Equation Conversion

To convert a rectangular equation to polar form, replace x with $r\cos\theta$ and y with $r\sin\theta$. For example, here is how to write the rectangular equation $y = x^2$ in polar form.

$y = x^2$ Rectangular equation

$r\sin\theta = (r\cos\theta)^2$ Polar equation

$r = \sec\theta\tan\theta$ Solve for r.

Converting a polar equation to rectangular form requires considerable ingenuity. Example 5 demonstrates several polar-to-rectangular conversions that enable you to sketch the graphs of some polar equations.

EXAMPLE 5 Converting Polar Equations to Rectangular Form

See LarsonPrecalculus.com for an interactive version of this type of example.

a. The graph of the polar equation $r = 2$ consists of all points that are two units from the pole. In other words, this graph is a circle centered at the origin with a radius of 2, as shown in Figure 10.51. Confirm this by converting to rectangular form, using the relationship $r^2 = x^2 + y^2$.

$$\underbrace{r = 2}_{\text{Polar equation}} \Rightarrow r^2 = 2^2 \Rightarrow \underbrace{x^2 + y^2 = 2^2}_{\text{Rectangular equation}}$$

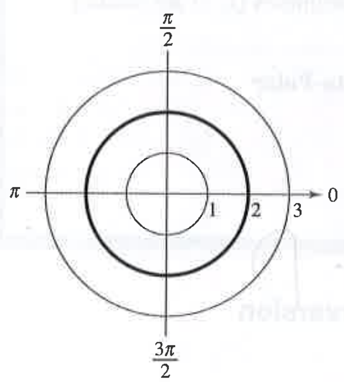


Figure 10.51

b. The graph of the polar equation $\theta = \pi/3$ consists of all points on the line that makes an angle of $\pi/3$ with the polar axis and passes through the pole, as shown in Figure 10.52. To convert to rectangular form, use the relationship $\tan\theta = y/x$.

$$\underbrace{\theta = \pi/3}_{\text{Polar equation}} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \underbrace{y = \sqrt{3}x}_{\text{Rectangular equation}}$$

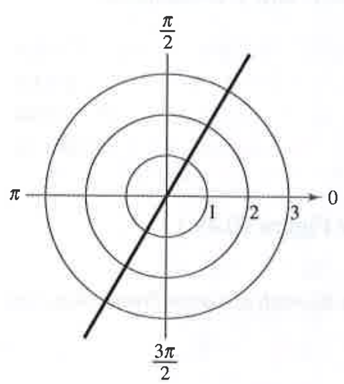


Figure 10.52

c. The graph of the polar equation $r = \sec\theta$ is not evident by inspection, so convert to rectangular form using the relationship $r\cos\theta = x$.

$$\underbrace{r = \sec\theta}_{\text{Polar equation}} \Rightarrow r\cos\theta = 1 \Rightarrow \underbrace{x = 1}_{\text{Rectangular equation}}$$

The graph is a vertical line, as shown in Figure 10.53.

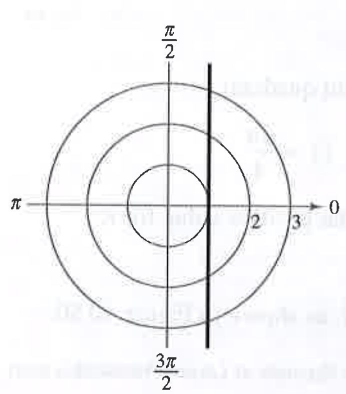


Figure 10.53

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Describe the graph of each polar equation and find the corresponding rectangular equation.

a. $r = 7$ **b.** $\theta = \pi/4$ **c.** $r = 6\sin\theta$

Summarize (Section 10.7)

1. Explain how to plot the point (r, θ) in the polar coordinate system *(page 745)*. For examples of plotting points in the polar coordinate system, see Examples 1 and 2.
2. Explain how to convert points from rectangular to polar form and vice versa *(page 747)*. For examples of converting between forms, see Examples 3 and 4.
3. Explain how to convert equations from rectangular to polar form and vice versa *(page 748)*. For an example of converting polar equations to rectangular form, see Example 5.

10.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. The origin of the polar coordinate system is called the ________.
2. For the point (r, θ), r is the ________ ________ from O to P and θ is the ________ ________, counterclockwise from the polar axis to the line segment $\overline{OP}$.
3. To plot the point (r, θ), use the ________ coordinate system.
4. The polar coordinates (r, θ) and the rectangular coordinates (x, y) are related as follows:
   $x =$ ________ $y =$ ________ $\tan\theta =$ ________ $r^2 =$ ________

Skills and Applications

Plotting a Point in the Polar Coordinate System In Exercises 5–18, plot the point given in polar coordinates and find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$.

5. $(2, \pi/6)$
6. $(3, 5\pi/4)$
7. $(4, -\pi/3)$
8. $(1, -3\pi/4)$
9. $(2, 3\pi)$
10. $(4, 5\pi/2)$
11. $(-2, 2\pi/3)$
12. $(-3, 11\pi/6)$
13. $(0, 7\pi/6)$
14. $(0, -7\pi/2)$
15. $(\sqrt{2}, 2.36)$
16. $(2\sqrt{2}, 4.71)$
17. $(-3, -1.57)$
18. $(-5, -2.36)$

Polar-to-Rectangular Conversion In Exercises 19–28, a point is given in polar coordinates. Convert the point to rectangular coordinates.

19. $(0, \pi)$
20. $(0, -\pi)$
21. $(3, \pi/2)$
22. $(3, 3\pi/2)$
23. $(2, 3\pi/4)$
24. $(1, 5\pi/4)$
25. $(-2, 7\pi/6)$
26. $(-3, 5\pi/6)$
27. $(-3, -\pi/3)$
28. $(-2, -4\pi/3)$

Using a Graphing Utility to Find Rectangular Coordinates In Exercises 29–38, use a graphing utility to find the rectangular coordinates of the point given in polar coordinates. Round your results to two decimal places.

29. $(2, 7\pi/8)$
30. $(3/2, 6\pi/5)$
31. $(1, 5\pi/12)$
32. $(4, 7\pi/9)$
33. $(-2.5, 1.1)$
34. $(-2, 5.76)$
35. $(2.5, -2.9)$
36. $(8.75, -6.5)$
37. $(-3.1, 7.92)$
38. $(-2.04, -5.3)$

Rectangular-to-Polar Conversion In Exercises 39–50, a point is given in rectangular coordinates. Convert the point to polar coordinates. (There are many correct answers.)

39. $(1, 1)$
40. $(2, 2)$
41. $(-3, -3)$
42. $(-4, -4)$
43. $(3, 0)$
44. $(-6, 0)$
45. $(0, -5)$
46. $(0, 8)$
47. $(-\sqrt{3}, -\sqrt{3})$
48. $(-\sqrt{3}, \sqrt{3})$
49. $(\sqrt{3}, -1)$
50. $(-1, \sqrt{3})$

Using a Graphing Utility to Find Polar Coordinates In Exercises 51–58, use a graphing utility to find one set of polar coordinates of the point given in rectangular coordinates. Round your results to two decimal places.

51. $(3, -2)$
52. $(6, 3)$
53. $(-5, 2)$
54. $(7, -2)$
55. $(-\sqrt{3}, -4)$
56. $(5, -\sqrt{2})$
57. $\left(\frac{5}{2}, \frac{4}{3}\right)$
58. $\left(-\frac{7}{9}, -\frac{3}{4}\right)$

Converting a Rectangular Equation to Polar Form In Exercises 59–78, convert the rectangular equation to polar form. Assume $a > 0$.

59. $x^2 + y^2 = 9$
60. $x^2 + y^2 = 16$
61. $y = x$
62. $y = -x$
63. $x = 10$
64. $y = -2$
65. $3x - y + 2 = 0$
66. $3x + 5y - 2 = 0$
67. $xy = 16$
68. $2xy = 1$
69. $x = a$
70. $y = a$
71. $x^2 + y^2 = a^2$
72. $x^2 + y^2 = 9a^2$
73. $x^2 + y^2 - 2ax = 0$
74. $x^2 + y^2 - 2ay = 0$
75. $(x^2 + y^2)^2 = x^2 - y^2$
76. $(x^2 + y^2)^2 = 9(x^2 - y^2)$
77. $y^3 = x^2$
78. $y^2 = x^3$

Converting a Polar Equation to Rectangular Form In Exercises 79–100, convert the polar equation to rectangular form.

79. $r = 5$
80. $r = -7$
81. $\theta = 2\pi/3$
82. $\theta = -5\pi/3$
83. $\theta = \pi/2$
84. $\theta = 3\pi/2$
85. $r = 4 \csc \theta$
86. $r = 2 \csc \theta$
87. $r = -3 \sec \theta$
88. $r = -\sec \theta$
89. $r = -2 \cos \theta$
90. $r = 4 \sin \theta$
91. $r^2 = \cos \theta$
92. $r^2 = 2 \sin \theta$
93. $r^2 = \sin 2\theta$
94. $r^2 = \cos 2\theta$
95. $r = 2 \sin 3\theta$
96. $r = 3 \cos 2\theta$
97. $r = \dfrac{2}{1 + \sin \theta}$
98. $r = \dfrac{1}{1 - \cos \theta}$
99. $r = \dfrac{6}{2 - 3 \sin \theta}$
100. $r = \dfrac{5}{\sin \theta - 4 \cos \theta}$

Converting a Polar Equation to Rectangular Form In Exercises 101–108, describe the graph of the polar equation and find the corresponding rectangular equation.

101. $r = 6$
102. $r = 8$
103. $\theta = \pi/6$
104. $\theta = 3\pi/4$
105. $r = 3 \sec \theta$
106. $r = 2 \csc \theta$
107. $r = 2 \sin \theta$
108. $r = -6 \cos \theta$

109. Ferris Wheel The center of a Ferris wheel lies at the pole of the polar coordinate system, where the distances are in feet. Passengers enter a car at $(30, -\pi/2)$. It takes 45 seconds for the wheel to complete one clockwise revolution.

(a) Write a polar equation that models the possible positions of a passenger car.

(b) Passengers enter a car. Find and interpret their coordinates after 15 seconds of rotation.

(c) Convert the point in part (b) to rectangular coordinates. Interpret the coordinates.

110. Ferris Wheel Repeat Exercise 109 when the distance from a passenger car to the center is 35 feet and it takes 60 seconds to complete one clockwise revolution.

David Lehner/Shutterstock.com

Exploration

True or False? In Exercises 111 and 112, determine whether the statement is true or false. Justify your answer.

111. If $\theta_1 = \theta_2 + 2\pi n$ for some integer n, then (r, θ_1) and (r, θ_2) represent the same point in the polar coordinate system.

112. If $|r_1| = |r_2|$, then (r_1, θ) and (r_2, θ) represent the same point in the polar coordinate system.

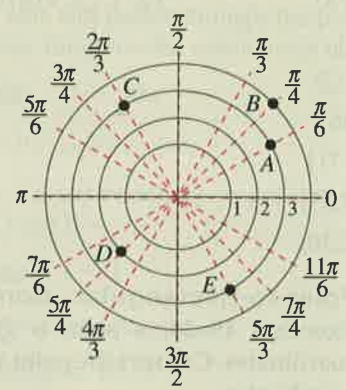
113. Error Analysis Describe the error in converting the rectangular coordinates $(1, -\sqrt{3})$ to polar form.

$$\tan \theta = -\sqrt{3}/1 \implies \theta = \frac{2\pi}{3}$$

$$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$$

$$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$$

114. HOW DO YOU SEE IT? Use the polar coordinate system shown below.



(a) Identify the polar coordinates of points A–E.

(b) Which points lie on the graph of $r = 3$?

(c) Which points lie on the graph of $\theta = \pi/4$?

115. Think About It

(a) Convert the polar equation

$$r = 2(h \cos \theta + k \sin \theta)$$

to rectangular form and verify that it represents a circle.

(b) Use the result of part (a) to convert

$$r = \cos \theta + 3 \sin \theta$$

to rectangular form and find the center and radius of the circle it represents.

10.8 Graphs of Polar Equations

Graphs of polar equations are often useful visual tools in mathematical modeling. For example, in Exercise 69 on page 758, you will use the graph of a polar equation to analyze the pickup pattern of a microphone.

- Graph polar equations by point plotting.
- Use symmetry, zeros, and maximum r-values to sketch graphs of polar equations.
- Recognize special polar graphs.

Introduction

In previous chapters, you sketched graphs in the rectangular coordinate system. You began with the basic point-plotting method. Then you used sketching aids such as symmetry, intercepts, asymptotes, periods, and shifts to further investigate the natures of graphs. This section approaches curve sketching in the polar coordinate system similarly, beginning with a demonstration of point plotting.

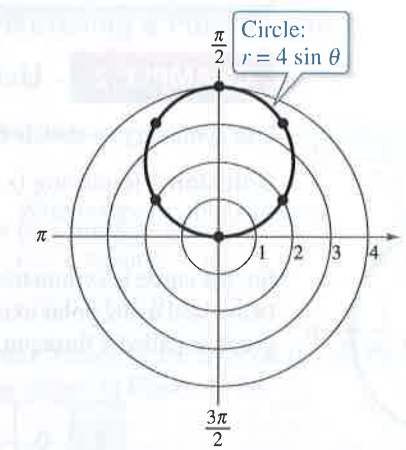
EXAMPLE 1 **Graphing a Polar Equation by Point Plotting**

Sketch the graph of the polar equation $r = 4 \sin \theta$.

Solution The sine function is periodic, so to obtain a full range of r-values, consider values of θ in the interval $0 \le \theta \le 2\pi$, as shown in the table below.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
r	0	2	$2\sqrt{3}$	4	$2\sqrt{3}$	2	0	-2	-4	-2	0

By plotting these points, it appears that the graph is a circle of radius 2 whose center is at the point $(x, y) = (0, 2)$, as shown in the figure below.



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Sketch the graph of the polar equation $r = 6 \cos \theta$.

One way to confirm the graph in Example 1 is to convert the polar equation to rectangular form and then sketch the graph of the rectangular equation. You can also use a graphing utility set to *polar* mode and graph the polar equation, or use a graphing utility set to *parametric* mode and graph a parametric representation.

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