

θ	r
0	3
$\frac{\pi}{6}$	$\frac{3}{2}$
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	$-\frac{3}{2}$

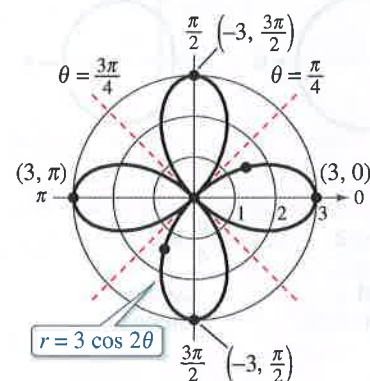


Figure 10.57

θ	$r = \pm 3\sqrt{\sin 2\theta}$
0	0
$\frac{\pi}{12}$	$\pm \frac{3}{\sqrt{2}}$
$\frac{\pi}{4}$	± 3
$\frac{5\pi}{12}$	$\pm \frac{3}{\sqrt{2}}$
$\frac{\pi}{2}$	0

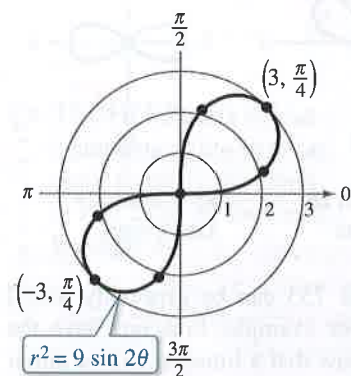


Figure 10.58

EXAMPLE 5 Sketching a Rose CurveSketch the graph of $r = 3 \cos 2\theta$.**Solution***Type of curve:* Rose curve with $2n = 4$ petals*Symmetry:* With respect to the line $\theta = \pi/2$, the polar axis, and the pole*Maximum value of $|r|$:* $|r| = 3$ when $\theta = 0, \pi/2, \pi, 3\pi/2$ *Zeros of r :* $r = 0$ when $\theta = \pi/4, 3\pi/4$

Using this information and plotting the additional points included in the table at the left, you obtain the graph shown in Figure 10.57.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.comSketch the graph of $r = 3 \cos 3\theta$.**EXAMPLE 6** Sketching a LemniscateSketch the graph of $r^2 = 9 \sin 2\theta$.**Solution***Type of curve:* Lemniscate*Symmetry:* With respect to the pole*Maximum value of $|r|$:* $|r| = 3$ when $\theta = \pi/4$ *Zeros of r :* $r = 0$ when $\theta = 0, \pi/2$ When $\sin 2\theta < 0$, this equation has no solution points. So, restrict the values of θ to those for which $\sin 2\theta \geq 0$.

$$0 \leq \theta \leq \frac{\pi}{2} \quad \text{or} \quad \pi \leq \theta \leq \frac{3\pi}{2}$$

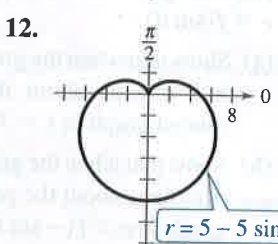
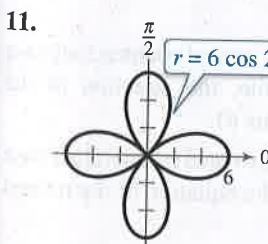
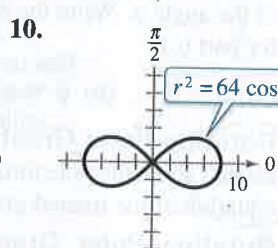
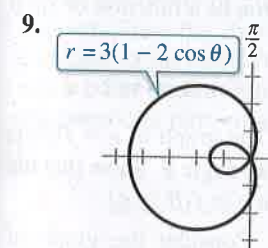
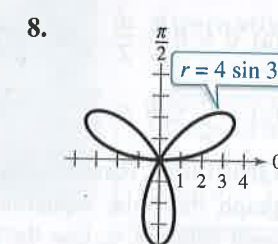
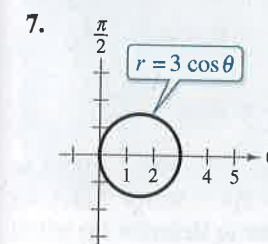
Using symmetry, you need to consider only the first of these two intervals. By finding a few additional points (included in the table at the left), you obtain the graph shown in Figure 10.58.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.comSketch the graph of $r^2 = 4 \cos 2\theta$.**Summarize (Section 10.8)**

1. Explain how to graph a polar equation by point plotting (page 751). For an example of graphing a polar equation by point plotting, see Example 1.
2. State the tests for symmetry in polar coordinates (page 752). For an example of using symmetry to sketch the graph of a polar equation, see Example 2.
3. Explain how to use zeros and maximum r -values to sketch the graph of a polar equation (page 753). For examples of using zeros and maximum r -values to sketch graphs of polar equations, see Examples 3 and 4.
4. State and give examples of the special polar graphs discussed in this lesson (page 755). For examples of sketching special polar graphs, see Examples 5 and 6.

10.8 ExercisesSee CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.**Vocabulary:** Fill in the blanks.

1. The graph of $r = f(\sin \theta)$ is symmetric with respect to the line _____.
2. The graph of $r = g(\cos \theta)$ is symmetric with respect to the _____.
3. The equation $r = 2 + \cos \theta$ represents a _____.
4. The equation $r = 2 \cos \theta$ represents a _____.
5. The equation $r^2 = 4 \sin 2\theta$ represents a _____.
6. The equation $r = 1 + \sin \theta$ represents a _____.

Skills and Applications**Identifying Types of Polar Graphs** In Exercises 7–12, identify the type of polar graph.

Sketching the Graph of a Polar Equation In Exercises 23–48, sketch the graph of the polar equation using symmetry, zeros, maximum r -values, and any other additional points.

23. $r = 5$
24. $r = -8$
25. $r = \pi/4$
26. $r = -2\pi/3$
27. $r = 3 \sin \theta$
28. $r = 4 \cos \theta$
29. $r = 3(1 - \cos \theta)$
30. $r = 4(1 - \sin \theta)$
31. $r = 4(1 + \sin \theta)$
32. $r = 6(1 + \cos \theta)$
33. $r = 5 + 2 \cos \theta$
34. $r = 5 - 2 \sin \theta$
35. $r = 1 - 3 \sin \theta$
36. $r = 2 - 5 \cos \theta$
37. $r = 3 - 6 \cos \theta$
38. $r = 4 + 6 \sin \theta$
39. $r = 5 \sin 2\theta$
40. $r = 2 \cos 2\theta$
41. $r = 6 \cos 3\theta$
42. $r = 3 \sin 3\theta$
43. $r = 2 \sec \theta$
44. $r = 5 \csc \theta$
45. $r = \frac{3}{\sin \theta - 2 \cos \theta}$
46. $r = \frac{6}{2 \sin \theta - 3 \cos \theta}$
47. $r^2 = 9 \cos 2\theta$
48. $r^2 = 16 \sin \theta$

Graphing a Polar Equation In Exercises 49–58, use a graphing utility to graph the polar equation.

49. $r = 9/4$
50. $r = -5/2$
51. $r = 5\pi/8$
52. $r = -\pi/10$
53. $r = 8 \cos \theta$
54. $r = \cos 2\theta$
55. $r = 3(2 - \sin \theta)$
56. $r = 2 \cos(3\theta - 2)$
57. $r = 8 \sin \theta \cos^2 \theta$
58. $r = 2 \csc \theta + 5$

Finding an Interval In Exercises 59–64, use a graphing utility to graph the polar equation. Find an interval for θ for which the graph is traced only once.

59. $r = 3 - 8 \cos \theta$
60. $r = 5 + 4 \cos \theta$
61. $r = 2 \cos(3\theta/2)$
62. $r = 3 \sin(5\theta/2)$
63. $r^2 = 16 \sin 2\theta$
64. $r^2 = 1/\theta$

Testing for Symmetry In Exercises 13–18, test for symmetry with respect to the line $\theta = \pi/2$, the polar axis, and the pole.

13. $r = 6 + 3 \cos \theta$
14. $r = 9 \cos 3\theta$
15. $r = \frac{2}{1 + \sin \theta}$
16. $r = \frac{3}{2 + \cos \theta}$
17. $r^2 = 36 \cos 2\theta$
18. $r^2 = 25 \sin 2\theta$

Finding the Maximum Value of $|r|$ and Zeros of r In Exercises 19–22, find the maximum value of $|r|$ and any zeros of r .

19. $r = 10 - 10 \sin \theta$
20. $r = 6 + 12 \cos \theta$
21. $r = 4 \cos 3\theta$
22. $r = 3 \sin 2\theta$

Asymptote of a Graph of a Polar Equation In Exercises 65–68, use a graphing utility to graph the polar equation and show that the given line is an asymptote of the graph.

Name of Graph	Polar Equation	Asymptote
65. Conchoid	$r = 2 - \sec \theta$	$x = -1$
66. Conchoid	$r = 2 + \csc \theta$	$y = 1$
67. Hyperbolic spiral	$r = \frac{3}{\theta}$	$y = 3$
68. Strophoid	$r = 2 \cos 2\theta \sec \theta$	$x = -2$

69. Microphone

The pickup pattern of a microphone is modeled by the polar equation

$$r = 5 + 5 \cos \theta$$

where $|r|$ measures how sensitive the microphone is to sounds coming from the angle θ .



- Sketch the graph of the model and identify the type of polar graph.
- At what angle is the microphone most sensitive to sound?

70. Area The total area of the region bounded by the lemniscate $r^2 = a^2 \cos 2\theta$ is a^2 .

- Sketch the graph of $r^2 = 16 \cos 2\theta$.
- Find the area of one loop of the graph from part (a).

Exploration

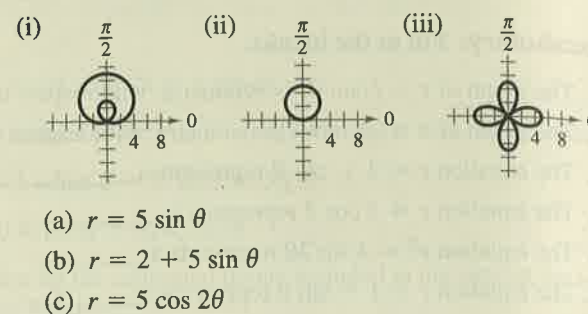
True or False? In Exercises 71 and 72, determine whether the statement is true or false. Justify your answer.

- The graph of $r = 10 \sin 5\theta$ is a rose curve with five petals.
- A rose curve is always symmetric with respect to the line $\theta = \pi/2$.

73. Graphing a Polar Equation Consider the equation $r = 3 \sin k\theta$.

- Use a graphing utility to graph the equation for $k = 1.5$. Find the interval for θ over which the graph is traced only once.
- Use the graphing utility to graph the equation for $k = 2.5$. Find the interval for θ over which the graph is traced only once.
- Is it possible to find an interval for θ over which the graph is traced only once for any rational number k ? Explain.

74. HOW DO YOU SEE IT? Match each polar equation with its graph.



75. Sketching the Graph of a Polar Equation Sketch the graph of $r = 10 \cos \theta$ over each interval. Describe the part of the graph obtained in each case.

- $0 \leq \theta \leq \frac{\pi}{2}$
- $\frac{\pi}{2} \leq \theta \leq \pi$
- $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
- $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

76. Graphical Reasoning Use a graphing utility to graph the polar equation $r = 6[1 + \cos(\theta - \phi)]$ for each value of ϕ . Use the graphs to describe the effect of the angle ϕ . Write the equation as a function of $\sin \theta$ for part (c).

- $\phi = 0$
- $\phi = \pi/4$
- $\phi = \pi/2$

77. Rotating Polar Graphs The graph of $r = f(\theta)$ is rotated about the pole through an angle ϕ . Show that the equation of the rotated graph is $r = f(\theta - \phi)$.

78. Rotating Polar Graphs Consider the graph of $r = f(\sin \theta)$.

- Show that when the graph is rotated counterclockwise $\pi/2$ radians about the pole, the equation of the rotated graph is $r = f(-\cos \theta)$.
- Show that when the graph is rotated counterclockwise π radians about the pole, the equation of the rotated graph is $r = f(-\sin \theta)$.
- Show that when the graph is rotated counterclockwise $3\pi/2$ radians about the pole, the equation of the rotated graph is $r = f(\cos \theta)$.

Rotating Polar Graphs In Exercises 79 and 80, use the results of Exercises 77 and 78.

79. Write an equation for the limaçon $r = 2 - \sin \theta$ after it is rotated through each angle.

- $\frac{\pi}{4}$
- $\frac{\pi}{2}$
- π
- $\frac{3\pi}{2}$

80. Write an equation for the rose curve $r = 2 \sin 2\theta$ after it is rotated through each angle.

- $\frac{\pi}{6}$
- $\frac{\pi}{2}$
- $\frac{2\pi}{3}$
- π

10.9 Polar Equations of Conics



Polar equation of conics can model the orbits of planets and satellites. For example, in Exercise 62 on page 764, you will use a polar equation to model the parabolic path of a satellite.

- Define conics in terms of eccentricity, and write and graph polar equations of conics.
- Use equations of conics in polar form to model real-life problems.

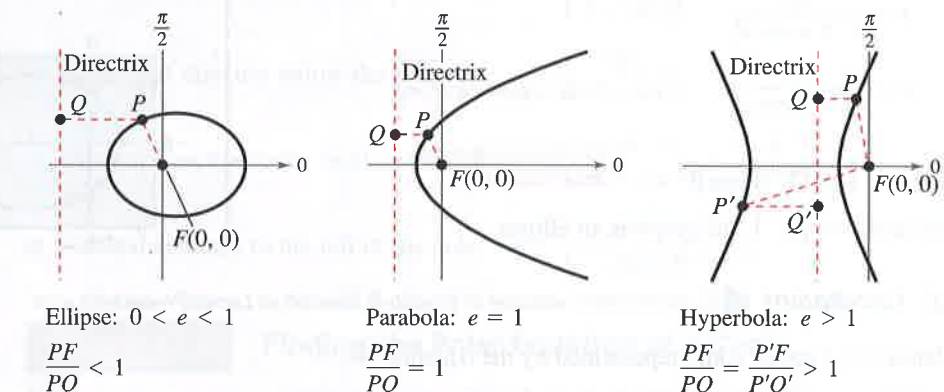
Alternative Definition and Polar Equations of Conics

In Sections 10.3 and 10.4, you learned that the rectangular equations of ellipses and hyperbolas take simpler forms when the origin lies at their *centers*. There are many important applications of conics in which it is more convenient to use a *focus* as the origin. In these cases, it is convenient to use polar coordinates.

To begin, consider an alternative definition of a conic that uses the concept of *eccentricity*.

Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic**. The constant ratio is the *eccentricity* of the conic and is denoted by e . Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)



In the figures, note that for each type of conic, a focus is at the pole. The benefit of locating a focus of a conic at the pole is that the equation of the conic takes on a simpler form.

Polar Equations of Conics

The graph of a polar equation of the form

$$1. r = \frac{ep}{1 \pm e \cos \theta} \quad \text{or} \quad 2. r = \frac{ep}{1 \pm e \sin \theta}$$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

For a proof of the polar equations of conics, see Proofs in Mathematics on page 774.