10.2 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

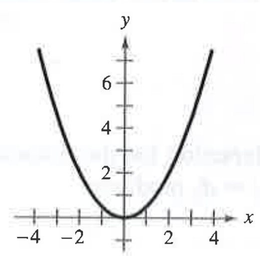
Vocabulary: Fill in the blanks.

1. A _______ is the intersection of a plane and a double-napped cone.
2. When a plane passes through the vertex of a double-napped cone, the intersection is a _______ _______.
3. A _______ of points is a collection of points satisfying a given geometric property.
4. A _______ is the set of all points (x, y) in a plane that are equidistant from a fixed line, called the _______, and a fixed point, called the _______, not on the line.
5. The line that passes through the focus and the vertex of a parabola is the _______ of the parabola.
6. The _______ of a parabola is the midpoint between the focus and the directrix.
7. A line segment that passes through the focus of a parabola and has endpoints on the parabola is a _______ _______.
8. A line is _______ to a parabola at a point on the parabola when the line intersects, but does not cross, the parabola at the point.

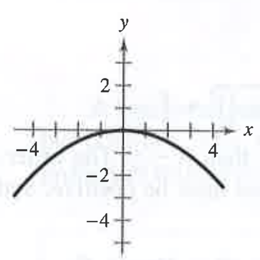
Skills and Applications

Matching In Exercises 9–12, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]

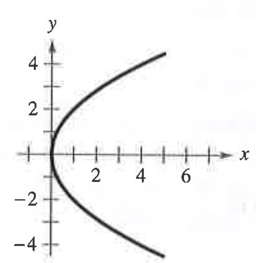
(a)



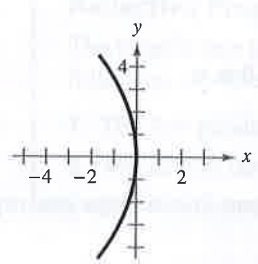
(b)



(c)



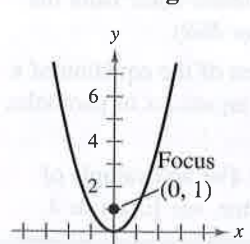
(d)



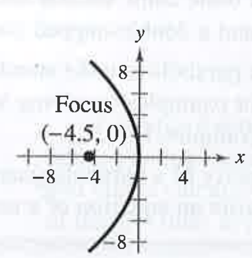
9. $y^2 = 4x$
10. $x^2 = 2y$
11. $x^2 = -8y$
12. $y^2 = -12x$

Finding the Standard Equation of a Parabola In Exercises 13–26, find the standard form of the equation of the parabola with the given characteristic(s) and vertex at the origin.

13.



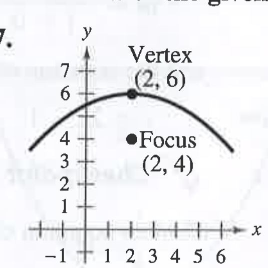
14.



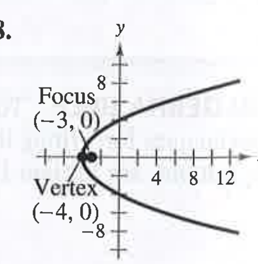
15. Focus: $\left(0, \frac{1}{2}\right)$
16. Focus: $\left(\frac{3}{2}, 0\right)$
17. Focus: $(-2, 0)$
18. Focus: $(0, -1)$
19. Directrix: $y = 2$
20. Directrix: $y = -4$
21. Directrix: $x = -1$
22. Directrix: $x = 3$
23. Vertical axis; passes through the point $(4, 6)$
24. Vertical axis; passes through the point $(-3, -3)$
25. Horizontal axis; passes through the point $(-2, 5)$
26. Horizontal axis; passes through the point $(3, -2)$

Finding the Standard Equation of a Parabola In Exercises 27–36, find the standard form of the equation of the parabola with the given characteristics.

27.



28.



29. Vertex: $(6, 3)$; focus: $(4, 3)$
30. Vertex: $(1, -8)$; focus: $(3, -8)$
31. Vertex: $(0, 2)$; directrix: $y = 4$
32. Vertex: $(1, 2)$; directrix: $y = -1$
33. Focus: $(2, 2)$; directrix: $x = -2$
34. Focus: $(0, 0)$; directrix: $y = 8$
35. Vertex: $(3, -3)$; vertical axis; passes through the point $(0, 0)$
36. Vertex: $(-1, 6)$; horizontal axis; passes through the point $(-9, 2)$

Finding the Vertex, Focus, and Directrix of a Parabola In Exercises 37–50, find the vertex, focus, and directrix of the parabola. Then sketch the parabola.

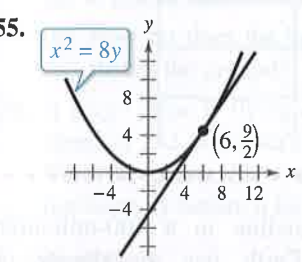
37. $y = \frac{1}{2}x^2$
38. $y = -4x^2$
39. $y^2 = -6x$
40. $y^2 = 3x$
41. $x^2 + 12y = 0$
42. $x + y^2 = 0$
43. $(x - 1)^2 + 8(y + 2) = 0$
44. $(x + 5) + (y - 1)^2 = 0$
45. $(y + 7)^2 = 4\left(x - \frac{3}{2}\right)$
46. $\left(x + \frac{1}{2}\right)^2 = 4(y - 1)$
47. $y = \frac{1}{4}(x^2 - 2x + 5)$
48. $x = \frac{1}{4}(y^2 + 2y + 33)$
49. $y^2 + 6y + 8x + 25 = 0$
50. $x^2 - 4x - 4y = 0$

Finding the Vertex, Focus, and Directrix of a Parabola In Exercises 51–54, find the vertex, focus, and directrix of the parabola. Use a graphing utility to graph the parabola.

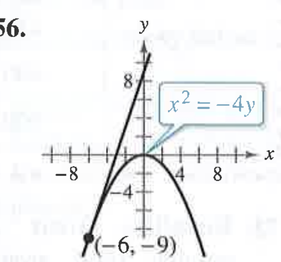
51. $x^2 + 4x - 6y = -10$
52. $x^2 - 2x + 8y = -9$
53. $y^2 + x + y = 0$
54. $y^2 - 4x - 4 = 0$

Finding the Tangent Line at a Point on a Parabola In Exercises 55–60, find an equation of the tangent line to the parabola at the given point.

55.



56.



57. $x^2 = 2y$, $(4, 8)$
58. $x^2 = 2y$, $\left(-3, \frac{9}{2}\right)$
59. $y = -2x^2$, $(-1, -2)$
60. $y = -2x^2$, $(2, -8)$

61. **Flashlight** The light bulb in a flashlight is at the focus of the parabolic reflector, 1.5 centimeters from the vertex of the reflector (see figure). Write an equation for a cross section of the flashlight's reflector with its focus on the positive x-axis and its vertex at the origin.

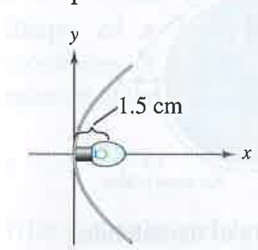


Figure for 61

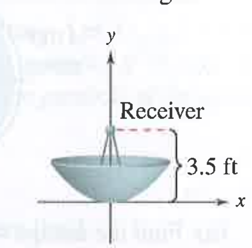
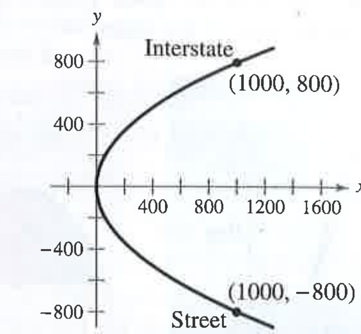


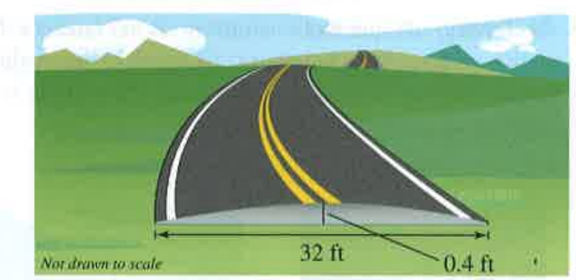
Figure for 62

62. **Satellite Dish** The receiver of a parabolic satellite dish is at the focus of the parabola (see figure). Write an equation for a cross section of the satellite dish.

63. **Highway Design** Highway engineers use a parabolic curve to design an entrance ramp from a straight street to an interstate highway (see figure). Write an equation of the parabola.



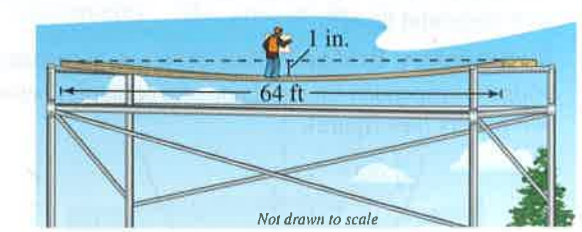
64. **Road Design** Roads are often designed with parabolic surfaces to allow rain to drain off. A particular road is 32 feet wide and 0.4 foot higher in the center than it is on the sides (see figure).



(a) Write an equation of the parabola with its vertex at the origin that models the road surface.

(b) How far from the center of the road is the road surface 0.1 foot lower than the center?

65. **Beam Deflection** A simply supported beam is 64 feet long and has a load at the center (see figure). The deflection of the beam at its center is 1 inch. The shape of the deflected beam is parabolic.



(a) Write an equation of the parabola with its vertex at the origin that models the shape of the beam.

(b) How far from the center of the beam is the deflection $\frac{1}{2}$ inch?

66. **Beam Deflection** Repeat Exercise 65 when the length of the beam is 36 feet and the deflection of the beam at its center is 2 inches.

67. Fluid Flow Water is flowing from a horizontal pipe 48 feet above the ground. The falling stream of water has the shape of a parabola whose vertex $(0, 48)$ is at the end of the pipe (see figure). The stream of water strikes the ocean at the point $(10\sqrt{3}, 0)$. Write an equation for the path of the water.

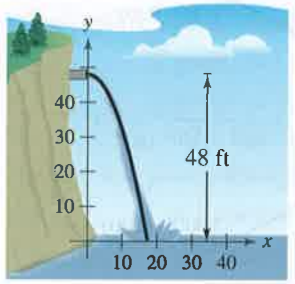


Figure for 67

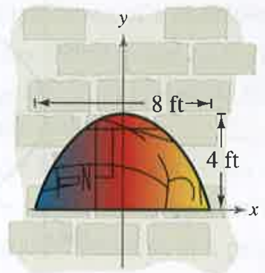


Figure for 68

68. Window Design A church window is bounded above by a parabola (see figure). Write an equation of the parabola.

69. Archway A parabolic archway is 12 meters high at the vertex. At a height of 10 meters, the width of the archway is 8 meters (see figure). How wide is the archway at ground level?

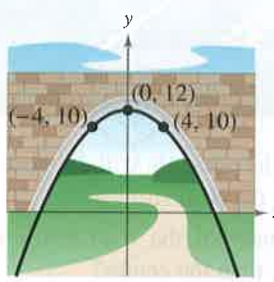


Figure for 69

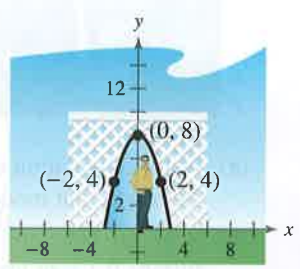
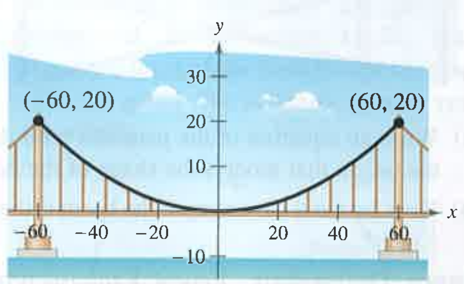


Figure for 70

70. Lattice Arch A parabolic lattice arch is 8 feet high at the vertex. At a height of 4 feet, the width of the lattice arch is 4 feet (see figure). How wide is the lattice arch at ground level?

71. Suspension Bridge Each cable of a suspension bridge is suspended (in the shape of a parabola) between two towers (see figure).



(a) Find the coordinates of the focus.

(b) Write an equation that models the cables.

72. Suspension Bridge

Each cable of the Golden Gate Bridge is suspended (in the shape of a parabola) between two towers that are 1280 meters apart. The top of each tower is 152 meters above the roadway. The cables touch the roadway at the midpoint between the towers.

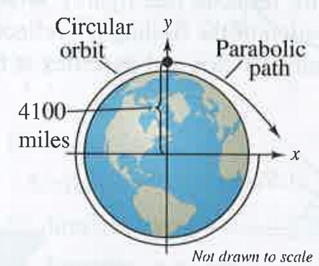
(a) Sketch the bridge on a rectangular coordinate system with the cables touching the roadway at the origin. Label the coordinates of the known points.

(b) Write an equation that models the cables.

(c) Complete the table by finding the height y of the cables over the roadway at a distance of x meters from the point where the cables touch the roadway.

Distance, x	Height, y
0	
100	
250	
400	
500	

73. Satellite Orbit A satellite in a 100-mile-high circular orbit around Earth has a velocity of approximately 17,500 miles per hour. When this velocity is multiplied by $\sqrt{2}$, the satellite has the minimum velocity necessary to escape Earth's gravity and follow a parabolic path with the center of Earth as the focus (see figure).



(a) Find the escape velocity of the satellite.

(b) Write an equation for the parabolic path of the satellite. (Assume that the radius of Earth is 4000 miles.)

74. Path of a Softball The path of a softball is modeled by

$$-12.5(y - 7.125) = (x - 6.25)^2$$

where x and y are measured in feet, with $x = 0$ corresponding to the position from which the ball was thrown.

(a) Use a graphing utility to graph the trajectory of the softball.

(b) Use the *trace* feature of the graphing utility to approximate the highest point and the range of the trajectory.

Projectile Motion In Exercises 75 and 76, consider the path of an object projected horizontally with a velocity of v feet per second at a height of s feet, where the model for the path is

$$x^2 = -\frac{v^2}{16}(y - s).$$

In this model (in which air resistance is disregarded), y is the height (in feet) of the projectile and x is the horizontal distance (in feet) the projectile travels.

75. A ball is thrown from the top of a 100-foot tower with a velocity of 28 feet per second.

(a) Write an equation for the parabolic path.

(b) How far does the ball travel horizontally before it strikes the ground?

76. A cargo plane is flying at an altitude of 500 feet and a speed of 255 miles per hour. A supply crate is dropped from the plane. How many *feet* will the crate travel horizontally before it hits the ground?

Exploration

True or False? In Exercises 77–79, determine whether the statement is true or false. Justify your answer.

77. It is possible for a parabola to intersect its directrix.

78. A tangent line to a parabola always intersects the directrix.

79. When the vertex and focus of a parabola are on a horizontal line, the directrix of the parabola is vertical.

80. Slope of a Tangent Line Let (x_1, y_1) be the coordinates of a point on the parabola $x^2 = 4py$. The equation of the line tangent to the parabola at the point is

$$y - y_1 = \frac{x_1}{2p}(x - x_1).$$

What is the slope of the tangent line?

81. Think About It Explain what each equation represents, and how equations (a) and (b) are equivalent.

(a) $y = a(x - h)^2 + k, \quad a \neq 0$

(b) $(x - h)^2 = 4p(y - k), \quad p \neq 0$

(c) $(y - k)^2 = 4p(x - h), \quad p \neq 0$

82. HOW DO YOU SEE IT? In parts (a)–(d), describe how a plane could intersect the double-napped cone to form each conic section (see figure).

(a) Circle (b) Ellipse

(c) Parabola (d) Hyperbola

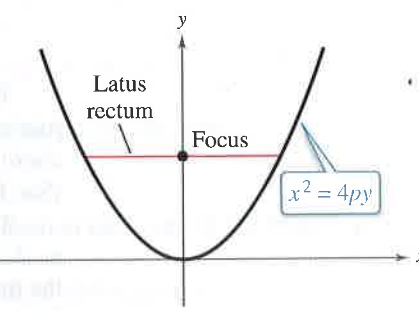
83. Think About It The graph of $x^2 + y^2 = 0$ is a degenerate conic. Sketch the graph of this equation and identify the degenerate conic. Describe the intersection of the plane and the double-napped cone for this conic.

84. Graphical Reasoning Consider the parabola $x^2 = 4py$.

(a) Use a graphing utility to graph the parabola for $p = 1$, $p = 2$, $p = 3$, and $p = 4$. Describe the effect on the graph when p increases.

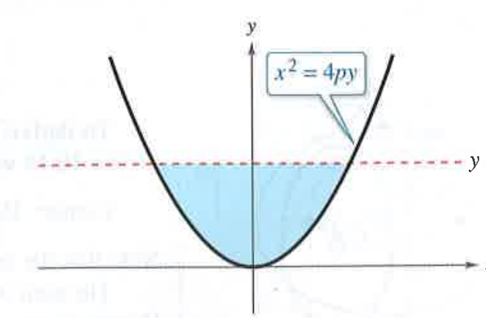
(b) Find the focus for each parabola in part (a).

(c) For each parabola in part (a), find the length of the latus rectum (see figure). How can the length of the latus rectum be determined directly from the standard form of the equation of the parabola?



(d) How can you use the result of part (c) as a sketching aid when graphing parabolas?

85. Geometry The area of the shaded region in the figure is $A = \frac{8}{3}p^{1/2}b^{3/2}$.



(a) Find the area when $p = 2$ and $b = 4$.

(b) Give a geometric explanation of why the area approaches 0 as p approaches 0.