1.9 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. If $f(g(x))$ and $g(f(x))$ both equal x, then the function g is the _______ function of the function f.
2. The inverse function of f is denoted by _______.
3. The domain of f is the _______ of f^{-1}, and the _______ of f^{-1} is the range of f.
4. The graphs of f and f^{-1} are reflections of each other in the line _______.
5. A function f is _______ when each value of the dependent variable corresponds to exactly one value of the independent variable.
6. A graphical test for the existence of an inverse function of f is the _______ Line Test.

Skills and Applications

Finding an Inverse Function Informally In Exercises 7–14, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

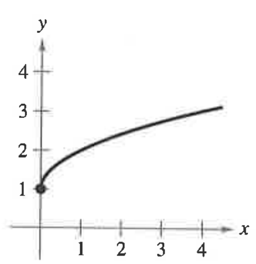
7. $f(x) = 6x$
8. $f(x) = \frac{1}{3}x$
9. $f(x) = 3x + 1$
10. $f(x) = \dfrac{x-3}{2}$
11. $f(x) = x^2 - 4,\ x \ge 0$
12. $f(x) = x^2 + 2,\ x \ge 0$
13. $f(x) = x^3 + 1$
14. $f(x) = \dfrac{x^5}{4}$

Verifying Inverse Functions In Exercises 15–18, verify that f and g are inverse functions algebraically.

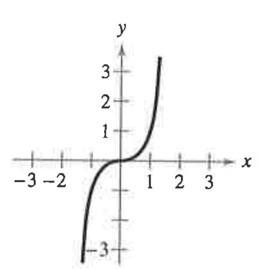
15. $f(x) = \dfrac{x-9}{4},\quad g(x) = 4x + 9$
16. $f(x) = -\dfrac{3}{2}x - 4,\quad g(x) = -\dfrac{2x+8}{3}$
17. $f(x) = \dfrac{x^3}{4},\quad g(x) = \sqrt[3]{4x}$
18. $f(x) = x^3 + 5,\quad g(x) = \sqrt[3]{x-5}$

Sketching the Graph of an Inverse Function In Exercises 19 and 20, use the graph of the function to sketch the graph of its inverse function $y = f^{-1}(x)$.

19.



20.



Verifying Inverse Functions In Exercises 21–32, verify that f and g are inverse functions (a) algebraically and (b) graphically.

21. $f(x) = x - 5,\quad g(x) = x + 5$
22. $f(x) = 2x,\quad g(x) = \dfrac{x}{2}$
23. $f(x) = 7x + 1,\quad g(x) = \dfrac{x-1}{7}$
24. $f(x) = 3 - 4x,\quad g(x) = \dfrac{3-x}{4}$
25. $f(x) = x^3,\quad g(x) = \sqrt[3]{x}$
26. $f(x) = \dfrac{x^3}{3},\quad g(x) = \sqrt[3]{3x}$
27. $f(x) = \sqrt{x+5},\quad g(x) = x^2 - 5,\quad x \ge 0$
28. $f(x) = 1 - x^3,\quad g(x) = \sqrt[3]{1-x}$
29. $f(x) = \dfrac{1}{x},\quad g(x) = \dfrac{1}{x}$
30. $f(x) = \dfrac{1}{1+x},\quad x \ge 0,\quad g(x) = \dfrac{1-x}{x},\quad 0 < x \le 1$
31. $f(x) = \dfrac{x-1}{x+5},\quad g(x) = -\dfrac{5x+1}{x-1}$
32. $f(x) = \dfrac{x+3}{x-2},\quad g(x) = \dfrac{2x+3}{x-1}$

Using a Table to Determine an Inverse Function In Exercises 33 and 34, does the function have an inverse function?

33.

x	−1	0	1	2	3	4
$f(x)$	−2	1	2	1	−2	−6

34.

x	−3	−2	−1	0	2	3
$f(x)$	10	6	4	1	−3	−10

Using a Table to Find an Inverse Function In Exercises 35 and 36, use the table of values for $y = f(x)$ to complete a table for $y = f^{-1}(x)$.

35.

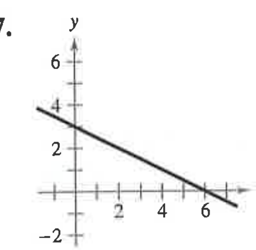
x	−1	0	1	2	3	4
$f(x)$	3	5	7	9	11	13

36.

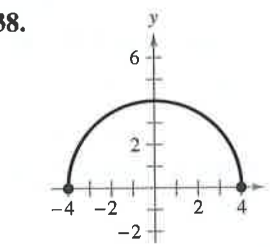
x	−3	−2	−1	0	1	2
$f(x)$	10	5	0	−5	−10	−15

Applying the Horizontal Line Test In Exercises 37–40, does the function have an inverse function?

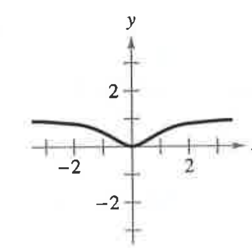
37.



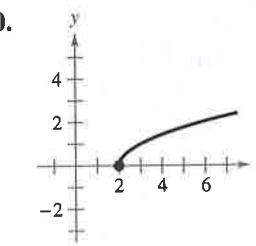
38.



39.



40.



Applying the Horizontal Line Test In Exercises 41–44, use a graphing utility to graph the function, and use the Horizontal Line Test to determine whether the function has an inverse function.

41. $g(x) = (x+3)^2 + 2$
42. $f(x) = \frac{1}{5}(x+2)^3$
43. $f(x) = x\sqrt{9-x^2}$
44. $h(x) = |x| - |x-4|$

Finding and Analyzing Inverse Functions In Exercises 45–54, (a) find the inverse function of f, (b) graph both f and f^{-1} on the same set of coordinate axes, (c) describe the relationship between the graphs of f and f^{-1}, and (d) state the domains and ranges of f and f^{-1}.

45. $f(x) = x^5 - 2$
46. $f(x) = x^3 + 8$
47. $f(x) = \sqrt{4-x^2},\quad 0 \le x \le 2$
48. $f(x) = x^2 - 2,\quad x \le 0$
49. $f(x) = \dfrac{4}{x}$
50. $f(x) = -\dfrac{2}{x}$
51. $f(x) = \dfrac{x+1}{x-2}$
52. $f(x) = \dfrac{x-2}{3x+5}$
53. $f(x) = \sqrt[3]{x-1}$
54. $f(x) = x^{3/5}$

Finding an Inverse Function In Exercises 55–70, determine whether the function has an inverse function. If it does, find the inverse function.

55. $f(x) = x^4$
56. $f(x) = \dfrac{1}{x^2}$
57. $g(x) = \dfrac{x+1}{6}$
58. $f(x) = 3x + 5$
59. $p(x) = -4$
60. $f(x) = 0$
61. $f(x) = (x+3)^2,\quad x \ge -3$
62. $q(x) = (x-5)^2$
63. $f(x) = \begin{cases} x+3, & x < 0 \\ 6-x, & x \ge 0 \end{cases}$
64. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$
65. $h(x) = |x+1| - 1$
66. $f(x) = |x-2|,\quad x \le 2$
67. $f(x) = \sqrt{2x+3}$
68. $f(x) = \sqrt{x-2}$
69. $f(x) = \dfrac{6x+4}{4x+5}$
70. $f(x) = \dfrac{5x-3}{2x+5}$

Restricting the Domain In Exercises 71–78, restrict the domain of the function f so that the function is one-to-one and has an inverse function. Then find the inverse function f^{-1}. State the domains and ranges of f and f^{-1}. Explain your results. (There are many correct answers.)

71. $f(x) = |x+2|$
72. $f(x) = |x-5|$
73. $f(x) = (x+6)^2$
74. $f(x) = (x-4)^2$
75. $f(x) = -2x^2 + 5$
76. $f(x) = \frac{1}{2}x^2 - 1$
77. $f(x) = |x-4| + 1$
78. $f(x) = -|x-1| - 2$

Composition with Inverses In Exercises 79–84, use the functions $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$ to find the value or function.

79. $(f^{-1} \circ g^{-1})(1)$
80. $(g^{-1} \circ f^{-1})(-3)$
81. $(f^{-1} \circ f^{-1})(4)$
82. $(g^{-1} \circ g^{-1})(-1)$
83. $(f \circ g)^{-1}$
84. $g^{-1} \circ f^{-1}$

Composition with Inverses In Exercises 85–88, use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the function.

85. $g^{-1} \circ f^{-1}$
86. $f^{-1} \circ g^{-1}$
87. $(f \circ g)^{-1}$
88. $(g \circ f)^{-1}$

89. Hourly Wage Your wage is \$10.00 per hour plus \$0.75 for each unit produced per hour. So, your hourly wage y in terms of the number of units produced x is $y = 10 + 0.75x$.

(a) Find the inverse function. What does each variable represent in the inverse function?

(b) Determine the number of units produced when your hourly wage is \$24.25.

90. Diesel Mechanics

The function

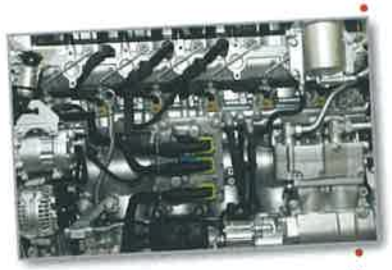
$$y = 0.03x^2 + 245.50, \quad 0 < x < 100$$

approximates the exhaust temperature y in degrees Fahrenheit, where x is the percent load for a diesel engine.

(a) Find the inverse function. What does each variable represent in the inverse function?

(b) Use a graphing utility to graph the inverse function.

(c) The exhaust temperature of the engine must not exceed 500 degrees Fahrenheit. What is the percent load interval?



Exploration

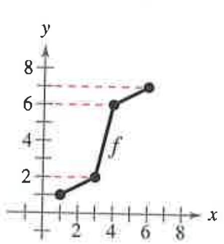
True or False? In Exercises 91 and 92, determine whether the statement is true or false. Justify your answer.

91. If f is an even function, then f^{-1} exists.

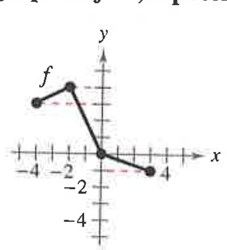
92. If the inverse function of f exists and the graph of f has a y-intercept, then the y-intercept of f is an x-intercept of f^{-1}.

Creating a Table In Exercises 93 and 94, use the graph of the function f to create a table of values for the given points. Then create a second table that can be used to find f^{-1}, and sketch the graph of f^{-1}, if possible.

93.



94.



95. Proof Prove that if f and g are one-to-one functions, then $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

96. Proof Prove that if f is a one-to-one odd function, then f^{-1} is an odd function.

97. Think About It The function $f(x) = k(2 - x - x^3)$ has an inverse function, and $f^{-1}(3) = -2$. Find k.

98. Think About It Consider the functions $f(x) = x + 2$ and $f^{-1}(x) = x - 2$. Evaluate $f(f^{-1}(x))$ and $f^{-1}(f(x))$ for the given values of x. What can you conclude about the functions?

x	-10	0	7	45
$f(f^{-1}(x))$				
$f^{-1}(f(x))$				

99. Think About It Restrict the domain of

$$f(x) = x^2 + 1$$

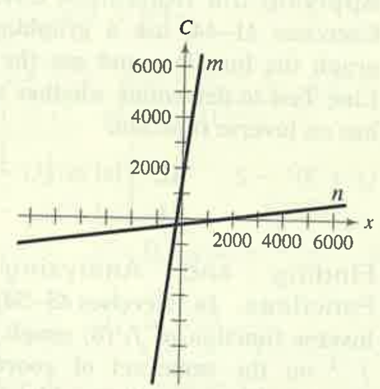
to $x \geq 0$. Use a graphing utility to graph the function. Does the restricted function have an inverse function? Explain.

100. HOW DO YOU SEE IT? The cost C for a business to make personalized T-shirts is given by

$$C(x) = 7.50x + 1500$$

where x represents the number of T-shirts.

(a) The graphs of C and C^{-1} are shown below. Match each function with its graph.



(b) Explain what $C(x)$ and $C^{-1}(x)$ represent in the context of the problem.

One-to-One Function Representation In Exercises 101 and 102, determine whether the situation can be represented by a one-to-one function. If so, write a statement that best describes the inverse function.

101. The number of miles n a marathon runner has completed in terms of the time t in hours

102. The depth of the tide d at a beach in terms of the time t over a 24-hour period

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1.10 Mathematical Modeling and Variation

Mathematical models have a wide variety of real-life applications. For example, in Exercise 71 on page 103, you will use variation to model ocean temperatures at various depths.

- **Use mathematical models to approximate sets of data points.**
- **Use the *regression* feature of a graphing utility to find equations of least squares regression lines.**
- **Write mathematical models for direct variation.**
- **Write mathematical models for direct variation as an *n*th power.**
- **Write mathematical models for inverse variation.**
- **Write mathematical models for combined variation.**
- **Write mathematical models for joint variation.**

Introduction

In this section, you will study two techniques for fitting models to data: *least squares regression* and *direct and inverse variation.*

EXAMPLE 1 Using a Mathematical Model

The table shows the populations y (in millions) of the United States from 2008 through 2015. *(Source: U.S. Census Bureau)*

DATA

Year	2008	2009	2010	2011	2012	2013	2014	2015
Population, y	304.1	306.8	309.3	311.7	314.1	316.5	318.9	321.2

Spreadsheet at LarsonPrecalculus.com

A linear model that approximates the data is

$$y = 2.43t + 284.9, \quad 8 \leq t \leq 15$$

where t represents the year, with $t = 8$ corresponding to 2008. Plot the actual data *and* the model on the same graph. How closely does the model represent the data?

Solution Figure 1.62 shows the actual data and the model plotted on the same graph. From the graph, it appears that the model is a "good fit" for the actual data. To see how well the model fits, compare the actual values of y with the values of y found using the model. The values found using the model are labeled y^* in the table below.

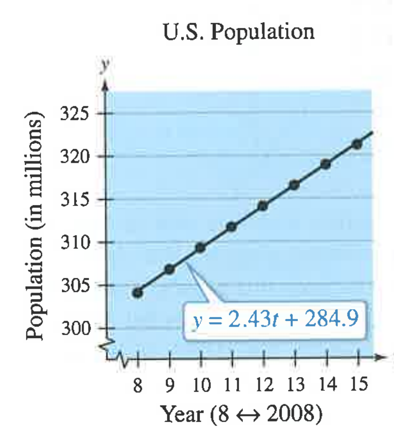

Figure 1.62

t	8	9	10	11	12	13	14	15
y	304.1	306.8	309.3	311.7	314.1	316.5	318.9	321.2
y^*	304.3	306.8	309.2	311.6	314.1	316.5	318.9	321.4

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The ordered pairs below give the median sales prices y (in thousands of dollars) of new homes sold in a neighborhood from 2009 through 2016. *(Spreadsheet at LarsonPrecalculus.com)*

DATA

(2009, 179.4)	(2011, 191.0)	(2013, 202.6)	(2015, 214.9)
(2010, 185.4)	(2012, 196.7)	(2014, 208.7)	(2016, 221.4)

A linear model that approximates the data is $y = 5.96t + 125.5$, $9 \leq t \leq 16$, where t represents the year, with $t = 9$ corresponding to 2009. Plot the actual data *and* the model on the same graph. How closely does the model represent the data?

Ase/Shutterstock.com