3.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: **Fill in the blanks.**

1. Polynomial and rational functions are examples of ________ functions.
2. Exponential and logarithmic functions are examples of nonalgebraic functions, also called ________ functions.
3. The ________ Property can be used to solve simple exponential equations.
4. The exponential function $f(x) = e^x$ is called the ________ ________ function, and the base e is called the ________ base.
5. To find the amount A in an account after t years with principal P and an annual interest rate r (in decimal form) compounded n times per year, use the formula ________.
6. To find the amount A in an account after t years with principal P and an annual interest rate r (in decimal form) compounded continuously, use the formula ________.

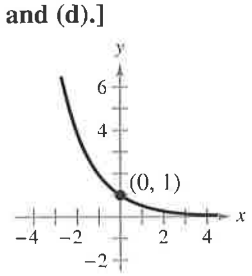
Skills and Applications

Evaluating an Exponential Function **In Exercises 7–12, evaluate the function at the given value of x. Round your result to three decimal places.**

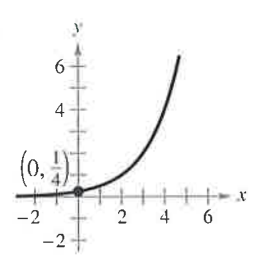
Function	Value
7. $f(x) = 0.9^x$	$x = 1.4$
8. $f(x) = 4.7^x$	$x = -\pi$
9. $f(x) = 3^x$	$x = \frac{2}{5}$
10. $f(x) = \left(\frac{2}{3}\right)^{5x}$	$x = \frac{3}{10}$
11. $f(x) = 5000(2^x)$	$x = -1.5$
12. $f(x) = 200(1.2)^{12x}$	$x = 24$

Matching an Exponential Function with Its Graph **In Exercises 13–16, match the exponential function with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

(a)



(b)



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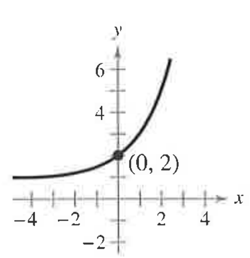
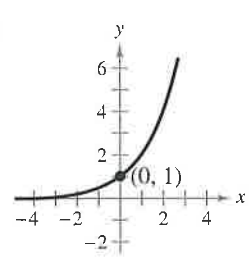


(d)



13. $f(x) = 2^x$
14. $f(x) = 2^x + 1$
15. $f(x) = 2^{-x}$
16. $f(x) = 2^{x-2}$

Graphing an Exponential Function **In Exercises 17–24, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.**

17. $f(x) = 7^x$
18. $f(x) = 7^{-x}$
19. $f(x) = \left(\frac{1}{4}\right)^{-x}$
20. $f(x) = \left(\frac{1}{4}\right)^x$
21. $f(x) = 4^{x-1}$
22. $f(x) = 4^{x+1}$
23. $f(x) = 2^{x+1} + 3$
24. $f(x) = 3^{x-2} + 1$

Using the One-to-One Property **In Exercises 25–28, use the One-to-One Property to solve the equation for x.**

25. $3^{x+1} = 27$
26. $2^{x-2} = 64$
27. $\left(\frac{1}{2}\right)^x = 32$
28. $5^{x-2} = \frac{1}{125}$

Transformations of the Graph of an Exponential Function **In Exercises 29–32, describe the transformation(s) of the graph of f that yield(s) the graph of g.**

29. $f(x) = 3^x$, $g(x) = 3^x + 1$
30. $f(x) = \left(\frac{7}{2}\right)^x$, $g(x) = -\left(\frac{7}{2}\right)^{-x}$
31. $f(x) = 10^x$, $g(x) = 10^{-x+3}$
32. $f(x) = 0.3^x$, $g(x) = -0.3^x + 5$

Evaluating a Natural Exponential Function **In Exercises 33–36, evaluate the function at the given value of x. Round your result to three decimal places.**

Function	Value
33. $f(x) = e^x$	$x = 1.9$
34. $f(x) = 1.5e^{x/2}$	$x = 240$
35. $f(x) = 5000e^{0.06x}$	$x = 6$
36. $f(x) = 250e^{0.05x}$	$x = 20$

Graphing a Natural Exponential Function **In Exercises 37–40, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.**

37. $f(x) = 3e^{x+4}$
38. $f(x) = 2e^{-1.5x}$
39. $f(x) = 2e^{x-2} + 4$
40. $f(x) = 2 + e^{x-5}$

Graphing a Natural Exponential Function **In Exercises 41–44, use a graphing utility to graph the exponential function.**

41. $s(t) = 2e^{0.5t}$
42. $s(t) = 3e^{-0.2t}$
43. $g(x) = 1 + e^{-x}$
44. $h(x) = e^{x-2}$

Using the One-to-One Property **In Exercises 45–48, use the One-to-One Property to solve the equation for x.**

45. $e^{3x+2} = e^3$
46. $e^{2x-1} = e^4$
47. $e^{x^2-3} = e^{2x}$
48. $e^{x^2+6} = e^{5x}$

Compound Interest **In Exercises 49–52, complete the table by finding the balance A when P dollars is invested at rate r for t years and compounded n times per year.**

n	1	2	4	12	365	Continuous
A						

49. $P = \$1500$, $r = 2\%$, $t = 10$ years
50. $P = \$2500$, $r = 3.5\%$, $t = 10$ years
51. $P = \$2500$, $r = 4\%$, $t = 20$ years
52. $P = \$1000$, $r = 6\%$, $t = 40$ years

Compound Interest **In Exercises 53–56, complete the table by finding the balance A when \$12,000 is invested at rate r for t years, compounded continuously.**

t	10	20	30	40	50
A					

53. $r = 4\%$
54. $r = 6\%$
55. $r = 6.5\%$
56. $r = 3.5\%$

57. **Trust Fund** On the day of a child's birth, a parent deposits \$30,000 in a trust fund that pays 5% interest, compounded continuously. Determine the balance in this account on the child's 25th birthday.

58. **Trust Fund** A philanthropist deposits \$5000 in a trust fund that pays 7.5% interest, compounded continuously. The balance will be given to the college from which the philanthropist graduated after the money has earned interest for 50 years. How much will the college receive?

59. **Inflation** Assuming that the annual rate of inflation averages 4% over the next 10 years, the approximate costs C of goods or services during any year in that decade can be modeled by $C(t) = P(1.04)^t$, where t is the time in years and P is the present cost. The price of an oil change for your car is presently \$29.88. Estimate the price 10 years from now.

60. **Computer Virus** The number V of computers infected by a virus increases according to the model $V(t) = 100e^{4.6052t}$, where t is the time in hours. Find the number of computers infected after (a) 1 hour, (b) 1.5 hours, and (c) 2 hours.

61. **Population Growth** The projected population of the United States for the years 2025 through 2055 can be modeled by $P = 307.58e^{0.0052t}$, where P is the population (in millions) and t is the time (in years), with $t = 25$ corresponding to 2025. *(Source: U.S. Census Bureau)*
    (a) Use a graphing utility to graph the function for the years 2025 through 2055.
    (b) Use the *table* feature of the graphing utility to create a table of values for the same time period as in part (a).
    (c) According to the model, during what year will the population of the United States exceed 430 million?

62. **Population** The population P (in millions) of Italy from 2003 through 2015 can be approximated by the model $P = 57.59e^{0.0051t}$, where t represents the year, with $t = 3$ corresponding to 2003. *(Source: U.S. Census Bureau)*
    (a) According to the model, is the population of Italy increasing or decreasing? Explain.
    (b) Find the populations of Italy in 2003 and 2015.
    (c) Use the model to predict the populations of Italy in 2020 and 2025.

63. **Radioactive Decay** Let Q represent a mass (in grams) of radioactive plutonium (^{239}Pu), whose half-life is 24,100 years. The quantity of plutonium present after t years is $Q = 16\left(\frac{1}{2}\right)^{t/24{,}100}$.
    (a) Determine the initial quantity (when $t = 0$).
    (b) Determine the quantity present after 75,000 years.
    (c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 150{,}000$.

64. **Radioactive Decay** Let Q represent a mass (in grams) of carbon (^{14}C), whose half-life is 5715 years. The quantity of carbon 14 present after t years is $Q = 10\left(\frac{1}{2}\right)^{t/5715}$.
    (a) Determine the initial quantity (when $t = 0$).
    (b) Determine the quantity present after 2000 years.
    (c) Sketch the graph of the function over the interval $t = 0$ to $t = 10{,}000$.

65. Depreciation The value of a wheelchair conversion van that originally cost \$49,810 depreciates so that each year it is worth $\frac{7}{8}$ of its value for the previous year.

(a) Find a model for $V(t)$, the value of the van after t years.

(b) Determine the value of the van 4 years after it was purchased.

66. Chemistry

Immediately following an injection, the concentration of a drug in the bloodstream is 300 milligrams per milliliter. After t hours, the concentration is 75% of the level of the previous hour.

(a) Find a model for $C(t)$, the concentration of the drug after t hours.

(b) Determine the concentration of the drug after 8 hours.

Exploration

True or False? **In Exercises 67 and 68, determine whether the statement is true or false. Justify your answer.**

67. The line $y = -2$ is an asymptote for the graph of $f(x) = 10^x - 2$.

68. $e = \dfrac{271{,}801}{99{,}990}$

Think About It **In Exercises 69–72, use properties of exponents to determine which functions (if any) are the same.**

69. $f(x) = 3^{x-2}$, $g(x) = 3^x - 9$, $h(x) = \frac{1}{9}(3^x)$

70. $f(x) = 4^x + 12$, $g(x) = 2^{2x+6}$, $h(x) = 64(4^x)$

71. $f(x) = 16(4^{-x})$, $g(x) = \left(\frac{1}{4}\right)^{x-2}$, $h(x) = 16(2^{-2x})$

72. $f(x) = e^{-x} + 3$, $g(x) = e^{3-x}$, $h(x) = -e^{x-3}$

73. Solving Inequalities Graph the functions $y = 3^x$ and $y = 4^x$ and use the graphs to solve each inequality.

(a) $4^x < 3^x$ (b) $4^x > 3^x$

74. Using Technology Use a graphing utility to graph each function. Use the graph to find where the function is increasing and decreasing, and approximate any relative maximum or minimum values.

(a) $f(x) = x^2e^{-x}$

(b) $g(x) = x2^{3-x}$

75. Graphical Reasoning Use a graphing utility to graph $y_1 = [1 + (1/x)]^x$ and $y_2 = e$ in the same viewing window. Using the *trace* feature, explain what happens to the graph of y_1 as x increases.

76. Graphical Reasoning Use a graphing utility to graph

$$f(x) = \left(1 + \frac{0.5}{x}\right)^x \quad \text{and} \quad g(x) = e^{0.5}$$

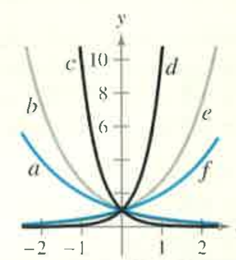
in the same viewing window. What is the relationship between f and g as x increases and decreases without bound?

77. Comparing Graphs Use a graphing utility to graph each pair of functions in the same viewing window. Describe any similarities and differences in the graphs.

(a) $y_1 = 2^x, y_2 = x^2$

(b) $y_1 = 3^x, y_2 = x^3$

78. HOW DO YOU SEE IT? The figure shows the graphs of $y = 2^x$, $y = e^x$, $y = 10^x$, $y = 2^{-x}$, $y = e^{-x}$, and $y = 10^{-x}$. Match each function with its graph. [The graphs are labeled (a) through (f).] Explain your reasoning.



79. Think About It Which functions are exponential?

(a) $f(x) = 3x$ (b) $g(x) = 3x^2$

(c) $h(x) = 3^x$ (d) $k(x) = 2^{-x}$

80. Compound Interest Use the formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

to calculate the balance A of an investment when $P = \$3000$, $r = 6\%$, and $t = 10$ years, and compounding is done (a) by the day, (b) by the hour, (c) by the minute, and (d) by the second. Does increasing the number of compoundings per year result in unlimited growth of the balance? Explain.

Project: Population per Square Mile To work an extended application analyzing the population per square mile of the United States, visit this text's website at *LarsonPrecalculus.com.* *(Source: U.S. Census Bureau)*

3.2 Logarithmic Functions and Their Graphs

Logarithmic functions can often model scientific observations. For example, in Exercise 83 on page 218, you will use a logarithmic function that models human memory.

- **Recognize and evaluate logarithmic functions with base *a*.**
- **Graph logarithmic functions.**
- **Recognize, evaluate, and graph natural logarithmic functions.**
- **Use logarithmic functions to model and solve real-life problems.**

Logarithmic Functions

In Section 3.1, you learned that the exponential function $f(x) = a^x$ is one-to-one. It follows that $f(x) = a^x$ must have an inverse function. This inverse function is the **logarithmic function with base *a*.**

Definition of Logarithmic Function with Base *a*

For $x > 0$, $a > 0$, and $a \neq 1$,

$y = \log_a x$ if and only if $x = a^y$.

The function

$f(x) = \log_a x$ Read as "log base a of x."

is the **logarithmic function with base *a*.**

The equations $y = \log_a x$ and $x = a^y$ are equivalent. For example, $2 = \log_3 9$ is equivalent to $9 = 3^2$, and $5^3 = 125$ is equivalent to $\log_5 125 = 3$.

When evaluating logarithms, remember that *a logarithm is an exponent.* This means that $\log_a x$ is the exponent to which a must be raised to obtain x. For example, $\log_2 8 = 3$ because 2 raised to the third power is 8.

EXAMPLE 1 Evaluating Logarithms

Evaluate each logarithm at the given value of x.

a. $f(x) = \log_2 x,\ x = 32$ **b.** $f(x) = \log_3 x,\ x = 1$

c. $f(x) = \log_4 x,\ x = 2$ **d.** $f(x) = \log_{10} x,\ x = \frac{1}{100}$

Solution

a. $f(32) = \log_2 32 = 5$ because $2^5 = 32$.

b. $f(1) = \log_3 1 = 0$ because $3^0 = 1$.

c. $f(2) = \log_4 2 = \frac{1}{2}$ because $4^{1/2} = \sqrt{4} = 2$.

d. $f\left(\frac{1}{100}\right) = \log_{10}\frac{1}{100} = -2$ because $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Evaluate each logarithm at the given value of x.

a. $f(x) = \log_6 x, x = 1$ **b.** $f(x) = \log_5 x, x = \frac{1}{125}$ **c.** $f(x) = \log_7 x, x = 343$

The logarithmic function with base 10 is called the **common logarithmic function.** It is denoted by $\log_{10}$ or simply log. On most calculators, it is denoted by LOG. Example 2 shows how to use a calculator to evaluate common logarithmic functions. You will learn how to use a calculator to calculate logarithms with any base in Section 3.3.