EXAMPLE 2 Evaluating Sine and Cosine

a. Because $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$, you have $\sin\frac{13\pi}{6} = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$.

b. Because $-\frac{7\pi}{2} = -4\pi + \frac{\pi}{2}$, you have

$$\cos\left(-\frac{7\pi}{2}\right) = \cos\left(-4\pi + \frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0.$$

c. For $\sin t = \frac{4}{5}$, $\sin(-t) = -\frac{4}{5}$ because the sine function is odd.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

a. Use the period of the cosine function to evaluate $\cos(9\pi/2)$.

b. Use the period of the sine function to evaluate $\sin(-7\pi/3)$.

c. Evaluate $\cos t$ given that $\cos(-t) = 0.3$.

When evaluating a trigonometric function with a calculator, set the calculator to the desired *mode* of measurement (*degree* or *radian*). Most calculators do not have keys for the cosecant, secant, and cotangent functions. To evaluate these functions, you can use the $\boxed{x^{-1}}$ key with their respective reciprocal functions: sine, cosine, and tangent. For example, to evaluate $\csc(\pi/8)$, use the fact that

$$\csc\frac{\pi}{8} = \frac{1}{\sin(\pi/8)}$$

and enter the keystroke sequence below in *radian* mode.

(SIN (π ÷ 8)) x^{-1} ENTER   Display 2.6131259

▷ TECHNOLOGY When evaluating trigonometric functions with a calculator, remember to enclose all fractional angle measures in parentheses. For example, to evaluate $\sin t$ for $t = \pi/6$, enter

SIN (π ÷ 6) ENTER.

These keystrokes yield the correct value of 0.5. Note that some calculators automatically place a left parenthesis after trigonometric functions.

EXAMPLE 3 Using a Calculator

Function	Mode	Calculator Keystrokes	Display
a. $\sin\frac{2\pi}{3}$	Radian	SIN (2 π ÷ 3) ENTER	0.8660254
b. cot 1.5	Radian	(TAN (1.5)) x^{-1} ENTER	0.0709148

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Use a calculator to evaluate (a) $\sin(5\pi/7)$ and (b) csc 2.0.

Summarize (Section 4.2)

1. Explain how to identify a unit circle and describe its relationship to real numbers *(page 270)*.
2. State the unit circle definitions of trigonometric functions *(page 271)*. For an example of evaluating trigonometric functions using the unit circle, see Example 1.
3. Explain how to use domain and period to evaluate sine and cosine functions *(page 273)*, and describe how to use a calculator to evaluate trigonometric functions *(page 274)*. For an example of using domain and period to evaluate sine and cosine functions, see Example 2. For an example of using a calculator to evaluate trigonometric functions, see Example 3.

4.2 Exercises

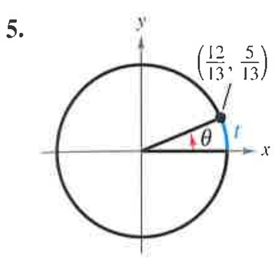
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

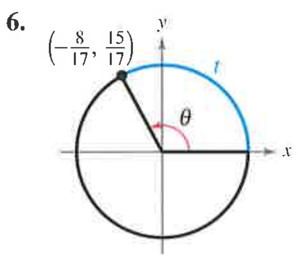
Vocabulary: Fill in the blanks.

1. Each real number t corresponds to a point (x, y) on the ________ ________.
2. A function f is ________ when there exists a positive real number c such that $f(t + c) = f(t)$ for all t in the domain of f.
3. The smallest number c for which a function f is periodic is the ________ of f.
4. A function f is ________ when $f(-t) = -f(t)$ and ________ when $f(-t) = f(t)$.

Skills and Applications

Evaluating Trigonometric Functions In Exercises 5–8, find the exact values of the six trigonometric functions of the real number t.

5.


6.


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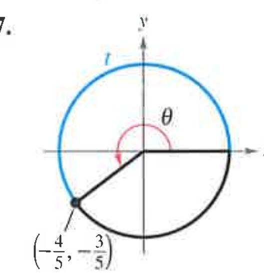
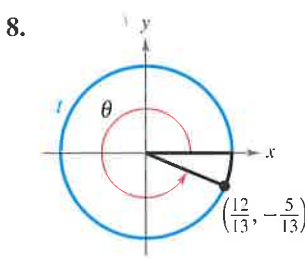


8.


Finding a Point on the Unit Circle In Exercises 9–12, find the point (x, y) on the unit circle that corresponds to the real number t.

9. $t = \pi/2$ 10. $t = \pi/4$
11. $t = 5\pi/6$ 12. $t = 4\pi/3$

Evaluating Sine, Cosine, and Tangent In Exercises 13–22, evaluate (if possible) the sine, cosine, and tangent at the real number.

13. $t = \frac{\pi}{4}$ 14. $t = \frac{\pi}{3}$
15. $t = -\frac{\pi}{6}$ 16. $t = -\frac{\pi}{4}$
17. $t = -\frac{7\pi}{4}$ 18. $t = -\frac{4\pi}{3}$
19. $t = \frac{11\pi}{6}$ 20. $t = \frac{5\pi}{3}$
21. $t = -\frac{3\pi}{2}$ 22. $t = -2\pi$

Evaluating Trigonometric Functions In Exercises 23–30, evaluate (if possible) the six trigonometric functions at the real number.

23. $t = 2\pi/3$ 24. $t = 5\pi/6$
25. $t = 4\pi/3$ 26. $t = 7\pi/4$
27. $t = -5\pi/3$ 28. $t = -3\pi/2$
29. $t = -\pi/2$ 30. $t = -\pi$

Using Period to Evaluate Sine and Cosine In Exercises 31–36, evaluate the trigonometric function using its period as an aid.

31. $\sin 4\pi$ 32. $\cos 3\pi$
33. $\cos(7\pi/3)$ 34. $\sin(9\pi/4)$
35. $\sin(19\pi/6)$ 36. $\sin(-8\pi/3)$

Using the Value of a Function In Exercises 37–42, use the given value to evaluate each function.

37. $\sin t = \frac{1}{2}$
 (a) $\sin(-t)$
 (b) $\csc(-t)$

38. $\sin(-t) = \frac{3}{8}$
 (a) $\sin t$
 (b) $\csc t$

39. $\cos(-t) = -\frac{1}{5}$
 (a) $\cos t$
 (b) $\sec(-t)$

40. $\cos t = -\frac{3}{4}$
 (a) $\cos(-t)$
 (b) $\sec(-t)$

41. $\sin t = \frac{4}{5}$
 (a) $\sin(\pi - t)$
 (b) $\sin(t + \pi)$

42. $\cos t = \frac{4}{5}$
 (a) $\cos(\pi - t)$
 (b) $\cos(t + \pi)$

Using a Calculator In Exercises 43–48, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

43. sin 0.6 44. $\cos(-2.8)$
45. $\tan(\pi/8)$ 46. $\tan(5\pi/7)$
47. sec 3.1 48. $\cot(-1.1)$

49. Harmonic Motion The displacement from equilibrium of an oscillating weight suspended by a spring is given by

$y(t) = \frac{1}{2}\cos 6t$

where y is the displacement in feet and t is the time in seconds. Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

50. Harmonic Motion
The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by

$y(t) = \frac{1}{4}e^{-t}\cos 6t$

where y is the displacement in feet and t is the time in seconds.

(a) Complete the table

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y					

(b) Use the *table* feature of a graphing utility to approximate the time when the weight reaches equilibrium.

(c) What appears to happen to the displacement as t increases?

Exploration

True or False? **In Exercises 51–54, determine whether the statement is true or false. Justify your answer.**

51. Because $\sin(-t) = -\sin t$, the sine of a negative angle is a negative number.

52. The real number 0 corresponds to the point (0, 1) on the unit circle.

53. $\tan a = \tan(a - 6\pi)$

54. $\cos\left(-\frac{7\pi}{2}\right) = \cos\left(\pi + \frac{\pi}{2}\right)$

55. Conjecture Let (x_1, y_1) and (x_2, y_2) be points on the unit circle corresponding to $t = t_1$ and $t = \pi - t_1$, respectively.

(a) Identify the symmetry of the points (x_1, y_1) and (x_2, y_2).

(b) Make a conjecture about any relationship between $\sin t_1$ and $\sin(\pi - t_1)$.

(c) Make a conjecture about any relationship between $\cos t_1$ and $\cos(\pi - t_1)$.

56. Using the Unit Circle Use the unit circle to verify that the cosine and secant functions are even and that the sine, cosecant, tangent, and cotangent functions are odd.

57. Error Analysis Describe the error.

Your classmate uses a calculator to evaluate $\tan(\pi/2)$ and gets a result of 0.0274224385.

58. Verifying Expressions Are Not Equal Verify that

$\sin(t_1 + t_2) \neq \sin t_1 + \sin t_2$

by approximating sin 0.25, sin 0.75, and sin 1.

59. Using Technology With a graphing utility in *radian* and *parametric* modes, enter the equations

$X_{1T} = \cos T$ and $Y_{1T} = \sin T$

and use the settings below.

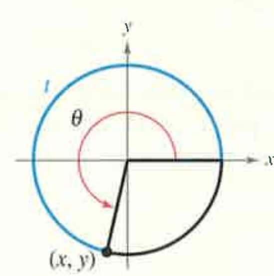
Tmin = 0, Tmax = 6.3, Tstep = 0.1
Xmin = −1.5, Xmax = 1.5, Xscl = 1
Ymin = −1, Ymax = 1, Yscl = 1

(a) Graph the entered equations and describe the graph.

(b) Use the *trace* feature to move the cursor around the graph. What do the t-values represent? What do the x- and y-values represent?

(c) What are the least and greatest values of x and y?

60. HOW DO YOU SEE IT? Use the figure below.



(a) Are all of the trigonometric functions of t defined? Explain.

(b) For those trigonometric functions that are defined, determine whether the sign of the trigonometric function is positive or negative. Explain.

61. Think About It Because $f(t) = \sin t$ is an odd function and $g(t) = \cos t$ is an even function, what can be said about the function $h(t) = f(t)g(t)$?

62. Think About It Because $f(t) = \sin t$ and $g(t) = \tan t$ are odd functions, what can be said about the function $h(t) = f(t)g(t)$?

Richard Megna/Fundamental Photographs

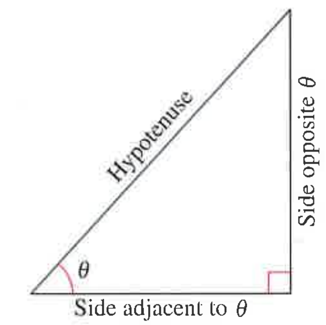
4.3 Right Triangle Trigonometry

Right triangle trigonometry has many real-life applications. For example, in Exercise 72 on page 287, you will use right triangle trigonometry to analyze the height of a helium-filled balloon.

- **Evaluate trigonometric functions of acute angles.**
- **Use fundamental trigonometric identities.**
- **Use trigonometric functions to model and solve real-life problems.**

The Six Trigonometric Functions

This section introduces the trigonometric functions from a *right triangle* perspective. Consider the right triangle shown below, in which one acute angle is labeled θ. Relative to the angle θ, the three sides of the triangle are the **hypotenuse,** the **opposite side** (the side opposite the angle θ), and the **adjacent side** (the side adjacent to the angle θ).



Using the lengths of these three sides, you can form six ratios that define the six trigonometric functions of the acute angle θ.

sine cosecant cosine secant tangent cotangent

In the definitions below,

$0° < \theta < 90°$

(θ lies in the first quadrant). For such angles, the value of each trigonometric function is *positive*.

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined below. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\cot\theta = \frac{\text{adj}}{\text{opp}}$

The abbreviations

opp, *adj,* and *hyp*

represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

adj = the length of the side *adjacent to* θ

hyp = the length of the *hypotenuse*

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