3.2 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: **Fill in the blanks.**

1. The inverse function of the exponential function $f(x) = a^x$ is the ________ function with base a.
2. The common logarithmic function has base ________.
3. The logarithmic function $f(x) = \ln x$ is the ________ logarithmic function and has base ________.
4. The Inverse Properties of logarithms state that $\log_a a^x = x$ and ________.
5. The One-to-One Property of natural logarithms states that if $\ln x = \ln y$, then ________.
6. The domain of the natural logarithmic function is the set of ________ ________ ________.

Skills and Applications

Writing an Exponential Equation **In Exercises 7–10, write the logarithmic equation in exponential form. For example, the exponential form of $\log_5 25 = 2$ is $5^2 = 25$.**

7. $\log_4 16 = 2$
8. $\log_9 \frac{1}{81} = -2$
9. $\log_{12} 12 = 1$
10. $\log_{32} 4 = \frac{2}{5}$

Writing a Logarithmic Equation **In Exercises 11–14, write the exponential equation in logarithmic form. For example, the logarithmic form of $2^3 = 8$ is $\log_2 8 = 3$.**

11. $5^3 = 125$
12. $9^{3/2} = 27$
13. $4^{-3} = \frac{1}{64}$
14. $24^0 = 1$

Evaluating a Logarithm **In Exercises 15–20, evaluate the logarithm at the given value of x without using a calculator.**

	Function	Value
15.	$f(x) = \log_2 x$	$x = 64$
16.	$f(x) = \log_{25} x$	$x = 5$
17.	$f(x) = \log_8 x$	$x = 1$
18.	$f(x) = \log x$	$x = 10$
19.	$g(x) = \log_a x$	$x = a^{-2}$
20.	$g(x) = \log_b x$	$x = \sqrt{b}$

Evaluating a Common Logarithm on a Calculator **In Exercises 21–24, use a calculator to evaluate $f(x) = \log x$ at the given value of x. Round your result to three decimal places.**

21. $x = \frac{7}{8}$
22. $x = \frac{1}{500}$
23. $x = 12.5$
24. $x = 96.75$

Using Properties of Logarithms **In Exercises 25–28, use the properties of logarithms to simplify the expression.**

25. $\log_8 8$
26. $\log_\pi \pi^2$
27. $\log_{7.5} 1$
28. $5^{\log_5 3}$

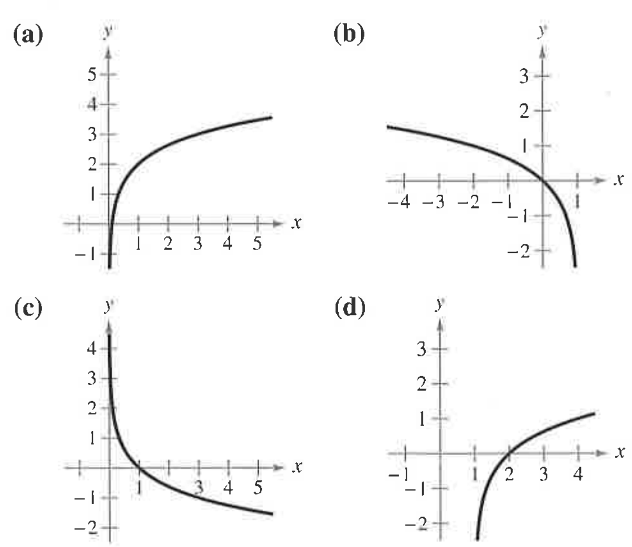
Using the One-to-One Property **In Exercises 29–32, use the One-to-One Property to solve the equation for x.**

29. $\log_5(x + 1) = \log_5 6$
30. $\log_2(x - 3) = \log_2 9$
31. $\log 11 = \log(x^2 + 7)$
32. $\log(x^2 + 6x) = \log 27$

Graphing Exponential and Logarithmic Functions **In Exercises 33–36, sketch the graphs of f and g in the same coordinate plane.**

33. $f(x) = 7^x$, $g(x) = \log_7 x$
34. $f(x) = 5^x$, $g(x) = \log_5 x$
35. $f(x) = 6^x$, $g(x) = \log_6 x$
36. $f(x) = 10^x$, $g(x) = \log x$

Matching a Logarithmic Function with Its Graph **In Exercises 37–40, use the graph of $g(x) = \log_3 x$ to match the given function with its graph. Then describe the relationship between the graphs of f and g. [The graphs are labeled (a), (b), (c), and (d).]**



37. $f(x) = \log_3 x + 2$
38. $f(x) = \log_3(x - 1)$
39. $f(x) = \log_3(1 - x)$
40. $f(x) = -\log_3 x$

Sketching the Graph of a Logarithmic Function **In Exercises 41–48, find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.**

41. $f(x) = \log_4 x$
42. $g(x) = \log_6 x$
43. $y = \log_3 x + 1$
44. $h(x) = \log_4(x - 3)$
45. $f(x) = -\log_6(x + 2)$
46. $y = \log_5(x - 1) + 4$
47. $y = \log \frac{x}{7}$
48. $y = \log(-2x)$

Writing a Natural Exponential Equation **In Exercises 49–52, write the logarithmic equation in exponential form.**

49. $\ln \frac{1}{2} = -0.693\ldots$
50. $\ln 7 = 1.945\ldots$
51. $\ln 250 = 5.521\ldots$
52. $\ln 1 = 0$

Writing a Natural Logarithmic Equation **In Exercises 53–56, write the exponential equation in logarithmic form.**

53. $e^2 = 7.3890\ldots$
54. $e^{-3/4} = 0.4723\ldots$
55. $e^{-4x} = \frac{1}{2}$
56. $e^{2x} = 3$

Evaluating a Logarithmic Function **In Exercises 57–60, use a calculator to evaluate the function at the given value of x. Round your result to three decimal places.**

	Function	Value
57.	$f(x) = \ln x$	$x = 18.42$
58.	$f(x) = 3 \ln x$	$x = 0.74$
59.	$g(x) = 8 \ln x$	$x = \sqrt{5}$
60.	$g(x) = -\ln x$	$x = \frac{1}{2}$

Using Properties of Natural Logarithms **In Exercises 61–66, use the properties of natural logarithms to simplify the expression.**

61. $e^{\ln 4}$
62. $\ln \frac{1}{e^2}$
63. $2.5 \ln 1$
64. $\frac{\ln e}{\pi}$
65. $\ln e^{\ln e}$
66. $e^{\ln(1/e)}$

Graphing a Natural Logarithmic Function **In Exercises 67–70, find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.**

67. $f(x) = \ln(x - 4)$
68. $h(x) = \ln(x + 5)$
69. $g(x) = \ln(-x)$
70. $f(x) = \ln(3 - x)$

Graphing a Natural Logarithmic Function **In Exercises 71–74, use a graphing utility to graph the function. Be sure to use an appropriate viewing window.**

71. $f(x) = \ln(x - 1)$
72. $f(x) = \ln(x + 2)$
73. $f(x) = -\ln x + 8$
74. $f(x) = 3 \ln x - 1$

Using the One-to-One Property **In Exercises 75–78, use the One-to-One Property to solve the equation for x.**

75. $\ln(x + 4) = \ln 12$
76. $\ln(x - 7) = \ln 7$
77. $\ln(x^2 - x) = \ln 6$
78. $\ln(x^2 - 2) = \ln 23$

79. **Monthly Payment** The model

$$t = 16.625 \ln \frac{x}{x - 750}, \quad x > 750$$

approximates the length of a home mortgage of \$150,000 at 6% in terms of the monthly payment. In the model, t is the length of the mortgage in years and x is the monthly payment in dollars.

(a) Approximate the lengths of a \$150,000 mortgage at 6% when the monthly payment is \$897.72 and when the monthly payment is \$1659.24.

(b) Approximate the total amounts paid over the term of the mortgage with a monthly payment of \$897.72 and with a monthly payment of \$1659.24. What amount of the total is interest costs in each case?

(c) What is the vertical asymptote for the model? Interpret its meaning in the context of the problem.

80. **Telephone Service** The percent P of households in the United States with wireless-only telephone service from 2005 through 2014 can be approximated by the model

$$P = -3.42 + 1.297t \ln t, \quad 5 \le t \le 14$$

where t represents the year, with $t = 5$ corresponding to 2005. *(Source: National Center for Health Statistics)*

(a) Approximate the percents of households with wireless-only telephone service in 2008 and 2012.

(b) Use a graphing utility to graph the function.

(c) Can the model be used to predict the percent of households with wireless-only telephone service in 2020? in 2030? Explain.

81. **Population** The time t (in years) for the world population to double when it is increasing at a continuous rate r (in decimal form) is given by $t = (\ln 2)/r$.

(a) Complete the table and interpret your results.

r	0.005	0.010	0.015	0.020	0.025	0.030
t						

(b) Use a graphing utility to graph the function.

82. Compound Interest A principal P, invested at $5\frac{1}{2}\%$ and compounded continuously, increases to an amount K times the original principal after t years, where $t = (\ln K)/0.055$.

(a) Complete the table and interpret your results.

K	1	2	4	6	8	10	12
t							

(b) Sketch a graph of the function.

83. Human Memory Model

Students in a mathematics class took an exam and then took a retest monthly with an equivalent exam. The average scores for the class are given by the human memory model

$f(t) = 80 - 17\log(t + 1), \quad 0 \le t \le 12$

where t is the time in months.

(a) Use a graphing utility to graph the model over the specified domain.

(b) What was the average score on the original exam $(t = 0)$?

(c) What was the average score after 4 months?

(d) What was the average score after 10 months?

84. Sound Intensity The relationship between the number of decibels β and the intensity of a sound I (in watts per square meter) is

$$\beta = 10\log\frac{I}{10^{-12}}.$$

(a) Determine the number of decibels of a sound with an intensity of 1 watt per square meter.

(b) Determine the number of decibels of a sound with an intensity of 10^{-2} watt per square meter.

(c) The intensity of the sound in part (a) is 100 times as great as that in part (b). Is the number of decibels 100 times as great? Explain.

Exploration

True or False? **In Exercises 85 and 86, determine whether the statement is true or false. Justify your answer.**

85. The graph of $f(x) = \log_6 x$ is a reflection of the graph of $g(x) = 6^x$ in the x-axis.

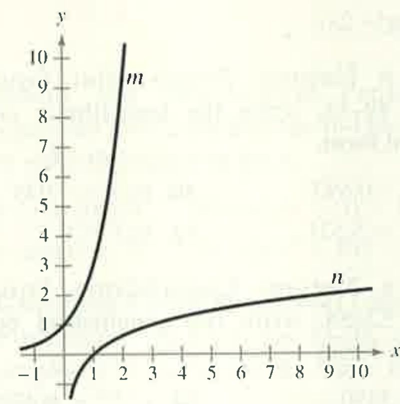
86. The graph of $f(x) = \ln(-x)$ is a reflection of the graph of $h(x) = e^{-x}$ in the line $y = -x$.

87. Graphical Reasoning Use a graphing utility to graph f and g in the same viewing window and determine which is increasing at the greater rate as x approaches $+\infty$. What can you conclude about the rate of growth of the natural logarithmic function?

(a) $f(x) = \ln x, \quad g(x) = \sqrt{x}$

(b) $f(x) = \ln x, \quad g(x) = \sqrt[4]{x}$

88. HOW DO YOU SEE IT? The figure shows the graphs of $f(x) = 3^x$ and $g(x) = \log_3 x$. [The graphs are labeled m and n.]



(a) Match each function with its graph.

(b) Given that $f(a) = b$, what is $g(b)$? Explain.

Error Analysis **In Exercises 89 and 90, describe the error.**

89.

x	1	2	8
y	0	1	3

From the table, you can conclude that y is an exponential function of x.

90.

x	1	2	5
y	2	4	32

From the table, you can conclude that y is a logarithmic function of x.

91. Numerical Analysis

(a) Complete the table for the function $f(x) = (\ln x)/x$.

x	1	5	10	10^2	10^4	10^6
$f(x)$						

(b) Use the table in part (a) to determine what value $f(x)$ approaches as x increases without bound.

(c) Use a graphing utility to confirm the result of part (b).

92. Writing Explain why $\log_a x$ is defined only for $0 < a < 1$ and $a > 1$.

3.3 Properties of Logarithms

Logarithmic functions have many real-life applications. For example, in Exercises 79–82 on page 224, you will use a logarithmic function that models the relationship between the number of decibels and the intensity of a sound.

- **Use the change-of-base formula to rewrite and evaluate logarithmic expressions.**
- **Use properties of logarithms to evaluate or rewrite logarithmic expressions.**
- **Use properties of logarithms to expand or condense logarithmic expressions.**
- **Use logarithmic functions to model and solve real-life problems.**

Change of Base

Most calculators have only two types of log keys, LOG for common logarithms (base 10) and LN for natural logarithms (base e). Although common logarithms and natural logarithms are the most frequently used, you may occasionally need to evaluate logarithms with other bases. To do this, use the **change-of-base formula.**

Change-of-Base Formula

Let a, b, and x be positive real numbers such that $a \ne 1$ and $b \ne 1$. Then $\log_a x$ can be converted to a different base as follows.

Base b	**Base 10**	**Base e**
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\log_a x = \dfrac{\log x}{\log a}$	$\log_a x = \dfrac{\ln x}{\ln a}$

One way to look at the change-of-base formula is that logarithms with base a are *constant multiples* of logarithms with base b. The constant multiplier is

$$\frac{1}{\log_b a}.$$

EXAMPLE 1 Changing Bases Using Common Logarithms

$$\log_4 25 = \frac{\log 25}{\log 4} \qquad \log_a x = \frac{\log x}{\log a}$$

$$\approx \frac{1.39794}{0.60206} \qquad \text{Use a calculator.}$$

$$\approx 2.3219 \qquad \text{Simplify.}$$

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Evaluate $\log_2 12$ using the change-of-base formula and common logarithms.

EXAMPLE 2 Changing Bases Using Natural Logarithms

$$\log_4 25 = \frac{\ln 25}{\ln 4} \qquad \log_a x = \frac{\ln x}{\ln a}$$

$$\approx \frac{3.21888}{1.38629} \qquad \text{Use a calculator.}$$

$$\approx 2.3219 \qquad \text{Simplify.}$$

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Evaluate $\log_2 12$ using the change-of-base formula and natural logarithms.