## 2.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Vocabulary:** **Fill in the blanks.**

1. The ________ ________ of ________ states that if $f(x)$ is a polynomial of degree $n$ $(n > 0)$, then $f$ has at least one zero in the complex number system.
2. The ________ ________ ________ states that if $f(x)$ is a polynomial of degree $n$ $(n > 0)$, then $f(x)$ has precisely $n$ linear factors, $f(x) = a_n(x - c_1)(x - c_2) \cdot \cdot \cdot (x - c_n)$, where $c_1, c_2, \ldots, c_n$ are complex numbers.
3. The test that gives a list of the possible rational zeros of a polynomial function is the ________ ________ Test.
4. If $a + bi$, where $b \neq 0$, is a complex zero of a polynomial with real coefficients, then so is its ________ ________, $a - bi$.
5. Every polynomial of degree $n > 0$ with real coefficients can be written as the product of ________ and ________ factors with real coefficients, where the ________ factors have no real zeros.
6. A quadratic factor that cannot be factored further as a product of linear factors containing real numbers is ________ over the ________.
7. The theorem that can be used to determine the possible numbers of positive and negative real zeros of a function is called ________ ________ of ________.
8. A real number $c$ is a ________ bound for the real zeros of $f$ when no real zeros are less than $c$, and is a ________ bound when no real zeros are greater than $c$.

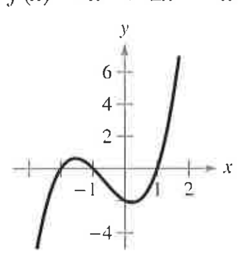
### Skills and Applications

**Zeros of Polynomial Functions** **In Exercises 9–14, determine the number of zeros of the polynomial function.**

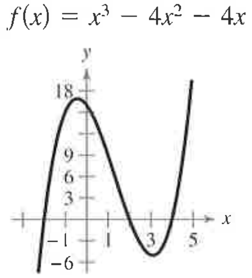
9. $f(x) = x^3 + 2x^2 + 1$
10. $f(x) = x^4 - 3x$
11. $g(x) = x^4 - x^5$
12. $f(x) = x^3 - x^6$
13. $f(x) = (x + 5)^2$
14. $h(t) = (t - 1)^2 - (t + 1)^2$

**Using the Rational Zero Test** **In Exercises 15–18, use the Rational Zero Test to list the possible rational zeros of $f$. Verify that the zeros of $f$ shown in the graph are contained in the list.**

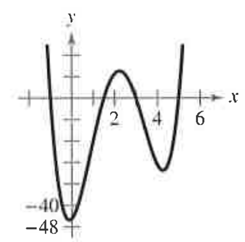
15. $f(x) = x^3 + 2x^2 - x - 2$



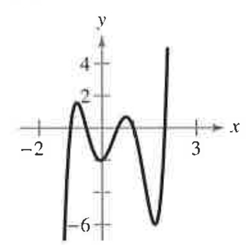

16. $f(x) = x^3 - 4x^2 - 4x + 16$




17. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$




18. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$




**Using the Rational Zero Test** **In Exercises 19–28, find (if possible) the rational zeros of the function.**

19. $f(x) = x^3 - 7x - 6$
20. $f(x) = x^3 - 13x + 12$
21. $g(t) = t^3 - 4t^2 + 4$
22. $h(x) = x^3 - 19x + 30$
23. $h(t) = t^3 + 8t^2 + 13t + 6$
24. $g(x) = x^3 + 8x^2 + 12x + 18$
25. $C(x) = 2x^3 + 3x^2 - 1$
26. $f(x) = 3x^3 - 19x^2 + 33x - 9$
27. $g(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$
28. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$



**Solving a Polynomial Equation** **In Exercises 29–32, find all real solutions of the polynomial equation.**

29. $-5x^3 + 11x^2 - 4x - 2 = 0$
30. $8x^3 + 10x^2 - 15x - 6 = 0$
31. $x^4 + 6x^3 + 3x^2 - 16x + 6 = 0$
32. $x^4 + 8x^3 + 14x^2 - 17x - 42 = 0$

**Using the Rational Zero Test** **In Exercises 33–36, (a) list the possible rational zeros of $f$, (b) sketch the graph of $f$ so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of $f$.**

33. $f(x) = x^3 + x^2 - 4x - 4$
34. $f(x) = -3x^3 + 20x^2 - 36x + 16$
35. $f(x) = -4x^3 + 15x^2 - 8x - 3$
36. $f(x) = 4x^3 - 12x^2 - x + 15$

**Using the Rational Zero Test** **In Exercises 37–40, (a) list the possible rational zeros of $f$, (b) use a graphing utility to graph $f$ so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of $f$.**

37. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$
38. $f(x) = 4x^4 - 17x^2 + 4$
39. $f(x) = 32x^3 - 52x^2 + 17x + 3$
40. $f(x) = 4x^3 + 7x^2 - 11x - 18$

**Finding a Polynomial Function with Given Zeros** **In Exercises 41–46, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)**

41. $1, 5i$
42. $4, -3i$
43. $2, 2, 1 + i$
44. $-1, 5, 3 - 2i$
45. $\frac{2}{3}, -1, 3 + \sqrt{2}i$
46. $-\frac{5}{2}, -5, 1 + \sqrt{3}i$

**Finding a Polynomial Function with Given Zeros** **In Exercises 47–50, find the polynomial function $f$ with real coefficients that has the given degree, zeros, and solution point.**

| | Degree | Zeros | Solution Point |
|---|---|---|---|
| 47. | 4 | $-2, 1, i$ | $f(0) = -4$ |
| 48. | 4 | $-1, 2, \sqrt{2}i$ | $f(1) = 12$ |
| 49. | 3 | $-3, 1 + \sqrt{3}i$ | $f(-2) = 12$ |
| 50. | 3 | $-2, 1 - \sqrt{2}i$ | $f(-1) = -12$ |

**Factoring a Polynomial** **In Exercises 51–54, write the polynomial (a) as the product of factors that are irreducible over the *rationals*, (b) as the product of linear and quadratic factors that are irreducible over the *reals*, and (c) in completely factored form.**

51. $f(x) = x^4 + 2x^2 - 8$
52. $f(x) = x^4 + 6x^2 - 27$
53. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$
(*Hint:* One factor is $x^2 - 6$.)
54. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$
(*Hint:* One factor is $x^2 + 4$.)

**Finding the Zeros of a Polynomial Function** **In Exercises 55–60, use the given zero to find all the zeros of the function.**

| | Function | Zero |
|---|---|---|
| 55. | $f(x) = x^3 - x^2 + 4x - 4$ | $2i$ |
| 56. | $f(x) = 2x^3 + 3x^2 + 18x + 27$ | $3i$ |
| 57. | $g(x) = x^3 - 8x^2 + 25x - 26$ | $3 + 2i$ |
| 58. | $g(x) = x^3 + 9x^2 + 25x + 17$ | $-4 + i$ |
| 59. | $h(x) = x^4 - 6x^3 + 14x^2 - 18x + 9$ | $1 - \sqrt{2}i$ |
| 60. | $h(x) = x^4 + x^3 - 3x^2 - 13x + 14$ | $-2 + \sqrt{3}i$ |

**Finding the Zeros of a Polynomial Function** **In Exercises 61–72, write the polynomial as the product of linear factors and list all the zeros of the function.**

61. $f(x) = x^2 + 36$
62. $f(x) = x^2 + 49$
63. $h(x) = x^2 - 2x + 17$
64. $g(x) = x^2 + 10x + 17$
65. $f(x) = x^4 - 16$
66. $f(y) = y^4 - 256$
67. $f(z) = z^2 - 2z + 2$
68. $h(x) = x^3 - 3x^2 + 4x - 2$
69. $g(x) = x^3 - 3x^2 + x + 5$
70. $f(x) = x^3 - x^2 + x + 39$
71. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$
72. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

**Finding the Zeros of a Polynomial Function** **In Exercises 73–78, find all the zeros of the function. When there is an extended list of possible rational zeros, use a graphing utility to graph the function in order to disregard any of the possible rational zeros that are obviously not zeros of the function.**

73. $f(x) = x^3 + 24x^2 + 214x + 740$
74. $f(s) = 2s^3 - 5s^2 + 12s - 5$
75. $f(x) = 16x^3 - 20x^2 - 4x + 15$
76. $f(x) = 9x^3 - 15x^2 + 11x - 5$
77. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$
78. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

**Using Descartes's Rule of Signs** In Exercises 79–86, use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of the function.

79. $g(x) = 2x^3 - 3x^2 - 3$
80. $h(x) = 4x^2 - 8x + 3$
81. $h(x) = 2x^3 + 3x^2 + 1$
82. $h(x) = 2x^4 - 3x - 2$
83. $g(x) = 6x^4 + 2x^3 - 3x^2 + 2$
84. $f(x) = 4x^3 - 3x^2 - 2x - 1$
85. $f(x) = 5x^3 + x^2 - x + 5$
86. $f(x) = 3x^3 - 2x^2 - x + 3$

**Verifying Upper and Lower Bounds** In Exercises 87–90, use synthetic division to verify the upper and lower bounds of the real zeros of $f$.

87. $f(x) = x^3 + 3x^2 - 2x + 1$
(a) Upper: $x = 1$ (b) Lower: $x = -4$
88. $f(x) = x^3 - 4x^2 + 1$
(a) Upper: $x = 4$ (b) Lower: $x = -1$
89. $f(x) = x^4 - 4x^3 + 16x - 16$
(a) Upper: $x = 5$ (b) Lower: $x = -3$
90. $f(x) = 2x^4 - 8x + 3$
(a) Upper: $x = 3$ (b) Lower: $x = -4$

**Finding Real Zeros of a Polynomial Function** In Exercises 91–94, find all real zeros of the function.

91. $f(x) = 16x^3 - 12x^2 - 4x + 3$
92. $f(z) = 12z^3 - 4z^2 - 27z + 9$
93. $f(y) = 4y^3 + 3y^2 + 8y + 6$
94. $g(x) = 3x^3 - 2x^2 + 15x - 10$

**Finding the Rational Zeros of a Polynomial** In Exercises 95–98, find the rational zeros of the polynomial function.

95. $P(x) = x^4 - \frac{25}{4}x^2 + 9 = \frac{1}{4}(4x^4 - 25x^2 + 36)$
96. $f(x) = x^3 - \frac{3}{2}x^2 - \frac{23}{2}x + 6 = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$
97. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4} = \frac{1}{4}(4x^3 - x^2 - 4x + 1)$
98. $f(z) = z^3 + \frac{11}{6}z^2 - \frac{1}{2}z - \frac{1}{3} = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

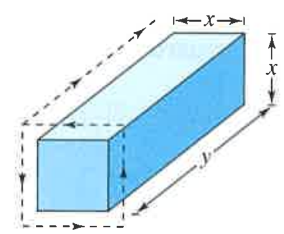
**Rational and Irrational Zeros** In Exercises 99–102, match the cubic function with the numbers of rational and irrational zeros.

(a) Rational zeros: 0; irrational zeros: 1
(b) Rational zeros: 3; irrational zeros: 0
(c) Rational zeros: 1; irrational zeros: 2
(d) Rational zeros: 1; irrational zeros: 0

99. $f(x) = x^3 - 1$
100. $f(x) = x^3 - 2$
101. $f(x) = x^3 - x$
102. $f(x) = x^3 - 2x$

103. **Geometry** You want to make an open box from a rectangular piece of material, 15 centimeters by 9 centimeters, by cutting equal squares from the corners and turning up the sides.
(a) Let $x$ represent the side length of each of the squares removed. Draw a diagram showing the squares removed from the original piece of material and the resulting dimensions of the open box.
(b) Use the diagram to write the volume $V$ of the box as a function of $x$. Determine the domain of the function.
(c) Sketch the graph of the function and approximate the dimensions of the box that yield a maximum volume.
(d) Find values of $x$ such that $V = 56$. Which of these values is a physical impossibility in the construction of the box? Explain.

104. **Geometry** A rectangular package to be sent by a delivery service (see figure) has a combined length and girth (perimeter of a cross section) of 120 inches.




(a) Use the diagram to write the volume $V$ of the package as a function of $x$.
(b) Use a graphing utility to graph the function and approximate the dimensions of the package that yield a maximum volume.
(c) Find values of $x$ such that $V = 13{,}500$. Which of these values is a physical impossibility in the construction of the package? Explain.

105. **Geometry**
A bulk food storage bin with dimensions 2 feet by 3 feet by 4 feet needs to be increased in size to hold five times as much food as the current bin.
(a) Assume each dimension is increased by the same amount. Write a function that represents the volume $V$ of the new bin.
(b) Find the dimensions of the new bin.





106. **Cost** The ordering and transportation cost $C$ (in thousands of dollars) for machine parts is given by

$$C(x) = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \geq 1$$

where $x$ is the order size (in hundreds). In calculus, it can be shown that the cost is a minimum when

$$3x^3 - 40x^2 - 2400x - 36{,}000 = 0.$$

Use a graphing utility to approximate the optimal order size to the nearest hundred units.

## Exploration

**True or False?** In Exercises 107 and 108, decide whether the statement is true or false. Justify your answer.

107. It is possible for a third-degree polynomial function with integer coefficients to have no real zeros.
108. If $x = -i$ is a zero of the function
$$f(x) = x^3 + ix^2 + ix - 1$$
then $x = i$ must also be a zero of $f$.

**Think About It** In Exercises 109–114, determine (if possible) the zeros of the function $g$ when the function $f$ has zeros at $x = r_1$, $x = r_2$, and $x = r_3$.

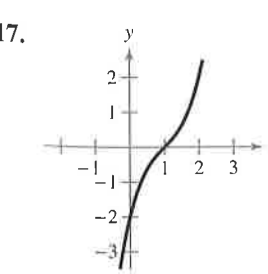
109. $g(x) = -f(x)$
110. $g(x) = 3f(x)$
111. $g(x) = f(x - 5)$
112. $g(x) = f(2x)$
113. $g(x) = 3 + f(x)$
114. $g(x) = f(-x)$

115. **Think About It** A cubic polynomial function $f$ has real zeros $-2$, $\frac{1}{2}$, and 3, and its leading coefficient is negative. Write an equation for $f$ and sketch its graph. How many different polynomial functions are possible for $f$?

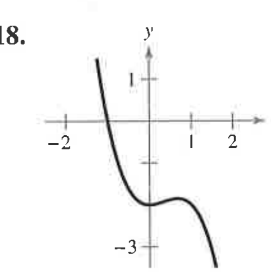
116. **Think About It** Sketch the graph of a fifth-degree polynomial function whose leading coefficient is positive and that has a zero at $x = 3$ of multiplicity 2.

**Writing an Equation** In Exercises 117 and 118, the graph of a cubic polynomial function $y = f(x)$ is shown. One of the zeros is $1 + i$. Write an equation for $f$.

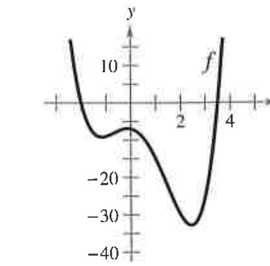
117.




118.




119. **Error Analysis** Describe the error.
The graph of a quartic (fourth-degree) polynomial $y = f(x)$ is shown. One of the zeros is $i$.




The function is $f(x) = (x + 2)(x - 3.5)(x - i)$. X

120. **HOW DO YOU SEE IT?** Use the information in the table to answer each question.

| Interval | Value of $f(x)$ |
|---|---|
| $(-\infty, -2)$ | Positive |
| $(-2, 1)$ | Negative |
| $(1, 4)$ | Negative |
| $(4, \infty)$ | Positive |

(a) What are the three real zeros of the polynomial function $f$?
(b) What can be said about the behavior of the graph of $f$ at $x = 1$?
(c) What is the least possible degree of $f$? Explain. Can the degree of $f$ ever be odd? Explain.
(d) Is the leading coefficient of $f$ positive or negative? Explain.
(e) Sketch a graph of a function that exhibits the behavior described in the table.

121. **Think About It** Let $y = f(x)$ be a quartic (fourth-degree) polynomial with leading coefficient $a = 1$ and
$$f(i) = f(2i) = 0.$$
Write an equation for $f$.

122. **Think About It** Let $y = f(x)$ be a cubic polynomial with leading coefficient $a = -1$ and
$$f(2) = f(i) = 0.$$
Write an equation for $f$.

123. **Writing an Equation** Write the equation for a quadratic function $f$ (with integer coefficients) that has the given zeros. Assume that $b$ is a positive integer.
(a) $\pm\sqrt{b}i$ (b) $a \pm bi$