2.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary

1. Two forms of the Division Algorithm are shown below. Identify and label each term or function.

$$f(x) = d(x)q(x) + r(x) \qquad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

In Exercises 2–6, fill in the blanks.

2. In the Division Algorithm, the rational expression $r(x)/d(x)$ is ________ because the degree of $r(x)$ is less than the degree of $d(x)$.
3. In the Division Algorithm, the rational expression $f(x)/d(x)$ is ________ because the degree of $f(x)$ is greater than or equal to the degree of $d(x)$.
4. A shortcut for long division of polynomials is ________ ________, in which the divisor must be of the form $x - k$.
5. The ________ Theorem states that a polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.
6. The ________ Theorem states that if a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Skills and Applications

Using the Division Algorithm **In Exercises 7 and 8, use long division to verify that $y_1 = y_2$.**

7. $y_1 = \dfrac{x^2}{x+2}, \quad y_2 = x - 2 + \dfrac{4}{x+2}$
8. $y_1 = \dfrac{x^3 - 3x^2 + 4x - 1}{x+3}, \quad y_2 = x^2 - 6x + 22 - \dfrac{67}{x+3}$

Using Technology **In Exercises 9 and 10, (a) use a graphing utility to graph the two equations in the same viewing window, (b) use the graphs to verify that the expressions are equivalent, and (c) use long division to verify the results algebraically.**

9. $y_1 = \dfrac{x^2 + 2x - 1}{x+3}, \quad y_2 = x - 1 + \dfrac{2}{x+3}$
10. $y_1 = \dfrac{x^4 + x^2 - 1}{x^2+1}, \quad y_2 = x^2 - \dfrac{1}{x^2+1}$

Long Division of Polynomials **In Exercises 11–24, use long division to divide.**

11. $(2x^2 + 10x + 12) \div (x + 3)$
12. $(5x^2 - 17x - 12) \div (x - 4)$
13. $(4x^3 - 7x^2 - 11x + 5) \div (4x + 5)$
14. $(6x^3 - 16x^2 + 17x - 6) \div (3x - 2)$
15. $(x^4 + 5x^3 + 6x^2 - x - 2) \div (x + 2)$
16. $(x^3 + 4x^2 - 3x - 12) \div (x - 3)$
17. $(6x + 5) \div (x + 1)$
18. $(9x - 4) \div (3x + 2)$
19. $(x^3 - 9) \div (x^2 + 1)$
20. $(x^5 + 7) \div (x^4 - 1)$
21. $(3x + 2x^3 - 9 - 8x^2) \div (x^2 + 1)$
22. $(5x^3 - 16 - 20x + x^4) \div (x^2 - x - 3)$
23. $\dfrac{x^4}{(x-1)^3}$
24. $\dfrac{2x^3 - 4x^2 - 15x + 5}{(x-1)^2}$

Using Synthetic Division **In Exercises 25–44, use synthetic division to divide.**

25. $(2x^3 - 10x^2 + 14x - 24) \div (x - 4)$
26. $(5x^3 + 18x^2 + 7x - 6) \div (x + 3)$
27. $(6x^3 + 7x^2 - x + 26) \div (x - 3)$
28. $(2x^3 + 12x^2 + 14x - 3) \div (x + 4)$
29. $(4x^3 - 9x + 8x^2 - 18) \div (x + 2)$
30. $(9x^3 - 16x - 18x^2 + 32) \div (x - 2)$
31. $(-x^3 + 75x - 250) \div (x + 10)$
32. $(3x^3 - 16x^2 - 72) \div (x - 6)$
33. $(x^3 - 3x^2 + 5) \div (x - 4)$
34. $(5x^3 + 6x + 8) \div (x + 2)$
35. $\dfrac{10x^4 - 50x^3 - 800}{x - 6}$
36. $\dfrac{x^5 - 13x^4 - 120x + 80}{x + 3}$
37. $\dfrac{x^3 + 512}{x + 8}$
38. $\dfrac{x^3 - 729}{x - 9}$
39. $\dfrac{-3x^4}{x - 2}$
40. $\dfrac{-2x^5}{x + 2}$
41. $\dfrac{180x - x^4}{x - 6}$
42. $\dfrac{5 - 3x + 2x^2 - x^3}{x + 1}$
43. $\dfrac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}}$
44. $\dfrac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}}$

Using the Remainder Theorem **In Exercises 45–50, write the function in the form $f(x) = (x - k)q(x) + r$ for the given value of k, and demonstrate that $f(k) = r$.**

45. $f(x) = x^3 - x^2 - 10x + 7, \quad k = 3$
46. $f(x) = x^3 - 4x^2 - 10x + 8, \quad k = -2$
47. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, \quad k = -\frac{2}{3}$
48. $f(x) = 10x^3 - 22x^2 - 3x + 4, \quad k = \frac{1}{5}$
49. $f(x) = -4x^3 + 6x^2 + 12x + 4, \quad k = 1 - \sqrt{3}$
50. $f(x) = -3x^3 + 8x^2 + 10x - 8, \quad k = 2 + \sqrt{2}$

Using the Remainder Theorem **In Exercises 51–54, use the Remainder Theorem and synthetic division to find each function value. Verify your answers using another method.**

51. $f(x) = 2x^3 - 7x + 3$
    (a) $f(1)$ (b) $f(-2)$ (c) $f(3)$ (d) $f(2)$
52. $g(x) = 2x^6 + 3x^4 - x^2 + 3$
    (a) $g(2)$ (b) $g(1)$ (c) $g(3)$ (d) $g(-1)$
53. $h(x) = x^3 - 5x^2 - 7x + 4$
    (a) $h(3)$ (b) $h(\frac{1}{2})$ (c) $h(-2)$ (d) $h(-5)$
54. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$
    (a) $f(1)$ (b) $f(-2)$ (c) $f(5)$ (d) $f(-10)$

Using the Factor Theorem **In Exercises 55–62, use synthetic division to show that x is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all real solutions of the equation.**

55. $x^3 + 6x^2 + 11x + 6 = 0, \quad x = -3$
56. $x^3 - 52x - 96 = 0, \quad x = -6$
57. $2x^3 - 15x^2 + 27x - 10 = 0, \quad x = \frac{1}{2}$
58. $48x^3 - 80x^2 + 41x - 6 = 0, \quad x = \frac{2}{3}$
59. $x^3 + 2x^2 - 3x - 6 = 0, \quad x = \sqrt{3}$
60. $x^3 + 2x^2 - 2x - 4 = 0, \quad x = \sqrt{2}$
61. $x^3 - 3x^2 + 2 = 0, \quad x = 1 + \sqrt{3}$
62. $x^3 - x^2 - 13x - 3 = 0, \quad x = 2 - \sqrt{5}$

Factoring a Polynomial **In Exercises 63–70, (a) verify the given factors of $f(x)$, (b) find the remaining factor(s) of $f(x)$, (c) use your results to write the complete factorization of $f(x)$, (d) list all real zeros of f, and (e) confirm your results by using a graphing utility to graph the function.**

	Function	Factors
63.	$f(x) = 2x^3 + x^2 - 5x + 2$	$(x + 2), (x - 1)$
64.	$f(x) = 3x^3 - x^2 - 8x - 4$	$(x + 1), (x - 2)$
65.	$f(x) = x^4 - 8x^3 + 9x^2 + 38x - 40$	$(x - 5), (x + 2)$
66.	$f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$	$(x + 2), (x - 4)$
67.	$f(x) = 6x^3 + 41x^2 - 9x - 14$	$(2x + 1), (3x - 2)$
68.	$f(x) = 10x^3 - 11x^2 - 72x + 45$	$(2x + 5), (5x - 3)$
69.	$f(x) = 2x^3 - x^2 - 10x + 5$	$(2x - 1), (x + \sqrt{5})$
70.	$f(x) = x^3 + 3x^2 - 48x - 144$	$(x + 4\sqrt{3}), (x + 3)$

Approximating Zeros **In Exercises 71–76, (a) use the *zero* or *root* feature of a graphing utility to approximate the zeros of the function accurate to three decimal places, (b) determine the exact value of one of the zeros, and (c) use synthetic division to verify your result from part (b), and then factor the polynomial completely.**

71. $f(x) = x^3 - 2x^2 - 5x + 10$
72. $g(x) = x^3 + 3x^2 - 2x - 6$
73. $h(t) = t^3 - 2t^2 - 7t + 2$
74. $f(s) = s^3 - 12s^2 + 40s - 24$
75. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$
76. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

Simplifying Rational Expressions **In Exercises 77–80, simplify the rational expression by using long division or synthetic division.**

77. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$
78. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$
79. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2}$
80. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4}$

81. **Profit** A company that produces calculators estimates that the profit P (in dollars) from selling a specific model of calculator is given by

$$P = -152x^3 + 7545x^2 - 169{,}625, \quad 0 \le x \le 45$$

where x is the advertising expense (in tens of thousands of dollars). For this model of calculator, an advertising expense of \$400,000 ($x = 40$) results in a profit of \$2,174,375.

(a) Use a graphing utility to graph the profit function.
(b) Use the graph from part (a) to estimate another amount the company can spend on advertising that results in the same profit.
(c) Use synthetic division to confirm the result of part (b) algebraically.

82. Lyme Disease The numbers N of confirmed cases of Lyme disease in Maryland from 2007 through 2014 are shown in the table, where t represents the year, with $t = 7$ corresponding to 2007. *(Source: Centers for Disease Control and Prevention)*

DATA

Year, t	Number, N
7	2576
8	1746
9	1466
10	1163
11	938
12	1113
13	801
14	957

Spreadsheet at LarsonPrecalculus.com

(a) Use a graphing utility to create a scatter plot of the data.

(b) Use the *regression* feature of the graphing utility to find a *quartic* model for the data. (A quartic model has the form $at^4 + bt^3 + ct^2 + dt + e$, where a, b, c, d, and e are constant and t is variable.) Graph the model in the same viewing window as the scatter plot.

(c) Use the model to create a table of estimated values of N. Compare the model with the original data.

(d) Use synthetic division to confirm algebraically your estimated value for the year 2014.

Exploration

True or False? **In Exercises 83–86, determine whether the statement is true or false. Justify your answer.**

83. If $(7x + 4)$ is a factor of some polynomial function $f(x)$, then $\frac{4}{7}$ is a zero of f.

84. $(2x - 1)$ is a factor of the polynomial

$$6x^6 + x^5 - 92x^4 + 45x^3 + 184x^2 + 4x - 48.$$

85. The rational expression $\dfrac{x^3 + 2x^2 - 7x + 4}{x^2 - 4x - 12}$ is improper.

86. The equation

$$\frac{x^3 - 3x^2 + 4}{x + 1} = x^2 - 4x + 4$$

is true for all values of x.

Think About It **In Exercises 87 and 88, perform the division. Assume that n is a positive integer.**

87. $\dfrac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3}$ **88.** $\dfrac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2}$

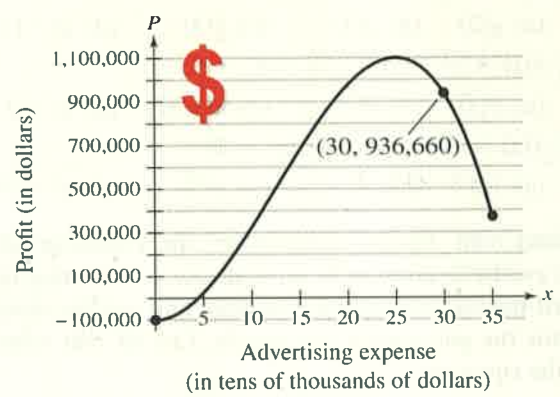
89. Error Analysis Describe the error.

Use synthetic division to find the remainder when $x^2 + 3x - 5$ is divided by $x + 1$.

$$\begin{array}{r|rrr} 1 & 1 & 3 & -5 \\ & & 1 & 4 \\ \hline & 1 & 4 & -1 \end{array}$$

Remainder: -1 ✗

90. HOW DO YOU SEE IT? The graph below shows a company's estimated profits for different advertising expenses. The company's actual profit was \$936,660 for an advertising expense of \$300,000.



(a) From the graph, it appears that the company could have obtained the same profit for a lesser advertising expense. Use the graph to estimate this expense.

(b) The company's model is

$$P = -140.75x^3 + 5348.3x^2 - 76{,}560,$$
$$0 \le x \le 35$$

where P is the profit (in dollars) and x is the advertising expense (in tens of thousands of dollars). Explain how you could verify the lesser expense from part (a) algebraically.

Exploration **In Exercises 91 and 92, find the constant c such that the denominator will divide evenly into the numerator.**

91. $\dfrac{x^3 + 4x^2 - 3x + c}{x - 5}$ **92.** $\dfrac{x^5 - 2x^2 + x + c}{x + 2}$

93. Think About It Find the value of k such that $x - 4$ is a factor of $x^3 - kx^2 + 2kx - 8$.

Dariusz Majgier/Shutterstock.com

2.4 Complex Numbers

Complex numbers are often used in electrical engineering. For example, in Exercise 87 on page 151, you will use complex numbers to find the impedance of an electrical circuit.

- **Use the imaginary unit i to write complex numbers.**
- **Add, subtract, and multiply complex numbers.**
- **Use complex conjugates to write the quotient of two complex numbers in standard form.**
- **Find complex solutions of quadratic equations.**

The Imaginary Unit i

You have learned that some quadratic equations have no real solutions. For example, the quadratic equation

$$x^2 + 1 = 0$$

has no real solution because there is no real number x that can be squared to produce -1. To overcome this deficiency, mathematicians created an expanded system of numbers using the **imaginary unit i,** defined as

$$i = \sqrt{-1} \qquad \text{Imaginary unit}$$

where $i^2 = -1$. By adding real numbers to real multiples of this imaginary unit, you obtain the set of **complex numbers.** Each complex number can be written in the **standard form $a + bi$.** For example, the standard form of the complex number $-5 + \sqrt{-9}$ is $-5 + 3i$ because

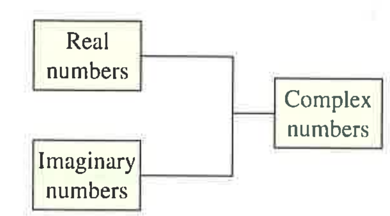
$$-5 + \sqrt{-9} = -5 + \sqrt{3^2(-1)} = -5 + 3\sqrt{-1} = -5 + 3i.$$

Definition of a Complex Number

Let a and b be real numbers. The number $a + bi$ is a **complex number** written in **standard form.** The real number a is the **real part** and the number bi (where b is a real number) is the **imaginary part** of the complex number.

When $b = 0$, the number $a + bi$ is a real number. When $b \neq 0$, the number $a + bi$ is an **imaginary number.** A number of the form bi, where $b \neq 0$, is a **pure imaginary number.**

Every real number a can be written as a complex number using $b = 0$. That is, for every real number a, $a = a + 0i$. So, the set of real numbers is a subset of the set of complex numbers, as shown in the figure below.



Equality of Complex Numbers

Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other

$$a + bi = c + di \qquad \text{Equality of two complex numbers}$$

if and only if $a = c$ and $b = d$.

iStockphoto.com/gerenme