2.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A ________ number has the form $a + bi$, where $a \neq 0$, $b = 0$.
2. An ________ number has the form $a + bi$, where $a \neq 0$, $b \neq 0$.
3. A ________ ________ number has the form $a + bi$, where $a = 0$, $b \neq 0$.
4. The imaginary unit i is defined as $i =$ ________, where $i^2 =$ ________.
5. When a is a positive real number, the ________ ________ root of $-a$ is defined as $\sqrt{-a} = \sqrt{a}i$.
6. The numbers $a + bi$ and $a - bi$ are called ________ ________, and their product is a real number $a^2 + b^2$.

Skills and Applications

Equality of Complex Numbers In Exercises 7–10, find real numbers a and b such that the equation is true.

7. $a + bi = 9 + 8i$
8. $a + bi = 10 - 5i$
9. $(a - 2) + (b + 1)i = 6 + 5i$
10. $(a + 2) + (b - 3)i = 4 + 7i$

Writing a Complex Number in Standard Form In Exercises 11–22, write the complex number in standard form.

11. $2 + \sqrt{-25}$
12. $4 + \sqrt{-49}$
13. $1 - \sqrt{-12}$
14. $2 - \sqrt{-18}$
15. $\sqrt{-40}$
16. $\sqrt{-27}$
17. 23
18. 50
19. $-6i + i^2$
20. $-2i^2 + 4i$
21. $\sqrt{-0.04}$
22. $\sqrt{-0.0025}$

Adding or Subtracting Complex Numbers In Exercises 23–30, perform the operation and write the result in standard form.

23. $(5 + i) + (2 + 3i)$
24. $(13 - 2i) + (-5 + 6i)$
25. $(9 - i) - (8 - i)$
26. $(3 + 2i) - (6 + 13i)$
27. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50})$
28. $(8 + \sqrt{-18}) - (4 + 3\sqrt{2}i)$
29. $13i - (14 - 7i)$
30. $25 + (-10 + 11i) + 15i$

Multiplying Complex Numbers In Exercises 31–38, perform the operation and write the result in standard form.

31. $(1 + i)(3 - 2i)$
32. $(7 - 2i)(3 - 5i)$
33. $12i(1 - 9i)$
34. $-8i(9 + 4i)$
35. $(\sqrt{2} + 3i)(\sqrt{2} - 3i)$
36. $(4 + \sqrt{7}i)(4 - \sqrt{7}i)$
37. $(6 + 7i)^2$
38. $(5 - 4i)^2$

Multiplying Conjugates In Exercises 39–46, write the complex conjugate of the complex number. Then multiply the number by its complex conjugate.

39. $9 + 2i$
40. $8 - 10i$
41. $-1 - \sqrt{5}i$
42. $-3 + \sqrt{2}i$
43. $\sqrt{-20}$
44. $\sqrt{-15}$
45. $\sqrt{6}$
46. $1 + \sqrt{8}$

A Quotient of Complex Numbers in Standard Form In Exercises 47–54, write the quotient in standard form.

47. $\dfrac{2}{4 - 5i}$
48. $\dfrac{13}{1 - i}$
49. $\dfrac{5 + i}{5 - i}$
50. $\dfrac{6 - 7i}{1 - 2i}$
51. $\dfrac{9 - 4i}{i}$
52. $\dfrac{8 + 16i}{2i}$
53. $\dfrac{3i}{(4 - 5i)^2}$
54. $\dfrac{5i}{(2 + 3i)^2}$

Performing Operations with Complex Numbers In Exercises 55–58, perform the operation and write the result in standard form.

55. $\dfrac{2}{1 + i} - \dfrac{3}{1 - i}$
56. $\dfrac{2i}{2 + i} + \dfrac{5}{2 - i}$
57. $\dfrac{i}{3 - 2i} + \dfrac{2i}{3 + 8i}$
58. $\dfrac{1 + i}{i} - \dfrac{3}{4 - i}$

Writing a Complex Number in Standard Form In Exercises 59–66, write the complex number in standard form.

59. $\sqrt{-6}\sqrt{-2}$
60. $\sqrt{-5}\sqrt{-10}$
61. $(\sqrt{-15})^2$
62. $(\sqrt{-75})^2$
63. $\sqrt{-8} + \sqrt{-50}$
64. $\sqrt{-45} - \sqrt{-5}$
65. $(3 + \sqrt{-5})(7 - \sqrt{-10})$
66. $(2 - \sqrt{-6})^2$

Complex Solutions of a Quadratic Equation In Exercises 67–76, use the Quadratic Formula to solve the quadratic equation.

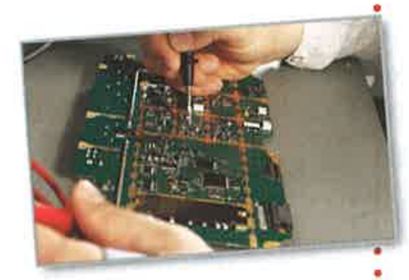
67. $x^2 - 2x + 2 = 0$
68. $x^2 + 6x + 10 = 0$
69. $4x^2 + 16x + 17 = 0$
70. $9x^2 - 6x + 37 = 0$
71. $4x^2 + 16x + 21 = 0$
72. $16t^2 - 4t + 3 = 0$
73. $\frac{3}{2}x^2 - 6x + 9 = 0$
74. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$
75. $1.4x^2 - 2x + 10 = 0$
76. $4.5x^2 - 3x + 12 = 0$

Simplifying a Complex Number In Exercises 77–86, simplify the complex number and write it in standard form.

77. $-6i^3 + i^2$
78. $4i^2 - 2i^3$
79. $-14i^5$
80. $(-i)^3$
81. $(\sqrt{-72})^3$
82. $(\sqrt{-2})^6$
83. $\dfrac{1}{i^3}$
84. $\dfrac{1}{(2i)^3}$
85. $(3i)^4$
86. $(-i)^6$

87. **Impedance of a Circuit** The opposition to current in an electrical circuit is called its impedance. The impedance z in a parallel circuit with two pathways satisfies the equation

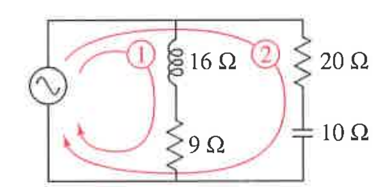
$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}$$



where z_1 is the impedance (in ohms) of pathway 1 and z_2 is the impedance (in ohms) of pathway 2.

(a) The impedance of each pathway in a parallel circuit is found by adding the impedances of all components in the pathway. Use the table to find z_1 and z_2.

	Resistor	Inductor	Capacitor
Symbol	$a\,\Omega$	$b\,\Omega$	$c\,\Omega$
Impedance	a	bi	$-ci$



(b) Find the impedance z.

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88. **Cube of a Complex Number** Cube each complex number.
(a) $-1 + \sqrt{3}i$ (b) $-1 - \sqrt{3}i$

Exploration

True or False? In Exercises 89–92, determine whether the statement is true or false. Justify your answer.

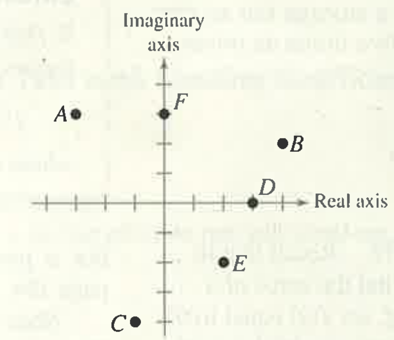
89. The sum of two complex numbers is always a real number.
90. There is no complex number that is equal to its complex conjugate.
91. $-i\sqrt{6}$ is a solution of $x^4 - x^2 + 14 = 56$.
92. $i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = -1$

93. **Pattern Recognition** Find the missing values.

$i^1 = i$ $\quad i^2 = -1$ $\quad i^3 = -i$ $\quad i^4 = 1$
$i^5 = \square$ $\quad i^6 = \square$ $\quad i^7 = \square$ $\quad i^8 = \square$
$i^9 = \square$ $\quad i^{10} = \square$ $\quad i^{11} = \square$ $\quad i^{12} = \square$

What pattern do you see? Write a brief description of how you would find i raised to any positive integer power.

94. **HOW DO YOU SEE IT?** The coordinate system shown below is called the complex plane. In the complex plane, the point (a, b) corresponds to the complex number $a + bi$.



Match each complex number with its corresponding point.
(i) 3 (ii) $3i$ (iii) $4 + 2i$
(iv) $2 - 2i$ (v) $-3 + 3i$ (vi) $-1 - 4i$

95. **Error Analysis** Describe the error.

$$\sqrt{-6}\sqrt{-6} = \sqrt{(-6)(-6)} = \sqrt{36} = 6 \quad \text{X}$$

96. **Proof** Prove that the complex conjugate of the product of two complex numbers $a_1 + b_1 i$ and $a_2 + b_2 i$ is the product of their complex conjugates.

97. **Proof** Prove that the complex conjugate of the sum of two complex numbers $a_1 + b_1 i$ and $a_2 + b_2 i$ is the sum of their complex conjugates.