1.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary

In Exercises 1–3, fill in the blanks.

1. Horizontal shifts, vertical shifts, and reflections are _____ transformations.
2. A reflection in the x-axis of the graph of $y = f(x)$ is represented by $h(x) =$ _____, while a reflection in the y-axis of the graph of $y = f(x)$ is represented by $h(x) =$ _____.
3. A nonrigid transformation of the graph of $y = f(x)$ represented by $g(x) = cf(x)$ is a _____ _____ when $c > 1$ and a _____ _____ when $0 < c < 1$.
4. Match each function h with the transformation it represents, where $c > 0$.

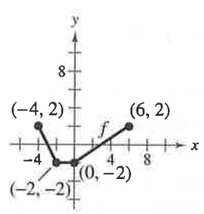
(a) $h(x) = f(x) + c$	(i) A horizontal shift of f, c units to the right
(b) $h(x) = f(x) - c$	(ii) A vertical shift of f, c units down
(c) $h(x) = f(x + c)$	(iii) A horizontal shift of f, c units to the left
(d) $h(x) = f(x - c)$	(iv) A vertical shift of f, c units up

Skills and Applications

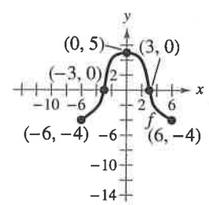
5. **Shifting the Graph of a Function** For each function, sketch the graphs of the function when $c = -2$, -1, 1, and 2 on the same set of coordinate axes.
   (a) $f(x) = |x| + c$ (b) $f(x) = |x - c|$
6. **Shifting the Graph of a Function** For each function, sketch the graphs of the function when $c = -3$, -2, 2, and 3 on the same set of coordinate axes.
   (a) $f(x) = \sqrt{x} + c$ (b) $f(x) = \sqrt{x - c}$
7. **Shifting the Graph of a Function** For each function, sketch the graphs of the function when $c = -4$, -1, 2, and 5 on the same set of coordinate axes.
   (a) $f(x) = [\![x]\!] + c$ (b) $f(x) = [\![x + c]\!]$
8. **Shifting the Graph of a Function** For each function, sketch the graphs of the function when $c = -3$, -2, 1, and 2 on the same set of coordinate axes.
   (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \geq 0 \end{cases}$
   (b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \geq 0 \end{cases}$

Sketching Transformations **In Exercises 9 and 10, use the graph of f to sketch each graph. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

9. (a) $y = f(-x)$
   (b) $y = f(x) + 4$
   (c) $y = 2f(x)$
   (d) $y = -f(x - 4)$
   (e) $y = f(x) - 3$
   (f) $y = -f(x) - 1$
   (g) $y = f(2x)$

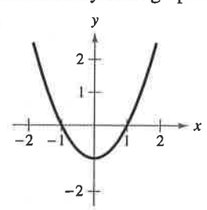


10. (a) $y = f(x - 5)$
    (b) $y = -f(x) + 3$
    (c) $y = \frac{1}{3}f(x)$
    (d) $y = -f(x + 1)$
    (e) $y = f(-x)$
    (f) $y = f(x) - 10$
    (g) $y = f(\frac{1}{3}x)$

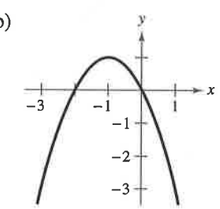


11. **Writing Equations from Graphs** Use the graph of $f(x) = x^2$ to write an equation for the function represented by each graph.

(a)

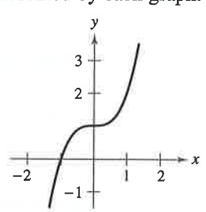


(b)



12. **Writing Equations from Graphs** Use the graph of $f(x) = x^3$ to write an equation for the function represented by each graph.

(a)



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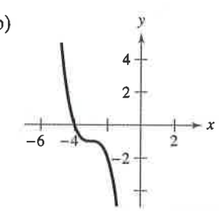
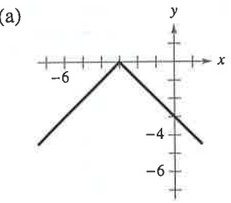
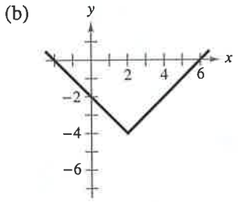


13. **Writing Equations from Graphs** Use the graph of $f(x) = |x|$ to write an equation for the function represented by each graph.

(a)

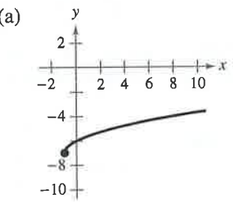


(b)

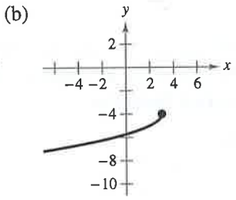


14. **Writing Equations from Graphs** Use the graph of $f(x) = \sqrt{x}$ to write an equation for the function represented by each graph.

(a)

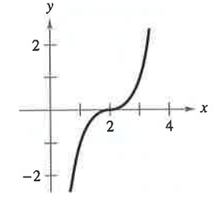


(b)



Writing Equations from Graphs **In Exercises 15–20, identify the parent function and the transformation represented by the graph. Write an equation for the function represented by the graph.**

15.



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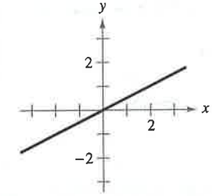

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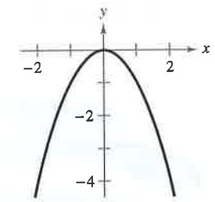

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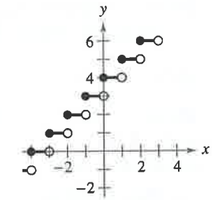

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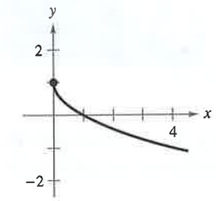

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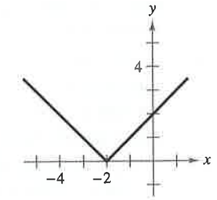


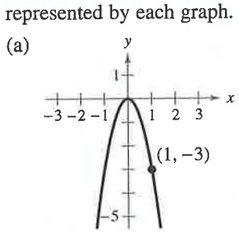
Describing Transformations **In Exercises 21–38, g is related to one of the parent functions described in Section 1.6. (a) Identify the parent function f. (b) Describe the sequence of transformations from f to g. (c) Sketch the graph of g. (d) Use function notation to write g in terms of f.**

21. $g(x) = x^2 + 6$
22. $g(x) = x^2 - 2$
23. $g(x) = -(x - 2)^3$
24. $g(x) = -(x + 1)^3$
25. $g(x) = -3 - (x + 1)^2$
26. $g(x) = 4 - (x - 2)^2$
27. $g(x) = |x - 1| + 2$
28. $g(x) = |x + 3| - 2$
29. $g(x) = 2\sqrt{x}$
30. $g(x) = \frac{1}{2}\sqrt{x}$
31. $g(x) = 2[\![x]\!] - 1$
32. $g(x) = -[\![x]\!] + 1$
33. $g(x) = |2x|$
34. $g(x) = |\frac{1}{2}x|$
35. $g(x) = -2x^2 + 1$
36. $g(x) = \frac{1}{2}x^2 - 2$
37. $g(x) = 3|x - 1| + 2$
38. $g(x) = -2|x + 1| - 3$

Writing an Equation from a Description **In Exercises 39–46, write an equation for the function whose graph is described.**

39. The shape of $f(x) = x^2$, but shifted three units to the right and seven units down
40. The shape of $f(x) = x^2$, but shifted two units to the left, nine units up, and then reflected in the x-axis
41. The shape of $f(x) = x^3$, but shifted 13 units to the right
42. The shape of $f(x) = x^3$, but shifted six units to the left, six units down, and then reflected in the y-axis
43. The shape of $f(x) = |x|$, but shifted 12 units up and then reflected in the x-axis
44. The shape of $f(x) = |x|$, but shifted four units to the left and eight units down
45. The shape of $f(x) = \sqrt{x}$, but shifted six units to the left and then reflected in both the x-axis and the y-axis
46. The shape of $f(x) = \sqrt{x}$, but shifted nine units down and then reflected in both the x-axis and the y-axis
47. **Writing Equations from Graphs** Use the graph of $f(x) = x^2$ to write an equation for the function represented by each graph.

(a)



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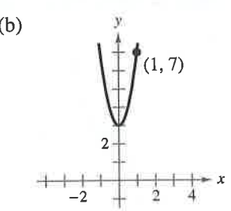
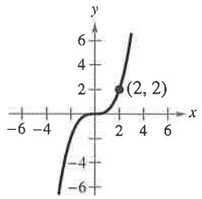


48. Writing Equations from Graphs Use the graph of

$f(x) = x^3$

to write an equation for the function represented by each graph.

(a)



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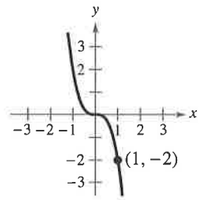
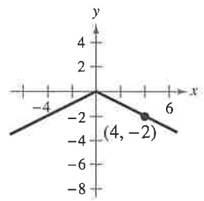


49. Writing Equations from Graphs Use the graph of

$f(x) = |x|$

to write an equation for the function represented by each graph.

(a)



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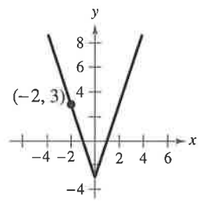
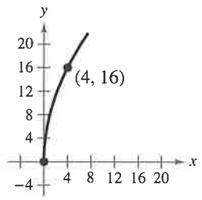


50. Writing Equations from Graphs Use the graph of

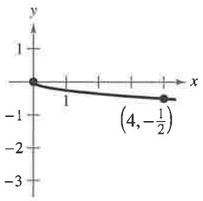
$f(x) = \sqrt{x}$

to write an equation for the function represented by each graph.

(a)

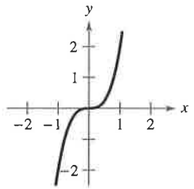


(b)



Writing Equations from Graphs **In Exercises 51–56, identify the parent function and the transformation represented by the graph. Write an equation for the function represented by the graph. Then use a graphing utility to verify your answer.**

51.



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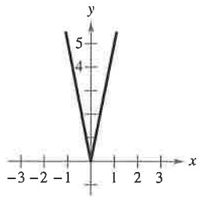

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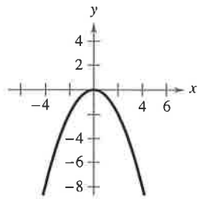

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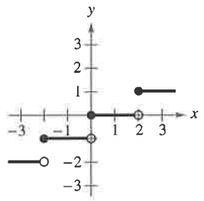

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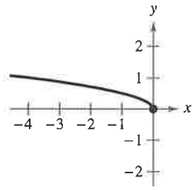

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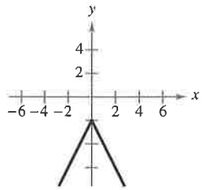
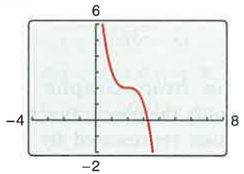
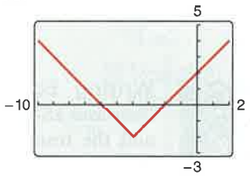


Writing Equations from Graphs **In Exercises 57–60, write an equation for the transformation of the parent function.**

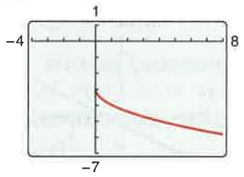
57.



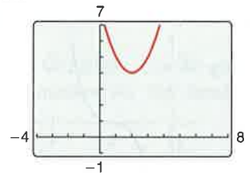
58.



59.



60.



61. Automobile Aerodynamics The horsepower H required to overcome wind drag on a particular automobile is given by

$H(x) = 0.00004636x^3$

where x is the speed of the car (in miles per hour).

(a) Use a graphing utility to graph the function.

(b) Rewrite the horsepower function so that x represents the speed in kilometers per hour. [Find $H(x/1.6)$.] Identify the type of transformation applied to the graph of the horsepower function.

Robert Young/Shutterstock.com

62. Households The number N (in millions) of households in the United States from 2000 through 2014 can be approximated by

$N(x) = -0.023(x - 33.12)^2 + 131, \quad 0 \le t \le 14$

where t represents the year, with $t = 0$ corresponding to 2000. *(Source: U.S. Census Bureau)*

(a) Describe the transformation of the parent function $f(x) = x^2$. Then use a graphing utility to graph the function over the specified domain.

(b) Find the average rate of change of the function from 2000 to 2014. Interpret your answer in the context of the problem.

(c) Use the model to predict the number of households in the United States in 2022. Does your answer seem reasonable? Explain.

Exploration

True or False? **In Exercises 63–66, determine whether the statement is true or false. Justify your answer.**

63. The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the x-axis.

64. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the y-axis.

65. The graphs of $f(x) = |x| + 6$ and $f(x) = |-x| + 6$ are identical.

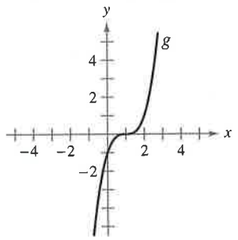
66. If the graph of the parent function $f(x) = x^2$ is shifted six units to the right, three units up, and reflected in the x-axis, then the point $(-2, 19)$ will lie on the graph of the transformation.

67. Finding Points on a Graph The graph of $y = f(x)$ passes through the points $(0, 1)$, $(1, 2)$, and $(2, 3)$. Find the corresponding points on the graph of $y = f(x + 2) - 1$.

68. Think About It Two methods of graphing a function are plotting points and translating a parent function as shown in this section. Which method of graphing do you prefer to use for each function? Explain.

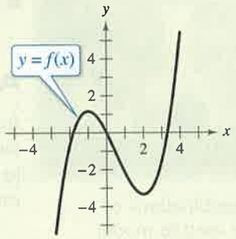
(a) $f(x) = 3x^2 - 4x + 1$ (b) $f(x) = 2(x - 1)^2 - 6$

69. Error Analysis Describe the error.



The graph of g is a right shift of one unit of the graph of $f(x) = x^3$. So, an equation for g is $g(x) = (x + 1)^3$.

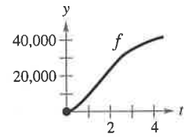
70. HOW DO YOU SEE IT? Use the graph of $y = f(x)$ to find the open intervals on which the graph of each transformation is increasing and decreasing. If not possible, state the reason.



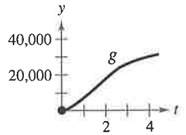
(a) $y = f(-x)$ (b) $y = -f(x)$ (c) $y = \frac{1}{2}f(x)$

(d) $y = -f(x - 1)$ (e) $y = f(x - 2) + 1$

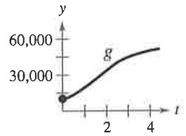
71. Describing Profits Management originally predicted that the profits from the sales of a new product could be approximated by the graph of the function f shown. The actual profits are represented by the graph of the function g along with a verbal description. Use the concepts of transformations of graphs to write g in terms of f.



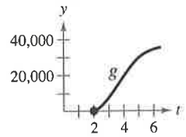
(a) The profits were only three-fourths as large as expected.



(b) The profits were consistently $10,000 greater than predicted.



(c) There was a two-year delay in the introduction of the product. After sales began, profits grew as expected.



72. Reversing the Order of Transformations Reverse the order of transformations in Example 2(a). Do you obtain the same graph? Do the same for Example 2(b). Do you obtain the same graph? Explain.