2.2 Exercises

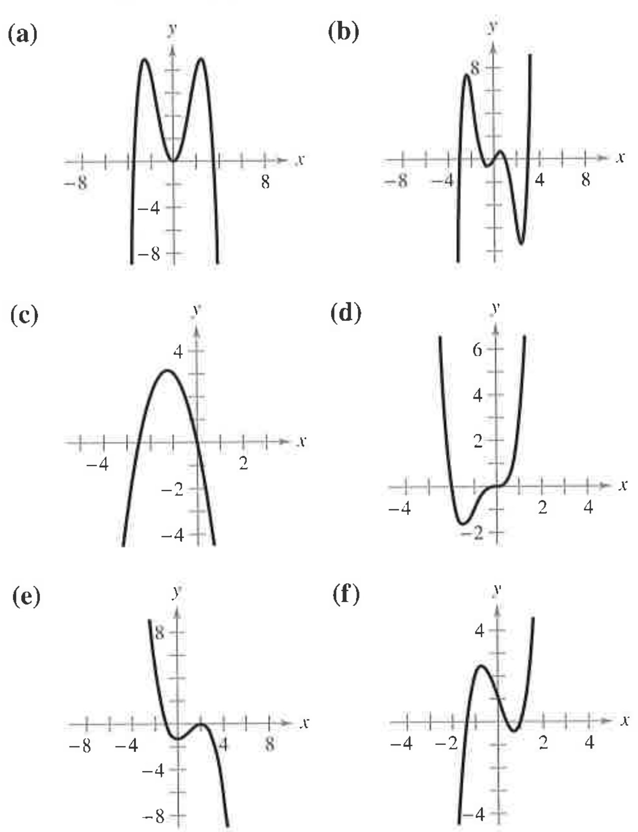
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: **Fill in the blanks.**

1. The graph of a polynomial function is ________, which means that the graph has no breaks, holes, or gaps.
2. The ________ ________ ________ is used to determine the left-hand and right-hand behavior of the graph of a polynomial function.
3. A polynomial function of degree n has at most ________ real zeros and at most ________ turning points.
4. When $x = a$ is a zero of a polynomial function f, the three statements below are true.
   (a) $x = a$ is a ________ of the polynomial equation $f(x) = 0$.
   (b) ________ is a factor of the polynomial $f(x)$.
   (c) $(a, 0)$ is an ________ of the graph of f.
5. When a real zero $x = a$ of a polynomial function f is of even multiplicity, the graph of f ________ the x-axis at $x = a$, and when it is of odd multiplicity, the graph of f ________ the x-axis at $x = a$.
6. A factor $(x - a)^k$, $k > 1$, yields a ________ ________ $x = a$ of ________ k.
7. A polynomial function is written in ________ form when its terms are written in descending order of exponents from left to right.
8. The ________ ________ Theorem states that if f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

Skills and Applications

Matching **In Exercises 9–14, match the polynomial function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]**



9. $f(x) = -2x^2 - 5x$
10. $f(x) = 2x^3 - 3x + 1$
11. $f(x) = -\frac{1}{4}x^4 + 3x^2$
12. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$
13. $f(x) = x^4 + 2x^3$
14. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

Sketching Transformations of Monomial Functions **In Exercises 15–18, sketch the graph of $y = x^n$ and each transformation.**

15. $y = x^3$
    (a) $f(x) = (x - 4)^3$ (b) $f(x) = x^3 - 4$
    (c) $f(x) = -\frac{1}{4}x^3$ (d) $f(x) = (x - 4)^3 - 4$
16. $y = x^5$
    (a) $f(x) = (x + 1)^5$ (b) $f(x) = x^5 + 1$
    (c) $f(x) = 1 - \frac{1}{2}x^5$ (d) $f(x) = -\frac{1}{2}(x + 1)^5$
17. $y = x^4$
    (a) $f(x) = (x + 3)^4$ (b) $f(x) = x^4 - 3$
    (c) $f(x) = 4 - x^4$ (d) $f(x) = \frac{1}{2}(x - 1)^4$
    (e) $f(x) = (2x)^4 + 1$ (f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$
18. $y = x^6$
    (a) $f(x) = (x - 5)^6$ (b) $f(x) = \frac{1}{8}x^6$
    (c) $f(x) = (x + 3)^6 - 4$ (d) $f(x) = -\frac{1}{4}x^6 + 1$
    (e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$ (f) $f(x) = (2x)^6 - 1$

Applying the Leading Coefficient Test **In Exercises 19–28, describe the left-hand and right-hand behavior of the graph of the polynomial function.**

19. $f(x) = 12x^3 + 4x$
20. $f(x) = 2x^2 - 3x + 1$
21. $g(x) = 5 - \frac{7}{2}x - 3x^2$
22. $h(x) = 1 - x^6$
23. $h(x) = 6x - 9x^3 + x^2$
24. $g(x) = 8 + \frac{1}{4}x^5 - x^4$
25. $f(x) = 9.8x^6 - 1.2x^3$
26. $h(x) = 1 - 0.5x^5 - 2.7x^3$
27. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$
28. $h(t) = -\frac{4}{3}(t - 6t^3 + 2t^4 + 9)$

Using Technology **In Exercises 29–32, use a graphing utility to graph the functions f and g in the same viewing window. Zoom out sufficiently far to show that the left-hand and right-hand behaviors of f and g appear identical.**

29. $f(x) = 3x^3 - 9x + 1$, $g(x) = 3x^3$
30. $f(x) = -\frac{1}{3}(x^3 - 3x + 2)$, $g(x) = -\frac{1}{3}x^3$
31. $f(x) = -(x^4 - 4x^3 + 16x)$, $g(x) = -x^4$
32. $f(x) = 3x^4 - 6x^2$, $g(x) = 3x^4$

Finding Real Zeros of a Polynomial Function **In Exercises 33–48, (a) find all real zeros of the polynomial function, (b) determine whether the multiplicity of each zero is even or odd, (c) determine the maximum possible number of turning points of the graph of the function, and (d) use a graphing utility to graph the function and verify your answers.**

33. $f(x) = x^2 - 36$
34. $f(x) = 81 - x^2$
35. $h(t) = t^2 - 6t + 9$
36. $f(x) = x^2 + 10x + 25$
37. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$
38. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$
39. $g(x) = 5x(x^2 - 2x - 1)$
40. $f(t) = t^2(3t^2 - 10t + 7)$
41. $f(x) = 3x^3 - 12x^2 + 3x$
42. $f(x) = x^4 - x^3 - 30x^2$
43. $g(t) = t^5 - 6t^3 + 9t$
44. $f(x) = x^5 + x^3 - 6x$
45. $f(x) = 3x^4 + 9x^2 + 6$
46. $f(t) = 2t^4 - 2t^2 - 40$
47. $g(x) = x^3 + 3x^2 - 4x - 12$
48. $f(x) = x^3 - 4x^2 - 25x + 100$

Using Technology **In Exercises 49–52, (a) use a graphing utility to graph the function, (b) use the graph to approximate any x-intercepts of the graph, (c) find any real zeros of the function algebraically, and (d) compare the results of part (c) with those of part (b).**

49. $y = 4x^3 - 20x^2 + 25x$
50. $y = 4x^3 + 4x^2 - 8x - 8$
51. $y = x^5 - 5x^3 + 4x$
52. $y = \frac{1}{5}x^5 - \frac{9}{5}x^3$

Finding a Polynomial Function **In Exercises 53–62, find a polynomial function that has the given zeros. (There are many correct answers.)**

53. 0, 7
54. −2, 5
55. 0, −2, −4
56. 0, 1, 6
57. 4, −3, 3, 0
58. −2, −1, 0, 1, 2
59. $1 + \sqrt{2}, 1 - \sqrt{2}$
60. $4 + \sqrt{3}, 4 - \sqrt{3}$
61. $2, 2 + \sqrt{5}, 2 - \sqrt{5}$
62. $3, 2 + \sqrt{7}, 2 - \sqrt{7}$

Finding a Polynomial Function **In Exercises 63–70, find a polynomial of degree n that has the given zero(s). (There are many correct answers.)**

	Zero(s)	Degree
63.	$x = -3$	$n = 2$
64.	$x = -\sqrt{2}, \sqrt{2}$	$n = 2$
65.	$x = -5, 0, 1$	$n = 3$
66.	$x = -2, 6$	$n = 3$
67.	$x = -5, 1, 2$	$n = 4$
68.	$x = -4, -1$	$n = 4$
69.	$x = 0, -\sqrt{3}, \sqrt{3}$	$n = 5$
70.	$x = -1, 4, 7, 8$	$n = 5$

Sketching the Graph of a Polynomial Function **In Exercises 71–84, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the real zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.**

71. $f(t) = \frac{1}{4}(t^2 - 2t + 15)$
72. $g(x) = -x^2 + 10x - 16$
73. $f(x) = x^3 - 25x$
74. $g(x) = -9x^2 + x^4$
75. $f(x) = -8 + \frac{1}{2}x^4$
76. $f(x) = 8 - x^3$
77. $f(x) = 3x^3 - 15x^2 + 18x$
78. $f(x) = -4x^3 + 4x^2 + 15x$
79. $f(x) = -5x^2 - x^3$
80. $f(x) = -48x^2 + 3x^4$
81. $f(x) = 9x^2(x + 2)^3$
82. $h(x) = \frac{1}{3}x^3(x - 4)^2$
83. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$
84. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

Using Technology **In Exercises 85–88, use a graphing utility to graph the function. Use the *zero* or *root* feature to approximate the real zeros of the function. Then determine whether the multiplicity of each zero is even or odd.**

85. $f(x) = x^3 - 16x$
86. $f(x) = \frac{1}{4}x^4 - 2x^2$
87. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$
88. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$

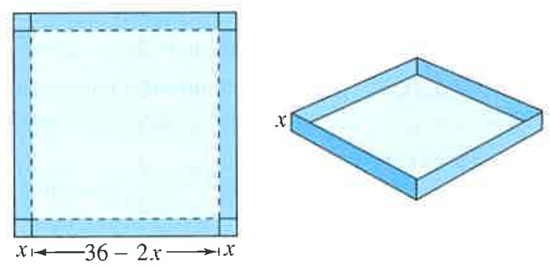
Using the Intermediate Value Theorem **In Exercises 89–92, (a) use the Intermediate Value Theorem and the *table* feature of a graphing utility to find intervals one unit in length in which the polynomial function is guaranteed to have a zero. (b) Adjust the table to approximate the zeros of the function to the nearest thousandth.**

89. $f(x) = x^3 - 3x^2 + 3$

90. $f(x) = 0.11x^3 - 2.07x^2 + 9.81x - 6.88$

91. $g(x) = 3x^4 + 4x^3 - 3$ 92. $h(x) = x^4 - 10x^2 + 3$

93. **Maximum Volume** You construct an open box from a square piece of material, 36 inches on a side, by cutting equal squares with sides of length x from the corners and turning up the sides (see figure).



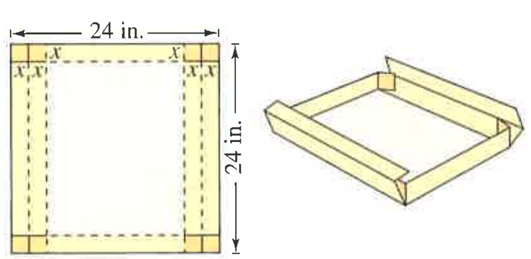
(a) Write a function V that represents the volume of the box.

(b) Determine the domain of the function V.

(c) Use a graphing utility to construct a table that shows the box heights x and the corresponding volumes $V(x)$. Use the table to estimate the dimensions that produce a maximum volume.

(d) Use the graphing utility to graph V and use the graph to estimate the value of x for which $V(x)$ is a maximum. Compare your result with that of part (c).

94. **Maximum Volume** You construct an open box with locking tabs from a square piece of material, 24 inches on a side, by cutting equal sections from the corners and folding along the dashed lines (see figure).



(a) Write a function V that represents the volume of the box.

(b) Determine the domain of the function V.

(c) Sketch a graph of the function and estimate the value of x for which $V(x)$ is a maximum.

95. **Revenue** The revenue R (in millions of dollars) for a software company from 2003 through 2016 can be modeled by

$$R = 6.212t^3 - 152.87t^2 + 990.2t - 414,\quad 3 \le t \le 16$$

where t represents the year, with $t = 3$ corresponding to 2003.

(a) Use a graphing utility to approximate any relative minima or maxima of the model over its domain.

(b) Use the graphing utility to approximate the intervals on which the revenue for the company is increasing and decreasing over its domain.

(c) Use the results of parts (a) and (b) to describe the company's revenue during this time period.

96. **Revenue** The revenue R (in millions of dollars) for a construction company from 2003 through 2010 can be modeled by

$$R = 0.1104t^4 - 5.152t^3 + 88.20t^2 - 654.8t + 1907,\quad 7 \le t \le 16$$

where t represents the year, with $t = 7$ corresponding to 2007.

(a) Use a graphing utility to approximate any relative minima or maxima of the model over its domain.

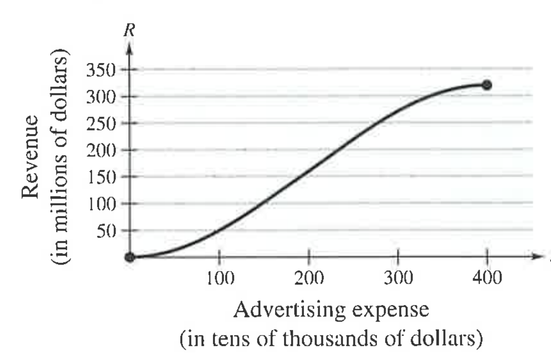
(b) Use the graphing utility to approximate the intervals on which the revenue for the company is increasing and decreasing over its domain.

(c) Use the results of parts (a) and (b) to describe the company's revenue during this time period.

97. **Revenue** The revenue R (in millions of dollars) for a beverage company is related to its advertising expense by the function

$$R = \frac{1}{100{,}000}(-x^3 + 600x^2),\quad 0 \le x \le 400$$

where x is the amount spent on advertising (in tens of thousands of dollars). Use the graph of this function to estimate the point on the graph at which the function is increasing most rapidly. This point is called the *point of diminishing returns* because any expense above this amount will yield less return per dollar invested in advertising.



98. **Arboriculture** The growth of a red oak tree is approximated by the function

$$G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839,\quad 2 \le t \le 34$$

where G is the height of the tree (in feet) and t is its age (in years).

(a) Use a graphing utility to graph the function.

(b) Estimate the age of the tree when it is growing most rapidly. This point is called the *point of diminishing returns* because the increase in size will be less with each additional year.

(c) Using calculus, the point of diminishing returns can be found by finding the vertex of the parabola

$$y = -0.009t^2 + 0.274t + 0.458.$$

Find the vertex of this parabola.

(d) Compare your results from parts (b) and (c).

Exploration

True or False? **In Exercises 99–102, determine whether the statement is true or false. Justify your answer.**

99. If the graph of a polynomial function falls to the right, then its leading coefficient is negative.

100. A fifth-degree polynomial function can have five turning points in its graph.

101. It is possible for a polynomial with an even degree to have a range of $(-\infty, \infty)$.

102. If f is a polynomial function of x such that $f(2) = -6$ and $f(6) = 6$, then f has at most one real zero between $x = 2$ and $x = 6$.

103. **Modeling Polynomials** Sketch the graph of a fourth-degree polynomial function that has a zero of multiplicity 2 and a negative leading coefficient. Sketch the graph of another polynomial function with the same characteristics except that the leading coefficient is positive.

104. **Modeling Polynomials** Sketch the graph of a fifth-degree polynomial function that has a zero of multiplicity 2 and a negative leading coefficient. Sketch the graph of another polynomial function with the same characteristics except that the leading coefficient is positive.

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105. **Graphical Reasoning** Sketch the graph of the function $f(x) = x^4$. Explain how the graph of each function g differs (if it does) from the graph of f. Determine whether g is even, odd, or neither.

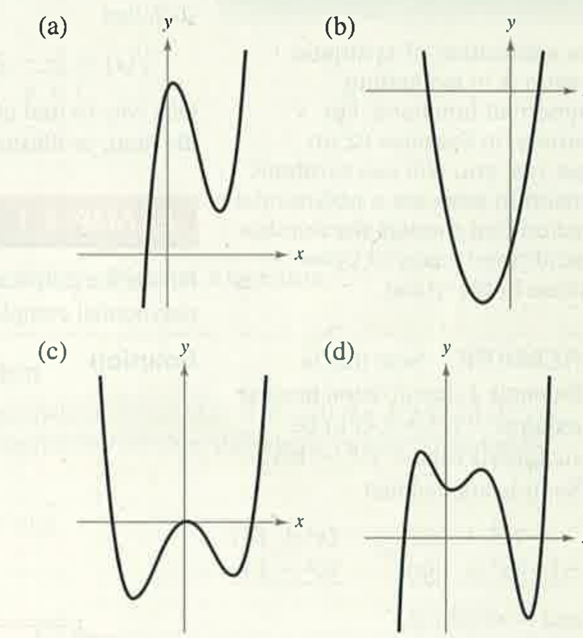
(a) $g(x) = f(x) + 2$ (b) $g(x) = f(x + 2)$

(c) $g(x) = f(-x)$ (d) $g(x) = -f(x)$

(e) $g(x) = f(\frac{1}{2}x)$ (f) $g(x) = \frac{1}{2}f(x)$

(g) $g(x) = f(x^{3/4})$ (h) $g(x) = (f \circ f)(x)$

106. **HOW DO YOU SEE IT?** For each graph, describe a polynomial function that could represent the graph. (Indicate the degree of the function and the sign of its leading coefficient.)



107. **Think About It** Use a graphing utility to graph the functions

$$y_1 = -\tfrac{1}{3}(x - 2)^5 + 1 \quad\text{and}\quad y_2 = \tfrac{3}{5}(x + 2)^5 - 3.$$

(a) Determine whether the graphs of y_1 and y_2 are increasing or decreasing. Explain.

(b) Will the graph of

$$g(x) = a(x - h)^5 + k$$

always be strictly increasing or strictly decreasing? If so, is this behavior determined by a, h, or k? Explain.

(c) Use a graphing utility to graph

$$f(x) = x^5 - 3x^2 + 2x + 1.$$

Use a graph and the result of part (b) to determine whether f can be written in the form $f(x) = a(x - h)^5 + k$. Explain.