10.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

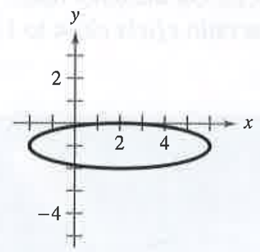
Vocabulary: **Fill in the blanks.**

1. An ________ is the set of all points (x, y) in a plane, the sum of whose distances from two distinct fixed points, called ________, is constant.
2. The chord joining the vertices of an ellipse is the ________ ________, and its midpoint is the ________ of the ellipse.
3. The chord perpendicular to the major axis at the center of an ellipse is the ________ ________ of the ellipse.
4. You can measure the "ovalness" of an ellipse by using the concept of ________.

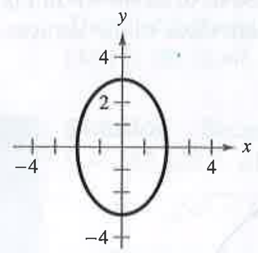
Skills and Applications

Matching **In Exercises 5–8, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

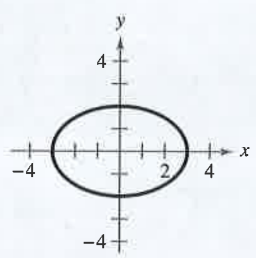
(a)



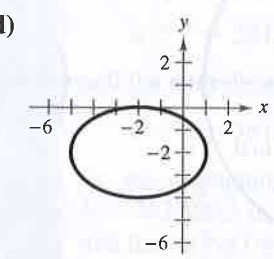
(b)



(c)



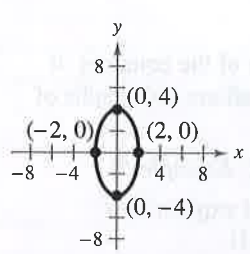
(d)



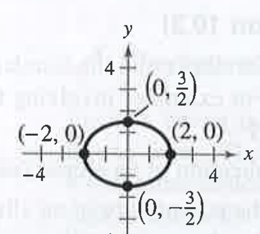
5. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$
6. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
7. $\dfrac{(x-2)^2}{16} + (y+1)^2 = 1$
8. $\dfrac{(x+2)^2}{9} + \dfrac{(y+2)^2}{4} = 1$

An Ellipse Centered at the Origin **In Exercises 9–18, find the standard form of the equation of the ellipse with the given characteristics and center at the origin.**

9.



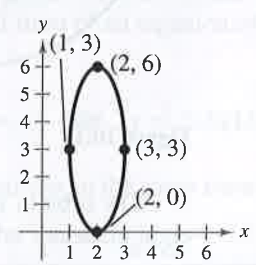
10.



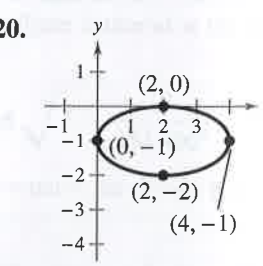
11. Vertices: $(\pm 7, 0)$; foci: $(\pm 2, 0)$
12. Vertices: $(0, \pm 8)$; foci: $(0, \pm 4)$
13. Foci: $(\pm 4, 0)$; major axis of length 10
14. Foci: $(0, \pm 3)$; major axis of length 8
15. Vertical major axis; passes through the points $(0, 6)$ and $(3, 0)$
16. Horizontal major axis; passes through the points $(5, 0)$ and $(0, 2)$
17. Vertices: $(\pm 6, 0)$; passes through the point $(4, 1)$
18. Vertices: $(0, \pm 8)$; passes through the point $(3, 4)$

Finding the Standard Equation of an Ellipse **In Exercises 19–30, find the standard form of the equation of the ellipse with the given characteristics.**

19.



20.



21. Vertices: $(2, 0)$, $(10, 0)$; minor axis of length 4
22. Vertices: $(3, 1)$, $(3, 11)$; minor axis of length 2
23. Foci: $(0, 0)$, $(4, 0)$; major axis of length 6
24. Foci: $(0, 0)$, $(0, 8)$; major axis of length 16
25. Center: $(1, 3)$; vertex: $(-2, 3)$; minor axis of length 4
26. Center: $(2, -1)$; vertex: $(2, \frac{1}{2})$; minor axis of length 2
27. Center: $(1, 4)$; $a = 2c$; vertices: $(1, 0)$, $(1, 8)$
28. Center: $(3, 2)$; $a = 3c$; foci: $(1, 2)$, $(5, 2)$
29. Vertices: $(0, 2)$, $(4, 2)$; endpoints of the minor axis: $(2, 3)$, $(2, 1)$
30. Vertices: $(5, 0)$, $(5, 12)$; endpoints of the minor axis: $(1, 6)$, $(9, 6)$

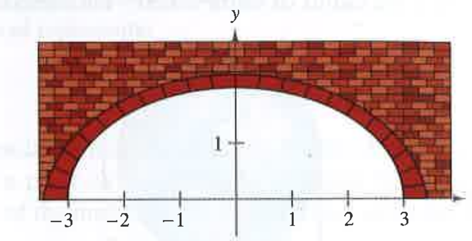
Sketching an Ellipse **In Exercises 31–46, find the center, vertices, foci, and eccentricity of the ellipse. Then sketch the ellipse.**

31. $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$
32. $\dfrac{x^2}{16} + \dfrac{y^2}{81} = 1$
33. $9x^2 + y^2 = 36$
34. $x^2 + 16y^2 = 64$
35. $\dfrac{(x-4)^2}{16} + \dfrac{(y+1)^2}{25} = 1$
36. $\dfrac{(x+3)^2}{12} + \dfrac{(y-2)^2}{16} = 1$
37. $\dfrac{(x+5)^2}{9/4} + (y-1)^2 = 1$
38. $(x+2)^2 + \dfrac{(y+4)^2}{1/4} = 1$
39. $9x^2 + 4y^2 + 36x - 24y + 36 = 0$
40. $9x^2 + 4y^2 - 54x + 40y + 37 = 0$
41. $x^2 + 5y^2 - 8x - 30y - 39 = 0$
42. $3x^2 + y^2 + 18x - 2y - 8 = 0$
43. $6x^2 + 2y^2 + 18x - 10y + 2 = 0$
44. $x^2 + 4y^2 - 6x + 20y - 2 = 0$
45. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$
46. $36x^2 + 9y^2 + 48x - 36y + 43 = 0$

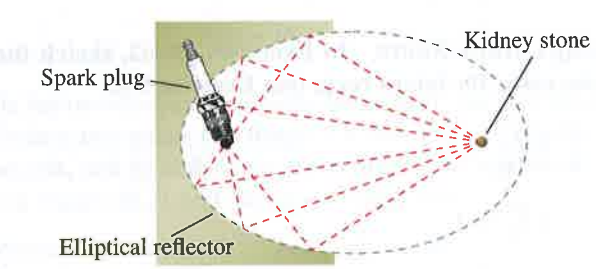
Graphing an Ellipse **In Exercises 47–50, use a graphing utility to graph the ellipse. Find the center, foci, and vertices.**

47. $5x^2 + 3y^2 = 15$
48. $3x^2 + 4y^2 = 12$
49. $x^2 + 9y^2 - 10x + 36y + 52 = 0$
50. $4x^2 + 3y^2 - 8x + 18y + 19 = 0$

51. **Using Eccentricity** Find an equation of the ellipse with vertices $(\pm 5, 0)$ and eccentricity $e = \frac{4}{5}$.
52. **Using Eccentricity** Find an equation of the ellipse with vertices $(0, \pm 8)$ and eccentricity $e = \frac{1}{2}$.
53. **Architecture** Statuary Hall is an elliptical room in the United States Capitol in Washington, D.C. The room is also called the Whispering Gallery because a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus. The dimensions of Statuary Hall are 46 feet wide by 97 feet long.
    (a) Find an equation of the shape of the room.
    (b) Determine the distance between the foci.
54. **Architecture** A mason is building a semielliptical fireplace arch that has a height of 2 feet at the center and a width of 6 feet along the base (see figure). The mason draws the semiellipse on the wall by the method shown on page 708. Find the positions of the thumbtacks and the length of the string.

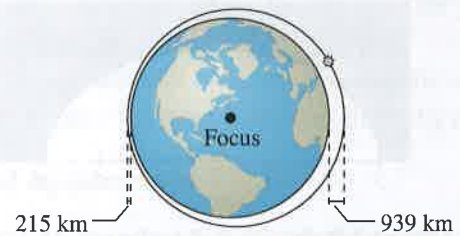


55. **Lithotripter** A lithotripter machine uses an elliptical reflector to break up kidney stones nonsurgically. A spark plug in the reflector generates energy waves at one focus of an ellipse. The reflector directs these waves toward the kidney stone, positioned at the other focus of the ellipse, with enough energy to break up the stone (see figure). The lengths of the major and minor axes of the ellipse are 280 millimeters and 160 millimeters, respectively. How far is the spark plug from the kidney stone?

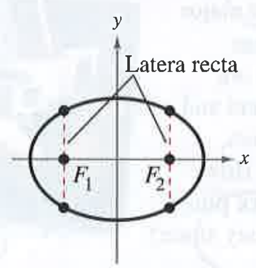


56. **Astronomy** Halley's comet has an elliptical orbit with the center of the sun at one focus. The eccentricity of the orbit is approximately 0.967. The length of the major axis of the orbit is approximately 35.88 astronomical units. (An astronomical unit is about 93 million miles.)
    (a) Find an equation of the orbit. Place the center of the orbit at the origin and place the major axis on the x-axis.
    (b) Use a graphing utility to graph the equation of the orbit.
    (c) Find the greatest and least distances (the aphelion and perihelion, respectively) from the sun's center to the comet's center.

57. Astronomy The first artificial satellite to orbit Earth was Sputnik I (launched by the former Soviet Union in 1957). Its highest point above Earth's surface was 939 kilometers, and its lowest point was 215 kilometers (see figure). The center of Earth was at one focus of the elliptical orbit. Find the eccentricity of the orbit. (Assume the radius of Earth is 6378 kilometers.)



58. Geometry A line segment through a focus of an ellipse with endpoints on the ellipse and perpendicular to the major axis is called a **latus rectum** of the ellipse. An ellipse has two latera recta. Knowing the length of the latera recta is helpful in sketching an ellipse because it yields other points on the curve (see figure). Show that the length of each latus rectum is $2b^2/a$.



Using Latera Recta In Exercises 59–62, sketch the ellipse using the latera recta (see Exercise 58).

59. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

60. $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

61. $5x^2 + 3y^2 = 15$

62. $9x^2 + 4y^2 = 36$

Exploration

True or False? In Exercises 63 and 64, determine whether the statement is true or false. Justify your answer.

63. The graph of $x^2 + 4y^4 - 4 = 0$ is an ellipse.

64. It is easier to distinguish the graph of an ellipse from the graph of a circle when the eccentricity of the ellipse is close to 1.

65. Think About It Find an equation of an ellipse such that for any point on the ellipse, the sum of the distances from the point to the points (2, 2) and (10, 2) is 36.

66. Think About It At the beginning of this section, you learned that an ellipse can be drawn using two thumbtacks, a string of fixed length (greater than the distance between the two thumbtacks), and a pencil. When the ends of the string are fastened to the thumbtacks and the string is drawn taut with the pencil, the path traced by the pencil is an ellipse.

(a) What is the length of the string in terms of a?

(b) Explain why the path is an ellipse.

67. Conjecture Consider the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a + b = 20.$$

(a) The area of the ellipse is given by $A = \pi ab$. Write the area of the ellipse as a function of a.

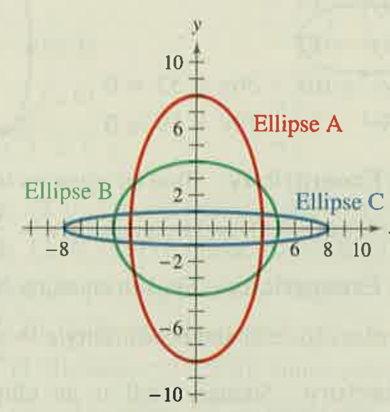
(b) Find the equation of an ellipse with an area of 264 square centimeters.

(c) Complete the table using your equation from part (a). Then make a conjecture about the shape of the ellipse with maximum area.

a	8	9	10	11	12	13
A						

(d) Use a graphing utility to graph the area function and use the graph to support your conjecture in part (c).

68. HOW DO YOU SEE IT? Without performing any calculations, order the eccentricities of the ellipses from least to greatest.



69. Proof Show that $a^2 = b^2 + c^2$ for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $a > 0$, $b > 0$, and the distance from the center of the ellipse (0, 0) to a focus is c.

10.4 Hyperbolas

Hyperbolas have many types of real-life applications. For example, in Exercise 53 on page 725, you will investigate the use of hyperbolas in long distance radio navigation for aircraft and ships.

- **Write equations of hyperbolas in standard form.**
- **Find asymptotes of and sketch hyperbolas.**
- **Use properties of hyperbolas to solve real-life problems.**
- **Classify conics from their general equations.**

Introduction

The definition of a **hyperbola** is similar to that of an ellipse. For an ellipse, the *sum* of the distances between the foci and a point on the ellipse is constant. For a hyperbola, the absolute value of the *difference* of the distances between the foci and a point on the hyperbola is constant.

Definition of a Hyperbola

A **hyperbola** is the set of all points (x, y) in a plane for which the absolute value of the difference of the distances from two distinct fixed points **(foci)** is constant. See Figure 10.20.

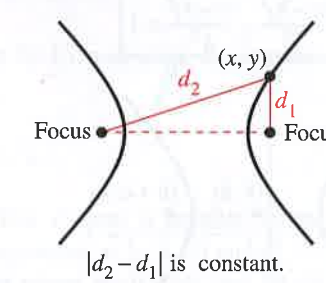


Figure 10.20

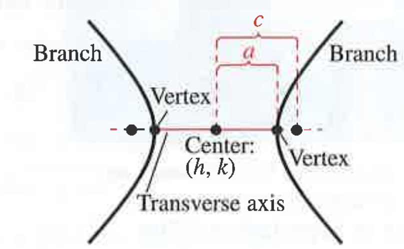


Figure 10.21

The graph of a hyperbola has two disconnected parts **(branches).** The line through the foci intersects the hyperbola at two points **(vertices).** The line segment connecting the vertices is the **transverse axis,** and its midpoint is the **center** of the hyperbola.

Consider the hyperbola in Figure 10.21 with the points listed below.

Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$

Note that the center is also the midpoint of the segment joining the foci.

The absolute value of the difference of the distances from *any* point on the hyperbola to the two foci is constant. Using a vertex point, this constant value is

$$|[2a + (c - a)] - (c - a)| = |2a| = 2a \qquad \text{Length of transverse axis}$$

which is the length of the transverse axis. Now, if you let (x, y) be *any* point on the hyperbola, then

$$|d_2 - d_1| = 2a$$

(see Figure 10.20). You would obtain the same result for a hyperbola with a vertical transverse axis.

The development of the standard form of the equation of a hyperbola is similar to that of an ellipse. Note in the definition on the next page that a, b, and c are related differently for hyperbolas than for ellipses. For a hyperbola, the distance between the foci and the center is greater than the distance between the vertices and the center.