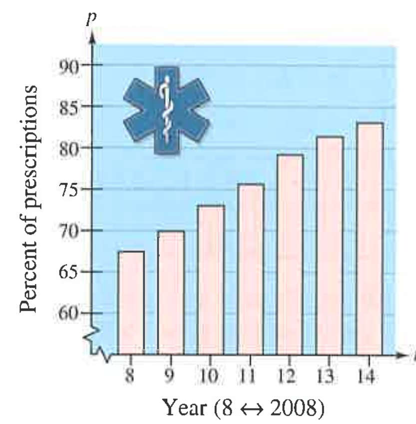

**64. Geometry** A rectangle is bounded by the $x$-axis and the semicircle $y = \sqrt{36 - x^2}$ (see figure). Write the area $A$ of the rectangle as a function of $x$, and graphically determine the domain of the function.




**65. Pharmacology** The percent $p$ of prescriptions filled with generic drugs at CVS Pharmacies from 2008 through 2014 (see figure) can be approximated by the model

$$p(t) = \begin{cases} 2.77t + 45.2, & 8 \le t \le 11 \\ 1.95t + 55.9, & 12 \le t \le 14 \end{cases}$$

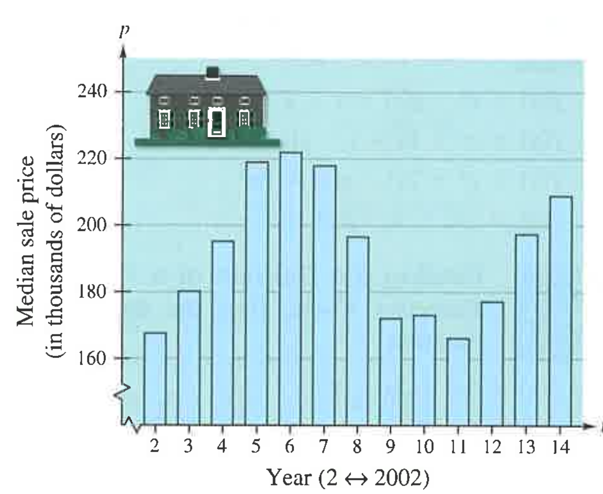
where $t$ represents the year, with $t = 8$ corresponding to 2008. Use this model to find the percent of prescriptions filled with generic drugs in each year from 2008 through 2014. (*Source: CVS Health*)




**66. Median Sale Price** The median sale price $p$ (in thousands of dollars) of an existing one-family home in the United States from 2002 through 2014 (see figure) can be approximated by the model

$$p(t) = \begin{cases} -0.757t^2 + 20.80t + 127.2, & 2 \le t \le 6 \\ 3.879t^2 - 82.50t + 605.8, & 7 \le t \le 11 \\ -4.171t^2 + 124.34t - 714.2, & 12 \le t \le 14 \end{cases}$$

where $t$ represents the year, with $t = 2$ corresponding to 2002. Use this model to find the median sale price of an existing one-family home in each year from 2002 through 2014. (*Source: National Association of Realtors*)






**67. Cost, Revenue, and Profit** A company produces a product for which the variable cost is \$12.30 per unit and the fixed costs are \$98,000. The product sells for \$17.98. Let $x$ be the number of units produced and sold.

(a) The total cost for a business is the sum of the variable cost and the fixed costs. Write the total cost $C$ as a function of the number of units produced.

(b) Write the revenue $R$ as a function of the number of units sold.

(c) Write the profit $P$ as a function of the number of units sold. (*Note:* $P = R - C$)

**68. Average Cost** The inventor of a new game believes that the variable cost for producing the game is \$0.95 per unit and the fixed costs are \$6000. The inventor sells each game for \$1.69. Let $x$ be the number of games produced.

(a) The total cost for a business is the sum of the variable cost and the fixed costs. Write the total cost $C$ as a function of the number of games produced.

(b) Write the average cost per unit $\overline{C} = \dfrac{C}{x}$ as a function of $x$.

**69. Height of a Balloon** A balloon carrying a transmitter ascends vertically from a point 3000 feet from the receiving station.

(a) Draw a diagram that gives a visual representation of the problem. Let $h$ represent the height of the balloon and let $d$ represent the distance between the balloon and the receiving station.

(b) Write the height of the balloon as a function of $d$. What is the domain of the function?

**70. Physics**

The function $F(y) = 149.76\sqrt{10}\,y^{5/2}$ estimates the force $F$ (in tons) of water against the face of a dam, where $y$ is the depth of the water (in feet).

(a) Complete the table. What can you conclude from the table?

| $y$ | 5 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| $F(y)$ | | | | | |

(b) Use the table to approximate the depth at which the force against the dam is 1,000,000 tons.

(c) Find the depth at which the force against the dam is 1,000,000 tons algebraically.

**71. Transportation** For groups of 80 or more people, a charter bus company determines the rate per person according to the formula

$$\text{Rate} = 8 - 0.05(n - 80), \quad n \ge 80$$

where the rate is given in dollars and $n$ is the number of people.

(a) Write the revenue $R$ for the bus company as a function of $n$.

(b) Use the function in part (a) to complete the table. What can you conclude?

| $n$ | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
|---|---|---|---|---|---|---|---|
| $R(n)$ | | | | | | | |

**72. E-Filing** The table shows the numbers of tax returns (in millions) made through e-file from 2007 through 2014. Let $f(t)$ represent the number of tax returns made through e-file in the year $t$. (*Source: eFile*)

DATA

| Year | Number of Tax Returns Made Through E-File |
|---|---|
| 2007 | 80.0 |
| 2008 | 89.9 |
| 2009 | 95.0 |
| 2010 | 98.7 |
| 2011 | 112.2 |
| 2012 | 112.1 |
| 2013 | 114.4 |
| 2014 | 125.8 |

Spreadsheet at LarsonPrecalculus.com

(a) Find $\dfrac{f(2014) - f(2007)}{2014 - 2007}$ and interpret the result in the context of the problem.

(b) Make a scatter plot of the data.

(c) Find a linear model for the data algebraically. Let $N$ represent the number of tax returns made through e-file and let $t = 7$ correspond to 2007.

(d) Use the model found in part (c) to complete the table.

| $t$ | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|
| $N$ | | | | | | | | |

(e) Compare your results from part (d) with the actual data.

(f) Use a graphing utility to find a linear model for the data. Let $x = 7$ correspond to 2007. How does the model you found in part (c) compare with the model given by the graphing utility?

**Evaluating a Difference Quotient** In Exercises 73–80, find the difference quotient and simplify your answer.

73. $f(x) = x^2 - 2x + 4,\quad \dfrac{f(2+h) - f(2)}{h},\quad h \neq 0$

74. $f(x) = 5x - x^2,\quad \dfrac{f(5+h) - f(5)}{h},\quad h \neq 0$

75. $f(x) = x^3 + 3x,\quad \dfrac{f(x+h) - f(x)}{h},\quad h \neq 0$

76. $f(x) = 4x^3 - 2x,\quad \dfrac{f(x+h) - f(x)}{h},\quad h \neq 0$

77. $g(x) = \dfrac{1}{x^2},\quad \dfrac{g(x) - g(3)}{x - 3},\quad x \neq 3$

78. $f(t) = \dfrac{1}{t-2},\quad \dfrac{f(t) - f(1)}{t - 1},\quad t \neq 1$

79. $f(x) = \sqrt{5x},\quad \dfrac{f(x) - f(5)}{x - 5},\quad x \neq 5$

80. $f(x) = x^{2/3} + 1,\quad \dfrac{f(x) - f(8)}{x - 8},\quad x \neq 8$

**Modeling Data** In Exercises 81–84, determine which of the following functions

$$f(x) = cx,\quad g(x) = cx^2,\quad h(x) = c\sqrt{|x|},\quad \text{and}\quad r(x) = \frac{c}{x}$$

can be used to model the data and determine the value of the constant $c$ that will make the function fit the data in the table.

81.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | $-4$ | $-1$ | $0$ | $1$ | $4$ |
| $y$ | $-32$ | $-2$ | $0$ | $-2$ | $-32$ |

82.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | $-4$ | $-1$ | $0$ | $1$ | $4$ |
| $y$ | $-1$ | $-\frac{1}{4}$ | $0$ | $\frac{1}{4}$ | $1$ |

83.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | $-4$ | $-1$ | $0$ | $1$ | $4$ |
| $y$ | $-8$ | $-32$ | Undefined | $32$ | $8$ |

84.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | $-4$ | $-1$ | $0$ | $1$ | $4$ |
| $y$ | $6$ | $3$ | $0$ | $3$ | $6$ |

## Exploration

**True or False?** In Exercises 85–88, determine whether the statement is true or false. Justify your answer.

85. Every relation is a function.

86. Every function is a relation.

87. For the function

$f(x) = x^4 - 1$

the domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.

88. The set of ordered pairs $\{(-8, -2), (-6, 0), (-4, 0), (-2, 2), (0, 4), (2, -2)\}$ represents a function.

89. **Error Analysis** Describe the error.

The functions

$f(x) = \sqrt{x - 1}$ and $g(x) = \dfrac{1}{\sqrt{x - 1}}$

have the same domain, which is the set of all real numbers $x$ such that $x \geq 1$.

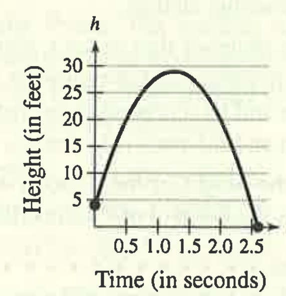
90. **Think About It** Consider

$f(x) = \sqrt{x - 2}$ and $g(x) = \sqrt[3]{x - 2}$.

Why are the domains of $f$ and $g$ different?

91. **Think About It** Given $f(x) = x^2$, is $f$ the independent variable? Why or why not?

92. **HOW DO YOU SEE IT?** The graph represents the height $h$ of a projectile after $t$ seconds.




(a) Explain why $h$ is a function of $t$.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of $h$.

(d) Is $t$ a function of $h$? Explain.

**Think About It** In Exercises 93 and 94, determine whether the statements use the word *function* in ways that are mathematically correct. Explain.

93. (a) The sales tax on a purchased item is a function of the selling price.

(b) Your score on the next algebra exam is a function of the number of hours you study the night before the exam.

94. (a) The amount in your savings account is a function of your salary.

(b) The speed at which a free-falling baseball strikes the ground is a function of the height from which it was dropped.



# 1.5 Analyzing Graphs of Functions

Graphs of functions can help you visualize relationships between variables in real life. For example, in Exercise 90 on page 59, you will use the graph of a function to visually represent the temperature in a city over a 24-hour period.

- **Use the Vertical Line Test for functions.**
- **Find the zeros of functions.**
- **Determine intervals on which functions are increasing or decreasing.**
- **Determine relative minimum and relative maximum values of functions.**
- **Determine the average rate of change of a function.**
- **Identify even and odd functions.**

## The Graph of a Function

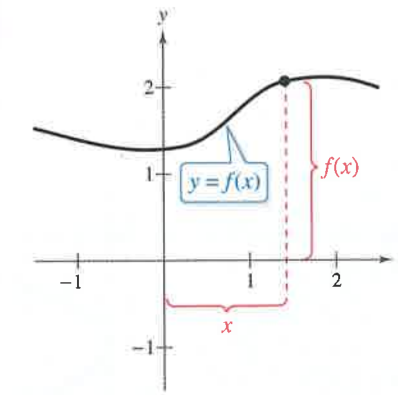
In Section 1.4, you studied functions from an algebraic point of view. In this section, you will study functions from a graphical perspective.

The **graph of a function** $f$ is the collection of ordered pairs $(x, f(x))$ such that $x$ is in the domain of $f$. As you study this section, remember that

$x =$ the directed distance from the $y$-axis

$y = f(x) =$ the directed distance from the $x$-axis

as shown in the figure at the right.




**EXAMPLE 1** **Finding the Domain and Range of a Function**

Use the graph of the function $f$, shown in Figure 1.32, to find (a) the domain of $f$, (b) the function values $f(-1)$ and $f(2)$, and (c) the range of $f$.

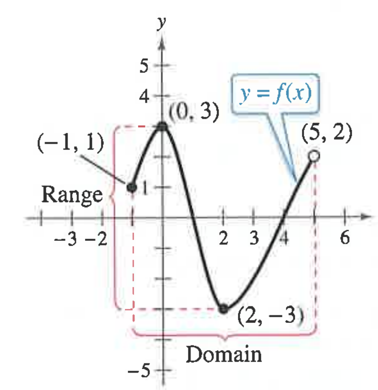



**Figure 1.32**

**Solution**

a. The closed dot at $(-1, 1)$ indicates that $x = -1$ is in the domain of $f$, whereas the open dot at $(5, 2)$ indicates that $x = 5$ is not in the domain. So, the domain of $f$ is all $x$ in the interval $[-1, 5)$.

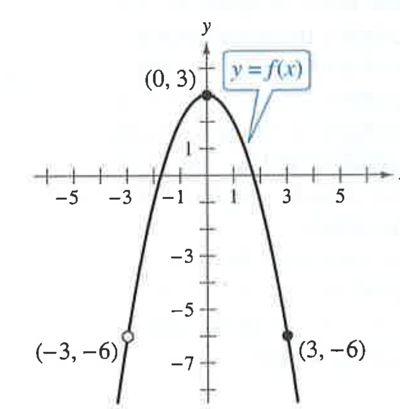
b. One point on the graph of $f$ is $(-1, 1)$, so $f(-1) = 1$. Another point on the graph of $f$ is $(2, -3)$, so $f(2) = -3$.

c. The graph does not extend below $f(2) = -3$ or above $f(0) = 3$, so the range of $f$ is the interval $[-3, 3]$.

**REMARK** The use of dots (open or closed) at the extreme left and right points of a graph indicates that the graph does not extend beyond these points. If such dots are not on the graph, then assume that the graph extends beyond these points.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the graph of the function $f$ to find (a) the domain of $f$, (b) the function values $f(0)$ and $f(3)$, and (c) the range of $f$.

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