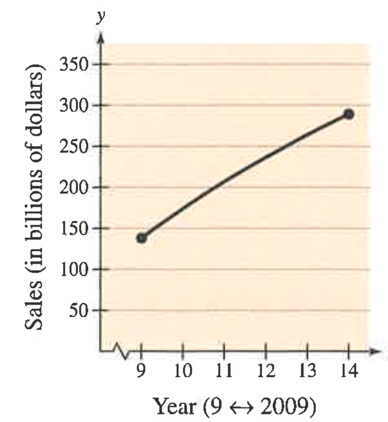
**EXAMPLE 11** **Retail Sales**

The retail sales $y$ (in billions of dollars) of e-commerce companies in the United States from 2009 through 2014 can be modeled by

$$y = -614 + 342.2 \ln t, \quad 9 \le t \le 14$$

where $t$ represents the year, with $t = 9$ corresponding to 2009 (see figure). During which year did the sales reach \$240 billion? *(Source: U.S. Census Bureau)*

Retail Sales of e-Commerce Companies




### Solution

| | |
|---|---|
| $-614 + 342.2 \ln t = y$ | Write original equation. |
| $-614 + 342.2 \ln t = 240$ | Substitute 240 for $y$. |
| $342.2 \ln t = 854$ | Add 614 to each side. |
| $\ln t = \dfrac{854}{342.2}$ | Divide each side by 342.2. |
| $e^{\ln t} = e^{854/342.2}$ | Exponentiate each side. |
| $t = e^{854/342.2}$ | Inverse Property |
| $t \approx 12$ | Use a calculator. |

The solution is $t \approx 12$. Because $t = 9$ represents 2009, it follows that the sales reached \$240 billion in 2012.

**✓ Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

In Example 11, during which year did the sales reach \$180 billion?

**Summarize** **(Section 3.4)**

1. State the One-to-One Properties and the Inverse Properties that are used to solve simple exponential and logarithmic equations *(page 226)*. For an example of solving simple exponential and logarithmic equations, see Example 1.
2. Describe strategies for solving exponential equations *(pages 227 and 228)*. For examples of solving exponential equations, see Examples 2–5.
3. Describe strategies for solving logarithmic equations *(pages 229 and 230)*. For examples of solving logarithmic equations, see Examples 6–9.
4. Describe examples of how to use exponential and logarithmic equations to model and solve real-life problems *(pages 231 and 232, Examples 10 and 11)*.



## 3.4 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

**Vocabulary:** **Fill in the blanks.**

1. To solve exponential and logarithmic equations, you can use the One-to-One and Inverse Properties below.
   (a) $a^x = a^y$ if and only if ________.
   (b) $\log_a x = \log_a y$ if and only if ________.
   (c) $a^{\log_a x} =$ ________
   (d) $\log_a a^x =$ ________
2. An ________ solution does not satisfy the original equation.

### Skills and Applications

**Determining Solutions** **In Exercises 3–6, determine whether each $x$-value is a solution (or an approximate solution) of the equation.**

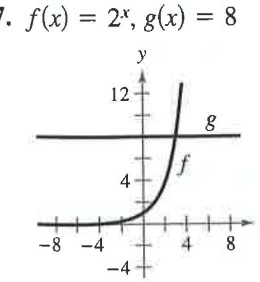
3. $4^{2x-7} = 64$
   (a) $x = 5$
   (b) $x = 2$
   (c) $x = \frac{1}{2}(\log_4 64 + 7)$
4. $4e^{x-1} = 60$
   (a) $x = 1 + \ln 15$
   (b) $x \approx 1.708$
   (c) $x = \ln 16$
5. $\log_2(x + 3) = 10$
   (a) $x = 1021$
   (b) $x = 17$
   (c) $x = 10^2 - 3$
6. $\ln(2x + 3) = 5.8$
   (a) $x = \frac{1}{2}(-3 + \ln 5.8)$
   (b) $x = \frac{1}{2}(-3 + e^{5.8})$
   (c) $x \approx 163.650$

**Solving a Simple Equation** **In Exercises 7–16, solve for $x$.**

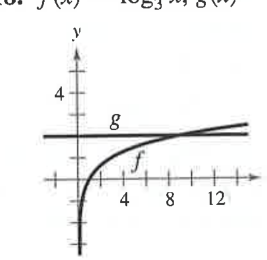
7. $4^x = 16$
8. $(\frac{1}{2})^x = 32$
9. $\ln x - \ln 2 = 0$
10. $\log x - \log 10 = 0$
11. $e^x = 2$
12. $e^x = \frac{1}{3}$
13. $\ln x = -1$
14. $\log x = -2$
15. $\log_4 x = 3$
16. $\log_5 x = \frac{1}{2}$

**Approximating a Point of Intersection** **In Exercises 17 and 18, approximate the point of intersection of the graphs of $f$ and $g$. Then solve the equation $f(x) = g(x)$ algebraically to verify your approximation.**

17. $f(x) = 2^x$, $g(x) = 8$




18. $f(x) = \log_3 x$, $g(x) = 2$




**Solving an Exponential Equation** **In Exercises 19–46, solve the exponential equation algebraically. Approximate the result to three decimal places, if necessary.**

19. $e^x = e^{x^2-2}$
20. $e^{x^2-3} = e^{x-2}$
21. $4(3^x) = 20$
22. $4e^x = 91$
23. $e^x - 8 = 31$
24. $5^x + 8 = 26$
25. $3^{2x} = 80$
26. $4^{-3t} = 0.10$
27. $3^{2-x} = 400$
28. $7^{-3-x} = 242$
29. $8(10^{3x}) = 12$
30. $8(3^{6-x}) = 40$
31. $e^{3x} = 12$
32. $500e^{-2x} = 125$
33. $7 - 2e^x = 5$
34. $-14 + 3e^x = 11$
35. $6(2^{3x-1}) - 7 = 9$
36. $8(4^{6-2x}) + 13 = 41$
37. $3^x = 2^{x-1}$
38. $e^{x+1} = 2^{x+2}$
39. $4^x = 5^{x^2}$
40. $3^{x^2} = 7^{6-x}$
41. $e^{2x} - 4e^x - 5 = 0$
42. $e^{2x} - 5e^x + 6 = 0$
43. $\dfrac{1}{1 - e^x} = 5$
44. $\dfrac{100}{1 + e^{2x}} = 1$
45. $\left(1 + \dfrac{0.065}{365}\right)^{365t} = 4$
46. $\left(1 + \dfrac{0.10}{12}\right)^{12t} = 2$

**Solving a Logarithmic Equation** **In Exercises 47–62, solve the logarithmic equation algebraically. Approximate the result to three decimal places, if necessary.**

47. $\ln x = -3$
48. $\ln x - 7 = 0$
49. $2.1 = \ln 6x$
50. $\log 3z = 2$
51. $3 - 4 \ln x = 11$
52. $3 + 8 \ln x = 7$
53. $6 \log_3 0.5x = 11$
54. $4 \log(x - 6) = 11$
55. $\ln x - \ln(x + 1) = 2$
56. $\ln x + \ln(x + 1) = 1$
57. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$
58. $\ln(x + 1) - \ln(x - 2) = \ln x$
59. $\log(3x + 4) = \log(x - 10)$
60. $\log_2 x + \log_2(x + 2) = \log_2(x + 6)$
61. $\log_4 x - \log_4(x - 1) = \frac{1}{2}$
62. $\log 8x - \log(1 + \sqrt{x}) = 2$

**Using Technology In Exercises 63–70, use a graphing utility to graphically solve the equation. Approximate the result to three decimal places. Verify your result algebraically.**

**63.** $5^x = 212$ **64.** $6e^{1-x} = 25$

**65.** $8e^{-2x/3} = 11$ **66.** $e^{0.09t} = 3$

**67.** $3 - \ln x = 0$ **68.** $10 - 4\ln(x - 2) = 0$

**69.** $2\ln(x + 3) = 3$ **70.** $\ln(x + 1) = 2 - \ln x$

**Compound Interest In Exercises 71 and 72, you invest \$2500 in an account at interest rate $r$, compounded continuously. Find the time required for the amount to (a) double and (b) triple.**

**71.** $r = 0.025$ **72.** $r = 0.0375$

**Algebra of Calculus In Exercises 73–80, solve the equation algebraically. Round your result to three decimal places, if necessary. Verify your answer using a graphing utility.**

**73.** $2x^2e^{2x} + 2xe^{2x} = 0$ **74.** $-x^2e^{-x} + 2xe^{-x} = 0$

**75.** $-xe^{-x} + e^{-x} = 0$ **76.** $e^{-2x} - 2xe^{-2x} = 0$

**77.** $\dfrac{1 + \ln x}{2} = 0$ **78.** $\dfrac{1 - \ln x}{x^2} = 0$

**79.** $2x\ln x + x = 0$ **80.** $2x\ln\left(\dfrac{1}{x}\right) - x = 0$

**81. Average Heights** The percent $m$ of American males between the ages of 20 and 29 who are under $x$ inches tall is modeled by

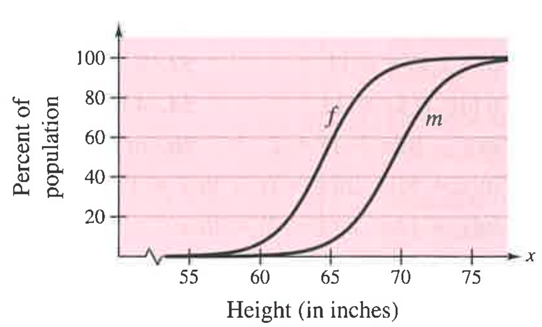
$$m(x) = \frac{100}{1 + e^{-0.5536(x-69.51)}}, \quad 64 \le x \le 78$$

and the percent $f$ of American females between the ages of 20 and 29 who are under $x$ inches tall is modeled by

$$f(x) = \frac{100}{1 + e^{-0.5834(x-64.49)}}, \quad 60 \le x \le 78.$$

*(Source: U.S. National Center for Health Statistics)*

(a) Use the graph to determine any horizontal asymptotes of the graphs of the functions. Interpret the meaning in the context of the problem.




(b) What is the average height of each sex?

Alexander Kuguchin/Shutterstock.com

**82. Demand** The demand equation for a smartphone is

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right).$$

Find the demand $x$ for each price.

(a) $p = \$169$

(b) $p = \$299$

**83. Ecology** The number $N$ of beavers in a given area after $x$ years can be approximated by

$$N = 5.5 \cdot 10^{0.23x}, \quad 0 \le x \le 10.$$

Use the model to approximate how many years it will take for the beaver population to reach 78.

**84. Ecology** The number $N$ of trees of a given species per acre is approximated by the model

$$N = 3500(10^{-0.12x}), \quad 3 \le x \le 30$$

where $x$ is the average diameter of the trees (in inches) 4.5 feet above the ground. Use the model to approximate the average diameter of the trees in a test plot when $N = 22$.

**85. Population** The population $P$ (in thousands) of Alaska in the years 2005 through 2015 can be modeled by

$$P = 75\ln t + 540, \quad 5 \le t \le 15$$

where $t$ represents the year, with $t = 5$ corresponding to 2005. During which year did the population of Alaska exceed 720 thousand? *(Source: U.S. Census Bureau)*

**86. Population** The population $P$ (in thousands) of Montana in the years 2005 through 2015 can be modeled by

$$P = 81\ln t + 807, \quad 5 \le t \le 15$$

where $t$ represents the year, with $t = 5$ corresponding to 2005. During which year did the population of Montana exceed 965 thousand? *(Source: U.S. Census Bureau)*

**87. Temperature** An object at a temperature of 80°C is placed in a room at 20°C. The temperature of the object is given by

$$T = 20 + 60e^{-0.06m}$$

where $m$ represents the number of minutes after the object is placed in the room. How long does it take the object to reach a temperature of 70°C?



**88. Temperature** An object at a temperature of 160°C was removed from a furnace and placed in a room at 20°C. The temperature $T$ of the object was measured each hour $h$ and recorded in the table. A model for the data is

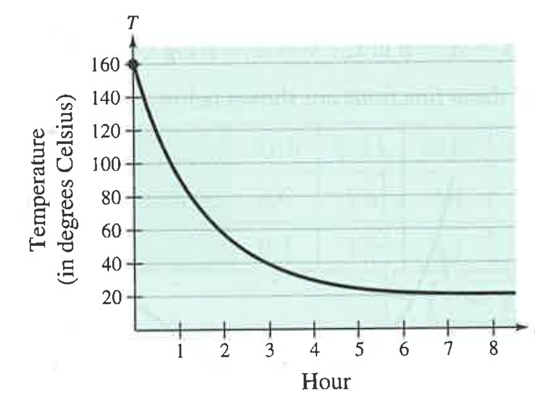
$$T = 20 + 140e^{-0.68h}.$$

DATA

Spreadsheet at LarsonPrecalculus.com

| Hour, $h$ | Temperature, $T$ |
|---|---|
| 0 | 160° |
| 1 | 90° |
| 2 | 56° |
| 3 | 38° |
| 4 | 29° |
| 5 | 24° |

(a) The figure below shows the graph of the model. Use the graph to identify the horizontal asymptote of the model and interpret the asymptote in the context of the problem.




(b) Use the model to approximate the time it took for the object to reach a temperature of 100°C.

## Exploration

**True or False? In Exercises 89–92, rewrite each verbal statement as an equation. Then decide whether the statement is true or false. Justify your answer.**

**89.** The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers.

**90.** The logarithm of the sum of two numbers is equal to the product of the logarithms of the numbers.

**91.** The logarithm of the difference of two numbers is equal to the difference of the logarithms of the numbers.

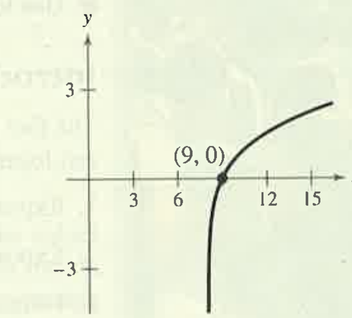
**92.** The logarithm of the quotient of two numbers is equal to the difference of the logarithms of the numbers.

**93. Think About It** Is it possible for a logarithmic equation to have more than one extraneous solution? Explain.

**94. HOW DO YOU SEE IT?** Solving $\log_3 x + \log_3(x - 8) = 2$ algebraically, the solutions appear to be $x = 9$ and $x = -1$. Use the graph of

$$y = \log_3 x + \log_3(x - 8) - 2$$

to determine whether each value is an actual solution of the equation. Explain.




**95. Finance** You are investing $P$ dollars at an annual interest rate of $r$, compounded continuously, for $t$ years. Which change below results in the highest value of the investment? Explain.

(a) Double the amount you invest.

(b) Double your interest rate.

(c) Double the number of years.

**96. Think About It** Are the times required for the investments in Exercises 71 and 72 to quadruple twice as long as the times for them to double? Give a reason for your answer and verify your answer algebraically.

**97. Effective Yield** The *effective yield* of an investment plan is the percent increase in the balance after 1 year. Find the effective yield for each investment plan. Which investment plan has the greatest effective yield? Which investment plan will have the highest balance after 5 years?

(a) 7% annual interest rate, compounded annually

(b) 7% annual interest rate, compounded continuously

(c) 7% annual interest rate, compounded quarterly

(d) 7.25% annual interest rate, compounded quarterly

**98. Graphical Reasoning** Let $f(x) = \log_a x$ and $g(x) = a^x$, where $a > 1$.

(a) Let $a = 1.2$ and use a graphing utility to graph the two functions in the same viewing window. What do you observe? Approximate any points of intersection of the two graphs.

(b) Determine the value(s) of $a$ for which the two graphs have one point of intersection.

(c) Determine the value(s) of $a$ for which the two graphs have two points of intersection.