

4.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.**Vocabulary:** Fill in the blanks.

In Exercises 1–6, let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.


1. $\sin \theta =$ _____ 2. $\frac{r}{y} =$ _____ 3. $\tan \theta =$ _____

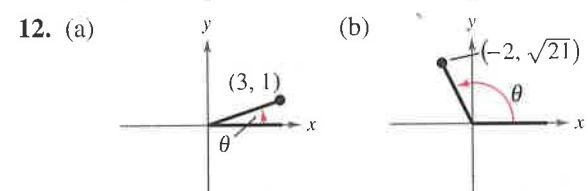
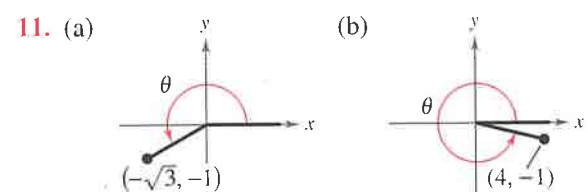
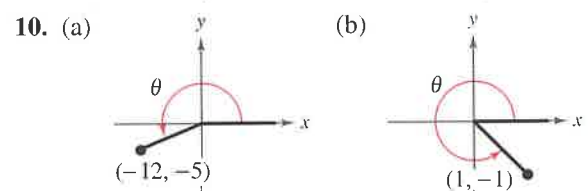
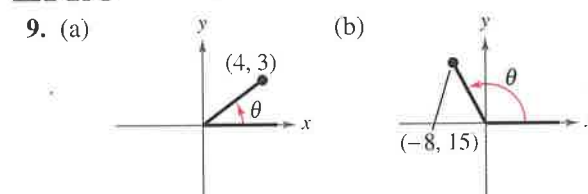
4. $\sec \theta =$ _____ 5. $\frac{x}{r} =$ _____ 6. $\frac{x}{y} =$ _____

7. Because $r = \sqrt{x^2 + y^2}$ cannot be _____, the sine and cosine functions are _____ for any real value of θ .

8. The acute angle formed by the terminal side of an angle θ in standard position and the horizontal axis is the _____ angle of θ and is denoted by θ' .

Skills and Applications

 **Evaluating Trigonometric Functions** In Exercises 9–12, find the exact values of the six trigonometric functions of each angle θ .




Evaluating Trigonometric Functions In Exercises 13–18, the point is on the terminal side of an angle in standard position. Find the exact values of the six trigonometric functions of the angle.


13. $(5, 12)$ 14. $(8, 15)$
 15. $(-5, -2)$ 16. $(-4, 10)$
 17. $(-5.4, 7.2)$ 18. $(3\frac{1}{2}, -2\sqrt{15})$

Determining a Quadrant In Exercises 19–22, determine the quadrant in which θ lies.

19. $\sin \theta > 0$, $\cos \theta > 0$ 20. $\sin \theta < 0$, $\cos \theta < 0$
 21. $\csc \theta > 0$, $\tan \theta < 0$ 22. $\sec \theta > 0$, $\cot \theta < 0$

 **Evaluating Trigonometric Functions** In Exercises 23–32, find the exact values of the remaining trigonometric functions of θ satisfying the given conditions.

23. $\tan \theta = \frac{15}{8}$, $\sin \theta > 0$
 24. $\cos \theta = \frac{8}{17}$, $\tan \theta < 0$
 25. $\sin \theta = 0.6$, θ lies in Quadrant II.
 26. $\cos \theta = -0.8$, θ lies in Quadrant III.
 27. $\cot \theta = -3$, $\cos \theta > 0$
 28. $\csc \theta = 4$, $\cot \theta < 0$
 29. $\cos \theta = 0$, $\csc \theta = 1$
 30. $\sin \theta = 0$, $\sec \theta = -1$
 31. $\cot \theta$ is undefined, $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$
 32. $\tan \theta$ is undefined, $\pi \leq \theta \leq 2\pi$

 **An Angle Formed by a Line Through the Origin** In Exercises 33–36, the terminal side of θ lies on the given line in the specified quadrant. Find the exact values of the six trigonometric functions of θ by finding a point on the line.

- | Line | Quadrant |
|------------------------|----------|
| 33. $y = -x$ | II |
| 34. $y = \frac{1}{3}x$ | III |
| 35. $2x - y = 0$ | I |
| 36. $4x + 3y = 0$ | IV |



Trigonometric Function of a Quadrantal Angle In Exercises 37–46, evaluate the trigonometric function of the quadrantal angle, if possible.

37. $\sin 0$ 38. $\csc \frac{3\pi}{2}$
 39. $\sec \frac{3\pi}{2}$ 40. $\sec \pi$
 41. $\sin \frac{\pi}{2}$ 42. $\cot 0$
 43. $\csc \pi$ 44. $\cot \frac{\pi}{2}$
 45. $\cos \frac{9\pi}{2}$ 46. $\tan\left(-\frac{\pi}{2}\right)$



Finding a Reference Angle In Exercises 47–54, find the reference angle θ' . Sketch θ in standard position and label θ' .

47. $\theta = 160^\circ$ 48. $\theta = 309^\circ$
 49. $\theta = -125^\circ$ 50. $\theta = -215^\circ$
 51. $\theta = \frac{2\pi}{3}$ 52. $\theta = \frac{7\pi}{6}$
 53. $\theta = 4.8$ 54. $\theta = 12.9$



Using a Reference Angle In Exercises 55–68, evaluate the sine, cosine, and tangent of the angle without using a calculator.

55. 225° 56. 300°
 57. 750° 58. 675°
 59. -120° 60. -570°
 61. $\frac{2\pi}{3}$ 62. $\frac{3\pi}{4}$
 63. $-\frac{\pi}{6}$ 64. $-\frac{2\pi}{3}$
 65. $\frac{11\pi}{4}$ 66. $\frac{13\pi}{6}$
 67. $-\frac{17\pi}{6}$ 68. $-\frac{23\pi}{4}$



Using a Trigonometric Identity In Exercises 69–74, use the function value to find the indicated trigonometric value in the specified quadrant.

- | Function Value | Quadrant | Trigonometric Value |
|----------------------------------|----------|---------------------|
| 69. $\sin \theta = -\frac{3}{5}$ | IV | $\cos \theta$ |
| 70. $\cot \theta = -3$ | II | $\csc \theta$ |
| 71. $\tan \theta = \frac{3}{2}$ | III | $\sec \theta$ |
| 72. $\csc \theta = -2$ | IV | $\cot \theta$ |
| 73. $\cos \theta = \frac{5}{8}$ | I | $\csc \theta$ |
| 74. $\sec \theta = -\frac{9}{4}$ | III | $\cot \theta$ |



Using a Calculator In Exercises 75–90, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

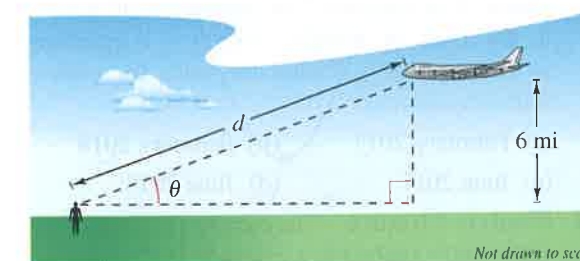
75. $\sin 10^\circ$ 76. $\tan 304^\circ$
 77. $\cos(-110^\circ)$ 78. $\sin(-330^\circ)$
 79. $\cot 178^\circ$ 80. $\sec 72^\circ$
 81. $\csc 405^\circ$ 82. $\cot(-560^\circ)$
 83. $\tan \frac{\pi}{9}$ 84. $\cos \frac{2\pi}{7}$
 85. $\sec \frac{11\pi}{8}$ 86. $\csc \frac{15\pi}{4}$
 87. $\sin(-0.65)$ 88. $\cos 1.35$
 89. $\csc(-10)$ 90. $\sec(-4.6)$



Solving for θ In Exercises 91–96, find two solutions of each equation. Give your answers in degrees ($0^\circ \leq \theta < 360^\circ$) and in radians ($0 \leq \theta < 2\pi$). Do not use a calculator.

91. (a) $\sin \theta = \frac{1}{2}$ 92. (a) $\cos \theta = \frac{\sqrt{2}}{2}$
 (b) $\sin \theta = -\frac{1}{2}$ (b) $\cos \theta = -\frac{\sqrt{2}}{2}$
 93. (a) $\cos \theta = \frac{1}{2}$ 94. (a) $\sin \theta = \frac{\sqrt{3}}{2}$
 (b) $\sec \theta = 2$ (b) $\csc \theta = \frac{2\sqrt{3}}{3}$
 95. (a) $\tan \theta = 1$ 96. (a) $\cot \theta = 0$
 (b) $\cot \theta = -\sqrt{3}$ (b) $\sec \theta = -\sqrt{2}$

97. **Distance** An airplane, flying at an altitude of 6 miles, is on a flight path that passes directly over an observer (see figure). Let θ be the angle of elevation from the observer to the plane. Find the distance d from the observer to the plane when (a) $\theta = 30^\circ$, (b) $\theta = 90^\circ$, and (c) $\theta = 120^\circ$.



98. **Harmonic Motion** The displacement from equilibrium of an oscillating weight suspended by a spring is given by $y(t) = 2 \cos 6t$, where y is the displacement in centimeters and t is the time in seconds. Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

99. Temperature

The table shows the average high temperatures (in degrees Fahrenheit) in Boston, Massachusetts (B), and Fairbanks, Alaska (F), for selected months in 2015. (Source: U.S. Climate Data)

Month	Boston, B	Fairbanks, F
January	33	1
March	41	31
June	72	71
August	83	62
November	56	17

- (a) Use the *regression* feature of a graphing utility to find a model of the form

$$y = a \sin(bt + c) + d$$

for each city. Let t represent the month, with $t = 1$ corresponding to January.

- (b) Use the models from part (a) to estimate the monthly average high temperatures for the two cities in February, April, May, July, September, October, and December.

- (c) Use a graphing utility to graph both models in the same viewing window. Compare the temperatures for the two cities.



100. **Sales** A company that produces snowboards forecasts monthly sales over the next 2 years to be

$$S = 23.1 + 0.442t + 4.3 \cos \frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ corresponding to January 2017. Predict the sales for each of the following months.

- (a) February 2017 (b) February 2018
(c) June 2017 (d) June 2018

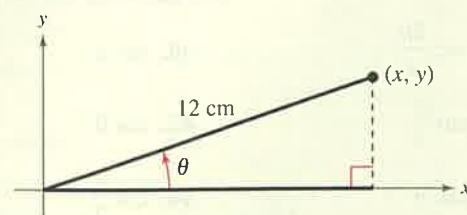
101. **Electric Circuits** The current I (in amperes) when 100 volts is applied to a circuit is given by

$$I = 5e^{-2t} \sin t$$

where t is the time (in seconds) after the voltage is applied. Approximate the current at $t = 0.7$ second after the voltage is applied.



102. **HOW DO YOU SEE IT?** Consider an angle in standard position with $r = 12$ centimeters, as shown in the figure. Describe the changes in the values of x , y , $\sin \theta$, $\cos \theta$, and $\tan \theta$ as θ increases continuously from 0° to 90° .



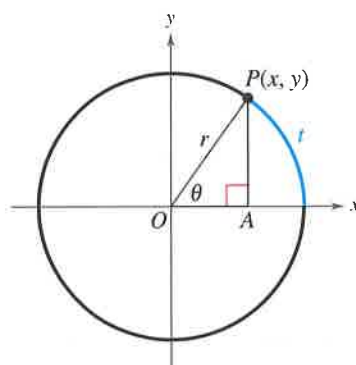
Exploration

True or False? In Exercises 103 and 104, determine whether the statement is true or false. Justify your answer.

103. In each of the four quadrants, the signs of the secant function and the sine function are the same.
104. The reference angle for an angle θ (in degrees) is the angle $\theta' = 360^\circ n - \theta$, where n is an integer and $0^\circ \leq \theta' \leq 360^\circ$.

105. **Writing** Write a short essay explaining to a classmate how to evaluate the six trigonometric functions of any angle θ in standard position. Include an explanation of reference angles and how to use them, the signs of the functions in each of the four quadrants, and the trigonometric values of common angles. Include figures or diagrams in your essay.

106. **Think About It** The figure shows point $P(x, y)$ on a unit circle and right triangle OAP .



- (a) Find $\sin t$ and $\cos t$ using the unit circle definitions of sine and cosine (from Section 4.2).
(b) What is the value of r ? Explain.
(c) Use the definitions of sine and cosine given in this section to find $\sin \theta$ and $\cos \theta$. Write your answers in terms of x and y .
(d) Based on your answers to parts (a) and (c), what can you conclude?

4.5 Graphs of Sine and Cosine Functions



Graphs of sine and cosine functions have many scientific applications. For example, in Exercise 80 on page 306, you will use the graph of a sine function to analyze airflow during a respiratory cycle.

- Sketch the graphs of basic sine and cosine functions.
- Use amplitude and period to help sketch the graphs of sine and cosine functions.
- Sketch translations of the graphs of sine and cosine functions.
- Use sine and cosine functions to model real-life data.

Basic Sine and Cosine Curves

In this section, you will study techniques for sketching the graphs of the sine and cosine functions. The graph of the sine function, shown in Figure 4.32, is a **sine curve**. In the figure, the black portion of the graph represents one period of the function and is **one cycle** of the sine curve. The gray portion of the graph indicates that the basic sine curve repeats indefinitely to the left and right. Figure 4.33 shows the graph of the cosine function.

Recall from Section 4.4 that the domain of the sine and cosine functions is the set of all real numbers. Moreover, the range of each function is the interval $[-1, 1]$, and each function has a period of 2π . This information is consistent with the basic graphs shown in Figures 4.32 and 4.33.

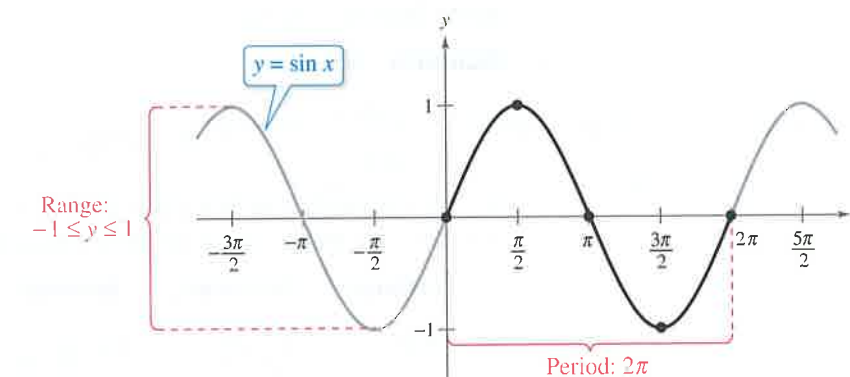


Figure 4.32

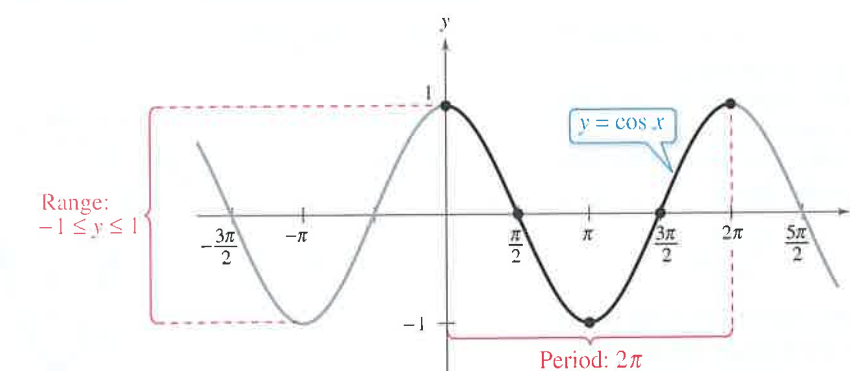


Figure 4.33

Note in Figures 4.32 and 4.33 that the sine curve is symmetric with respect to the *origin*, whereas the cosine curve is symmetric with respect to the *y-axis*. These properties of symmetry follow from the fact that the sine function is odd and the cosine function is even.