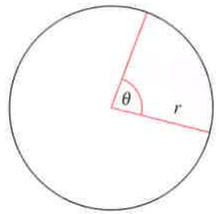


Figure 4.14

A **sector** of a circle is the region bounded by two radii of the circle and their intercepted arc (see Figure 4.14).

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle θ is

$$A = \frac{1}{2}r^2\theta$$

where θ is measured in radians.

EXAMPLE 8 **Area of a Sector of a Circle**

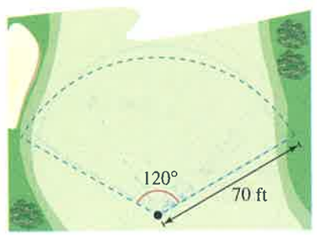


Figure 4.15

A sprinkler on a golf course fairway sprays water over a distance of 70 feet and rotates through an angle of 120° (see Figure 4.15). Find the area of the fairway watered by the sprinkler.

Solution

First convert 120° to radian measure.

$$\theta = 120^\circ$$

$$= (120\ \text{deg})\left(\frac{\pi\ \text{rad}}{180\ \text{deg}}\right) \qquad \text{Multiply by } \frac{\pi\ \text{rad}}{180^\circ}.$$

$$= \frac{2\pi}{3}\ \text{radians}$$

Then, using $\theta = 2\pi/3$ and $r = 70$, the area is

$$A = \frac{1}{2}r^2\theta \qquad \text{Formula for the area of a sector of a circle}$$

$$= \frac{1}{2}(70)^2\left(\frac{2\pi}{3}\right) \qquad \text{Substitute for } r \text{ and } \theta.$$

$$= \frac{4900\pi}{3} \qquad \text{Multiply.}$$

$$\approx 5131\ \text{square feet.} \qquad \text{Use a calculator.}$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

A sprinkler sprays water over a distance of 40 feet and rotates through an angle of 80°. Find the area watered by the sprinkler.

Summarize **(Section 4.1)**

1. Describe an angle *(page 260)*.
2. Explain how to use radian measure *(page 261)*. For examples involving radian measure, see Examples 1 and 2.
3. Explain how to use degree measure *(page 263)*. For examples involving degree measure, see Examples 3 and 4.
4. Describe real-life applications involving angles and their measure *(pages 264–266, Examples 5–8)*.

4.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

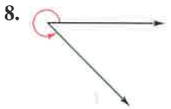
Vocabulary: **Fill in the blanks.**

1. Two angles that have the same initial and terminal sides are _____.
2. One _____ is the measure of a central angle that intercepts an arc equal in length to the radius of the circle.
3. Two positive angles that have a sum of $\pi/2$ are _____ angles, and two positive angles that have a sum of π are _____ angles.
4. The angle measure that is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about an angle's vertex is one _____.
5. The _____ speed of a particle is the ratio of the arc length traveled to the elapsed time, and the _____ speed of a particle is the ratio of the change in the central angle to the elapsed time.
6. The area A of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula _____.

Skills and Applications

Estimating an Angle **In Exercises 7–10, estimate the angle to the nearest one-half radian.**

7. 

8. 

9. 

10. 

Determining Quadrants **In Exercises 11 and 12, determine the quadrant in which each angle lies.**

11. (a) $\frac{\pi}{4}$ (b) $-\frac{5\pi}{4}$
12. (a) $-\frac{\pi}{6}$ (b) $\frac{11\pi}{9}$

Sketching Angles **In Exercises 13 and 14, sketch each angle in standard position.**

13. (a) $\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$
14. (a) $\frac{5\pi}{2}$ (b) 4

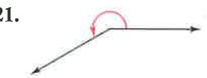
Finding Coterminal Angles **In Exercises 15 and 16, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.**

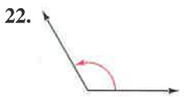
15. (a) $\frac{\pi}{6}$ (b) $-\frac{5\pi}{6}$
16. (a) $\frac{2\pi}{3}$ (b) $-\frac{9\pi}{4}$

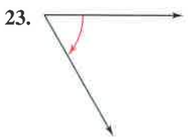
Complementary and Supplementary Angles **In Exercises 17–20, find (if possible) the complement and supplement of each angle.**

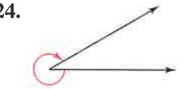
17. (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$
18. (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
19. (a) 1 (b) 2
20. (a) 3 (b) 1.5

Estimating an Angle **In Exercises 21–24, estimate the number of degrees in the angle.**

21. 

22. 

23. 

24. 

Determining Quadrants **In Exercises 25 and 26, determine the quadrant in which each angle lies.**

25. (a) 130° (b) $-8.3°$
26. (a) $-132° 50'$ (b) 3.4°

Sketching Angles **In Exercises 27 and 28, sketch each angle in standard position.**

27. (a) 270° (b) $-120°$
28. (a) 135° (b) $-750°$

Finding Coterminal Angles **In Exercises 29 and 30, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.**

29. (a) 120° (b) $-210°$
30. (a) 45° (b) $-420°$

Complementary and Supplementary Angles **In Exercises 31–34, find (if possible) the complement and supplement of each angle.**

31. (a) 18° (b) 85°
32. (a) 46° (b) 93°
33. (a) 24° (b) 126°
34. (a) 130° (b) 170°

Converting from Degrees to Radians **In Exercises 35 and 36, convert each degree measure to radian measure as a multiple of π. Do not use a calculator.**

35. (a) $120°$ (b) $-20°$

36. (a) $-60°$ (b) $144°$

Converting from Radians to Degrees **In Exercises 37 and 38, convert each radian measure to degree measure. Do not use a calculator.**

37. (a) $\frac{3\pi}{2}$ (b) $-\frac{7\pi}{6}$

38. (a) $-\frac{7\pi}{12}$ (b) $\frac{5\pi}{4}$

Converting from Degrees to Radians **In Exercises 39–42, convert the degree measure to radian measure. Round to three decimal places.**

39. $45°$

40. $-48.27°$

41. $-0.54°$

42. $345°$

Converting from Radians to Degrees **In Exercises 43–46, convert the radian measure to degree measure. Round to three decimal places, if necessary.**

43. $\frac{5\pi}{11}$

44. $\frac{15\pi}{8}$

45. -4.2π

46. -0.57

Converting to Decimal Degree Form **In Exercises 47 and 48, convert each angle measure to decimal degree form.**

47. (a) $54° 45'$ (b) $-128° 30'$

48. (a) $135° 10' 36''$ (b) $-408° 16' 20''$

Converting to D° M′ S″ Form **In Exercises 49 and 50, convert each angle measure to D° M′ S″ form.**

49. (a) $240.6°$ (b) $-145.8°$

50. (a) $345.12°$ (b) $-3.58°$

Finding Arc Length **In Exercises 51 and 52, find the length of the arc on a circle of radius r intercepted by a central angle θ.**

51. $r = 15$ inches, $\theta = 120°$

52. $r = 3$ meters, $\theta = 150°$

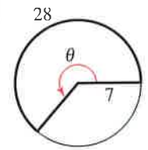
Finding the Central Angle **In Exercises 53 and 54, find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.**

53. $r = 80$ kilometers, $s = 150$ kilometers

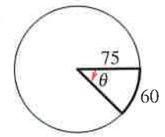
54. $r = 14$ feet, $s = 8$ feet

Finding the Central Angle **In Exercises 55 and 56, find the radian measure of the central angle.**

55.



56.



Area of a Sector of a Circle **In Exercises 57 and 58, find the area of the sector of a circle of radius r and central angle θ.**

57. $r = 6$ inches, $\theta = \frac{\pi}{3}$

58. $r = 2.5$ feet, $\theta = 225°$

Error Analysis **In Exercises 59 and 60, describe the error.**

59. $20° = (20 \text{ deg})\left(\frac{180 \text{ rad}}{\pi \text{ deg}}\right) = \frac{3600}{\pi} \text{ rad}$

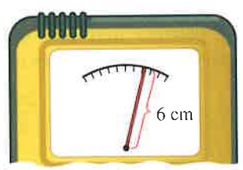
60. A circle has a radius of 6 millimeters. The length of the arc intercepted by a central angle of $72°$ is

$$\begin{aligned} s &= r\theta \\ &= 6(72) \\ &= 432 \text{ millimeters.} \end{aligned}$$

Earth-Space Science **In Exercises 61 and 62, find the distance between the cities. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (one city is due north of the other).**

	City	Latitude
61.	Dallas, Texas	32° 47′ 9″ N
	Omaha, Nebraska	41° 15′ 50″ N
62.	San Francisco, California	37° 47′ 36″ N
	Seattle, Washington	47° 37′ 18″ N

63. **Instrumentation** The pointer on a voltmeter is 6 centimeters in length (see figure). Find the number of degrees through which the pointer rotates when it moves 2.5 centimeters on the scale.



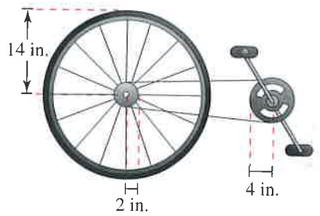
64. **Linear and Angular Speed** A $7\frac{1}{4}$-inch circular power saw blade rotates at 5200 revolutions per minute.
   (a) Find the angular speed of the saw blade in radians per minute.
   (b) Find the linear speed (in feet per minute) of the saw teeth as they contact the wood being cut.

65. **Linear and Angular Speed** A carousel with a 50-foot diameter makes 4 revolutions per minute.
   (a) Find the angular speed of the carousel in radians per minute.
   (b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

66. **Linear and Angular Speed** A Blu-ray disc is approximately 12 centimeters in diameter. The drive motor of a Blu-ray player is able to rotate up to 10,000 revolutions per minute.
   (a) Find the maximum angular speed (in radians per second) of a Blu-ray disc as it rotates.
   (b) Find the maximum linear speed (in meters per second) of a point on the outermost track as the disc rotates.

67. **Linear and Angular Speed** A computerized spin balance machine rotates a 25-inch-diameter tire at 480 revolutions per minute.
   (a) Find the road speed (in miles per hour) at which the tire is being balanced.
   (b) At what rate should the spin balance machine be set so that the tire is being tested for 55 miles per hour?

68. **Speed of a Bicycle** The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist pedals at a rate of 1 revolution per second.



   (a) Find the speed of the bicycle in feet per second and miles per hour.
   (b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

   (c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

Connel/Shutterstock.com

69. **Area** A sprinkler on a golf green is set to spray water over a distance of 15 meters and to rotate through an angle of $150°$. Draw a diagram that shows the region that can be irrigated with the sprinkler. Find the area of the region.

70. **Area** A car's rear windshield wiper rotates $125°$. The total length of the wiper mechanism is 25 inches and the length of the wiper blade is 14 inches. Find the area wiped by the wiper blade.

Exploration

True or False? **In Exercises 71–74, determine whether the statement is true or false. Justify your answer.**

71. An angle measure containing π must be in radian measure.

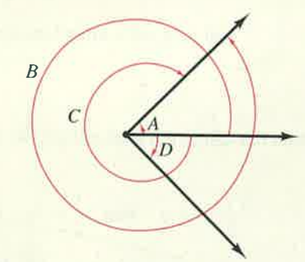
72. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

73. The difference between the measures of two coterminal angles is always a multiple of $360°$ when expressed in degrees and is always a multiple of 2π radians when expressed in radians.

74. An angle that measures $-1260°$ lies in Quadrant III.

75. **Writing** When the radius of a circle increases and the magnitude of a central angle is held constant, how does the length of the intercepted arc change? Explain.

76. **HOW DO YOU SEE IT?** Determine which angles in the figure are coterminal angles with angle A. Explain.



77. **Think About It** A fan motor turns at a given angular speed. How does the speed of the tips of the blades change when a fan of greater diameter is installed on the motor? Explain.

78. **Think About It** Is a degree or a radian the larger unit of measure? Explain.

79. **Proof** Prove that the area of a circular sector of radius r with central angle θ is $A = \frac{1}{2}\theta r^2$, where θ is measured in radians.