2.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

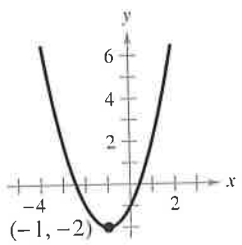
Vocabulary: Fill in the blanks.

1. Linear, constant, and squaring functions are examples of _______ functions.
2. A polynomial function of x with degree n has the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1x + a_0$ ($a_n \neq 0$), where n is a _______ _______ and $a_n, a_{n-1}, \ldots, a_1, a_0$ are _______ numbers.
3. A _______ function is a second-degree polynomial function, and its graph is called a _______.
4. When the graph of a quadratic function opens downward, its leading coefficient is _______ and the vertex of the graph is a _______.

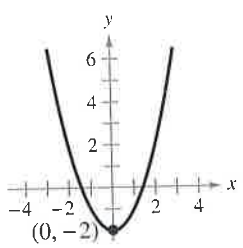
Skills and Applications

Matching **In Exercises 5–8, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

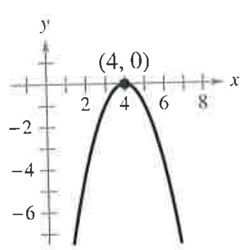
(a)



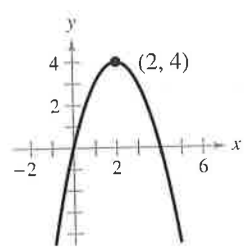
(b)



(c)



(d)



5. $f(x) = x^2 - 2$
6. $f(x) = (x + 1)^2 - 2$
7. $f(x) = -(x - 4)^2$
8. $f(x) = 4 - (x - 2)^2$

Sketching Graphs of Quadratic Functions **In Exercises 9–12, sketch the graph of each quadratic function and compare it with the graph of $y = x^2$.**

9. (a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = -\frac{1}{8}x^2$
 (c) $h(x) = \frac{3}{2}x^2$ (d) $k(x) = -3x^2$
10. (a) $f(x) = x^2 + 1$ (b) $g(x) = x^2 - 1$
 (c) $h(x) = x^2 + 3$ (d) $k(x) = x^2 - 3$
11. (a) $f(x) = (x - 1)^2$ (b) $g(x) = (3x)^2 + 1$
 (c) $h(x) = \left(\frac{1}{3}x\right)^2 - 3$ (d) $k(x) = (x + 3)^2$
12. (a) $f(x) = -\frac{1}{2}(x - 2)^2 + 1$
 (b) $g(x) = \left[\frac{1}{2}(x - 1)\right]^2 - 3$
 (c) $h(x) = -\frac{1}{2}(x + 2)^2 - 1$
 (d) $k(x) = [2(x + 1)]^2 + 4$

Using Standard Form to Graph a Parabola **In Exercises 13–26, write the quadratic function in standard form and sketch its graph. Identify the vertex, axis of symmetry, and x-intercept(s).**

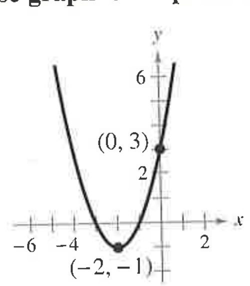
13. $f(x) = x^2 - 6x$
14. $g(x) = x^2 - 8x$
15. $h(x) = x^2 - 8x + 16$
16. $g(x) = x^2 + 2x + 1$
17. $f(x) = x^2 - 6x + 2$
18. $f(x) = x^2 + 16x + 61$
19. $f(x) = x^2 - 8x + 21$
20. $f(x) = x^2 + 12x + 40$
21. $f(x) = x^2 - x + \frac{5}{4}$
22. $f(x) = x^2 + 3x + \frac{1}{4}$
23. $f(x) = -x^2 + 2x + 5$
24. $f(x) = -x^2 - 4x + 1$
25. $h(x) = 4x^2 - 4x + 21$
26. $f(x) = 2x^2 - x + 1$

Using Technology **In Exercises 27–34, use a graphing utility to graph the quadratic function. Identify the vertex, axis of symmetry, and x-intercept(s). Then check your results algebraically by writing the quadratic function in standard form.**

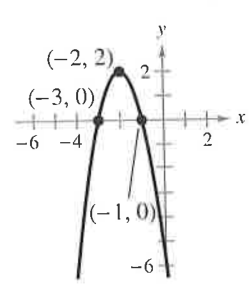
27. $f(x) = -(x^2 + 2x - 3)$
28. $f(x) = -(x^2 + x - 30)$
29. $g(x) = x^2 + 8x + 11$
30. $f(x) = x^2 + 10x + 14$
31. $f(x) = -2x^2 + 12x - 18$
32. $f(x) = -4x^2 + 24x - 41$
33. $g(x) = \frac{1}{2}(x^2 + 4x - 2)$
34. $f(x) = \frac{3}{5}(x^2 + 6x - 5)$

Writing a Quadratic Function **In Exercises 35 and 36, write the standard form of the quadratic function whose graph is the parabola shown.**

35.



36.

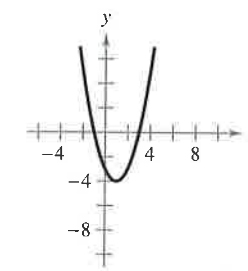


Writing a Quadratic Function **In Exercises 37–46, write the standard form of the quadratic function whose graph is a parabola with the given vertex and that passes through the given point.**

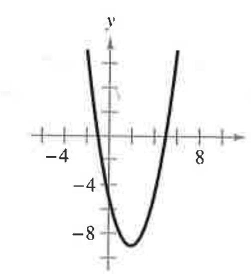
37. Vertex: $(-2, 5)$; point: $(0, 9)$
38. Vertex: $(-3, -10)$; point: $(0, 8)$
39. Vertex: $(1, -2)$; point: $(-1, 14)$
40. Vertex: $(2, 3)$; point: $(0, 2)$
41. Vertex: $(5, 12)$; point: $(7, 15)$
42. Vertex: $(-2, -2)$; point: $(-1, 0)$
43. Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right)$; point: $(-2, 0)$
44. Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; point: $(-2, 4)$
45. Vertex: $\left(-\frac{5}{2}, 0\right)$; point: $\left(-\frac{7}{2}, -\frac{16}{3}\right)$
46. Vertex: $(6, 6)$; point: $\left(\frac{61}{10}, \frac{3}{2}\right)$

Graphical Reasoning **In Exercises 47–50, determine the x-intercept(s) of the graph visually. Then find the x-intercept(s) algebraically to confirm your results.**

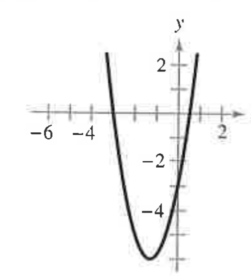
47. $y = x^2 - 2x - 3$



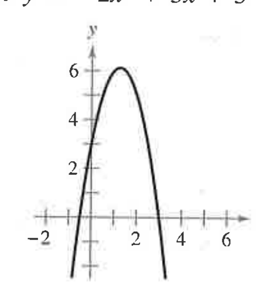
48. $y = x^2 - 4x - 5$



49. $y = 2x^2 + 5x - 3$



50. $y = -2x^2 + 5x + 3$



Using Technology **In Exercises 51–56, use a graphing utility to graph the quadratic function. Find the x-intercept(s) of the graph and compare them with the solutions of the corresponding quadratic equation when $f(x) = 0$.**

51. $f(x) = x^2 - 4x$
52. $f(x) = -2x^2 + 10x$
53. $f(x) = x^2 - 9x + 18$
54. $f(x) = x^2 - 8x - 20$
55. $f(x) = 2x^2 - 7x - 30$
56. $f(x) = \frac{7}{10}(x^2 + 12x - 45)$

Finding Quadratic Functions **In Exercises 57–62, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given x-intercepts. (There are many correct answers.)**

57. $(-3, 0), (3, 0)$
58. $(-5, 0), (5, 0)$
59. $(-1, 0), (4, 0)$
60. $(-2, 0), (3, 0)$
61. $(-3, 0), \left(-\frac{1}{2}, 0\right)$
62. $\left(-\frac{3}{2}, 0\right), (-5, 0)$

Number Problems **In Exercises 63–66, find two positive real numbers whose product is a maximum.**

63. The sum is 110.
64. The sum is S.
65. The sum of the first and twice the second is 24.
66. The sum of the first and three times the second is 42.

67. **Path of a Diver**
The path of a diver is modeled by

$$f(x) = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where $f(x)$ is the height (in feet) and x is the horizontal distance (in feet) from the end of the diving board. What is the maximum height of the diver?

68. **Height of a Ball** The path of a punted football is modeled by

$$f(x) = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$$

where $f(x)$ is the height (in feet) and x is the horizontal distance (in feet) from the point at which the ball is punted.
(a) How high is the ball when it is punted?
(b) What is the maximum height of the punt?
(c) How long is the punt?

69. **Minimum Cost** A manufacturer of lighting fixtures has daily production costs of $C = 800 - 10x + 0.25x^2$, where C is the total cost (in dollars) and x is the number of units produced. What daily production number yields a minimum cost?

70. **Maximum Profit** The profit P (in hundreds of dollars) that a company makes depends on the amount x (in hundreds of dollars) the company spends on advertising according to the model $P = 230 + 20x - 0.5x^2$. What expenditure for advertising yields a maximum profit?

71. Maximum Revenue The total revenue R earned (in thousands of dollars) from manufacturing handheld video games is given by $R(p) = -25p^2 + 1200p$, where p is the price per unit (in dollars).

(a) Find the revenues when the prices per unit are \$20, \$25, and \$30.

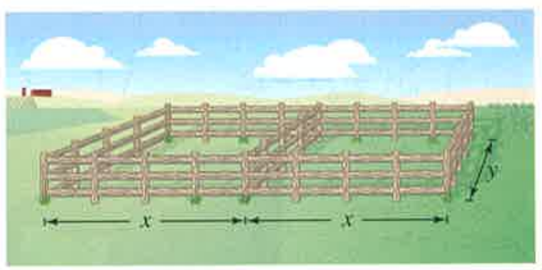
(b) Find the unit price that yields a maximum revenue. What is the maximum revenue? Explain.

72. Maximum Revenue The total revenue R earned per day (in dollars) from a pet-sitting service is given by $R(p) = -12p^2 + 150p$, where p is the price charged per pet (in dollars).

(a) Find the revenues when the prices per pet are \$4, \$6, and \$8.

(b) Find the unit price that yields a maximum revenue. What is the maximum revenue? Explain.

73. Maximum Area A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure).



(a) Write the area A of the corrals as a function of x.

(b) What dimensions produce a maximum enclosed area?

74. Maximum Area A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). The perimeter of the window is 16 feet.

(a) Write the area A of the window as a function of x.

(b) What dimensions produce a window of maximum area?

Exploration

True or False? **In Exercises 75 and 76, determine whether the statement is true or false. Justify your answer.**

75. The graph of $f(x) = -12x^2 - 1$ has no x-intercepts.

76. The graphs of $f(x) = -4x^2 - 10x + 7$ and $g(x) = 12x^2 + 30x + 1$ have the same axis of symmetry.

Think About It **In Exercises 77 and 78, find the values of b such that the function has the given maximum or minimum value.**

77. $f(x) = -x^2 + bx - 75$; Maximum value: 25

78. $f(x) = x^2 + bx - 25$; Minimum value: -50

79. Verifying the Vertex Write the quadratic function

$$f(x) = ax^2 + bx + c$$

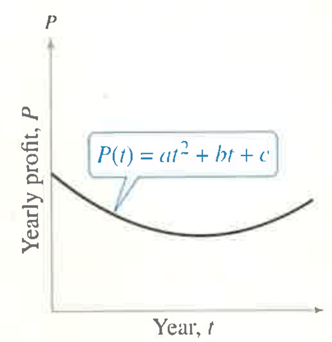
in standard form to verify that the vertex occurs at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

80. HOW DO YOU SEE IT? The graph shows a quadratic function of the form

$$P(t) = at^2 + bt + c$$

which represents the yearly profit for a company, where $P(t)$ is the profit in year t.



(a) Is the value of a positive, negative, or zero? Explain.

(b) Write an expression in terms of a and b that represents the year t when the company made the least profit.

(c) The company made the same yearly profits in 2008 and 2016. Estimate the year in which the company made the least profit.

81. Proof Assume that the function

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

has two real zeros. Prove that the x-coordinate of the vertex of the graph is the average of the zeros of f. (*Hint:* Use the Quadratic Formula.)

Project: Height of a Basketball To work an extended application analyzing the height of a dropped basketball, visit this text's website at *LarsonPrecalculus.com.*

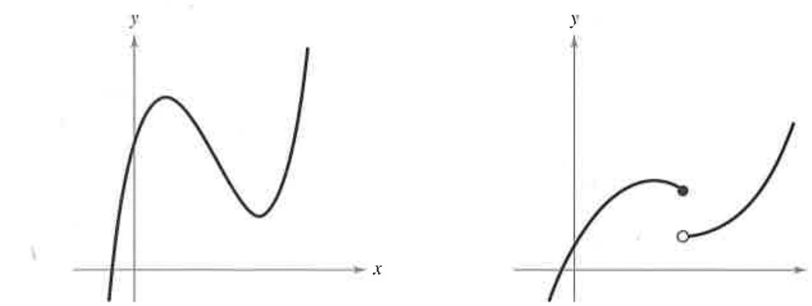
2.2 Polynomial Functions of Higher Degree

- Use transformations to sketch graphs of polynomial functions.
- Use the Leading Coefficient Test to determine the end behaviors of graphs of polynomial functions.
- Find real zeros of polynomial functions and use them as sketching aids.
- Use the Intermediate Value Theorem to help locate real zeros of polynomial functions.

Polynomial functions have many real-life applications. For example, in Exercise 98 on page 135, you will use a polynomial function to analyze the growth of a red oak tree.

Graphs of Polynomial Functions

In this section, you will study basic features of the graphs of polynomial functions. One feature is that the graph of a polynomial function is **continuous.** Essentially, this means that the graph of a polynomial function has no breaks, holes, or gaps, as shown in Figure 2.5(a). The graph shown in Figure 2.5(b) is an example of a piecewise-defined function that is not continuous.

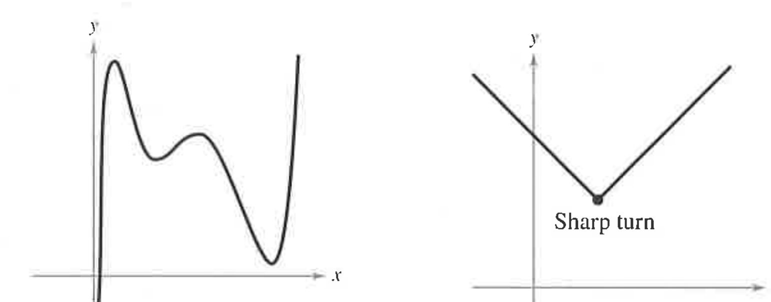


(a) Polynomial functions have continuous graphs.

(b) Functions with graphs that are not continuous are not polynomial functions.

Figure 2.5

Another feature of the graph of a polynomial function is that it has only smooth, rounded turns, as shown in Figure 2.6(a). The graph of a polynomial function cannot have a sharp turn, such as the one shown in Figure 2.6(b).



(a) Polynomial functions have graphs with smooth, rounded turns.

(b) Functions with graphs that have sharp turns are not polynomial functions.

Figure 2.6

Sketching graphs of polynomial functions of degree greater than 2 is often more involved than sketching graphs of polynomial functions of degree 0, 1, or 2. However, using the features presented in this section, along with your knowledge of point plotting, intercepts, and symmetry, you should be able to make reasonably accurate sketches by hand.

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