4.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. One period of a sine or cosine function is one _______ of the sine or cosine curve.
2. The ________ of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.
3. For the function $y = a\sin(bx - c)$, $\frac{c}{b}$ represents the ________ ________ of one cycle of the graph of the function.
4. For the function $y = d + a\cos(bx - c)$, d represents a ________ ________ of the basic curve.

Skills and Applications

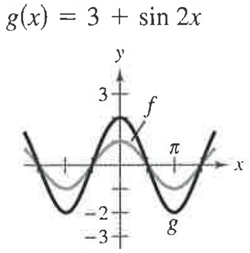
Finding the Period and Amplitude In Exercises 5–12, find the period and amplitude.

5. $y = 2\sin 5x$
6. $y = 3\cos 2x$
7. $y = \frac{3}{4}\cos\frac{\pi x}{2}$
8. $y = -5\sin\frac{\pi x}{3}$
9. $y = -\frac{1}{2}\sin\frac{5x}{4}$
10. $y = \frac{1}{4}\sin\frac{x}{6}$
11. $y = -\frac{5}{3}\cos\frac{\pi x}{12}$
12. $y = -\frac{2}{5}\cos 10\pi x$

Describing the Relationship Between Graphs In Exercises 13–24, describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

13. $f(x) = \cos x$
    $g(x) = \cos 5x$
14. $f(x) = \sin x$
    $g(x) = 2\sin x$
15. $f(x) = \cos 2x$
    $g(x) = -\cos 2x$
16. $f(x) = \sin 3x$
    $g(x) = \sin(-3x)$
17. $f(x) = \sin x$
    $g(x) = \sin(x - \pi)$
18. $f(x) = \cos x$
    $g(x) = \cos(x + \pi)$
19. $f(x) = \sin 2x$
    $g(x) = 3 + \sin 2x$
20. $f(x) = \cos 4x$
    $g(x) = -2 + \cos 4x$

21.



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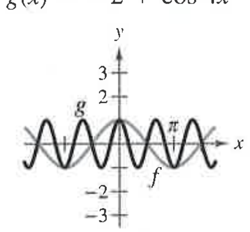

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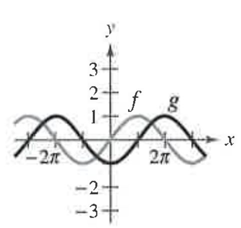

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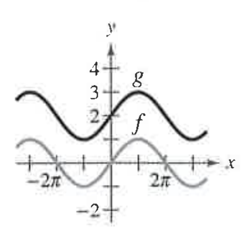


Sketching Graphs of Sine or Cosine Functions In Exercises 25–30, sketch the graphs of f and g in the same coordinate plane. (Include two full periods.)

25. $f(x) = \sin x$
    $g(x) = \sin\frac{x}{3}$
26. $f(x) = \sin x$
    $g(x) = 4\sin x$
27. $f(x) = \cos x$
    $g(x) = 2 + \cos x$
28. $f(x) = \cos x$
    $g(x) = \cos\left(x + \frac{\pi}{2}\right)$
29. $f(x) = -\cos x$
    $g(x) = -\cos(x - \pi)$
30. $f(x) = -\sin x$
    $g(x) = -3\sin x$

Sketching the Graph of a Sine or Cosine Function In Exercises 31–52, sketch the graph of the function. (Include two full periods.)

31. $y = 5\sin x$
32. $y = \frac{1}{4}\sin x$
33. $y = \frac{1}{3}\cos x$
34. $y = 4\cos x$
35. $y = \cos\frac{x}{2}$
36. $y = \sin 4x$
37. $y = \cos 2\pi x$
38. $y = \sin\frac{\pi x}{4}$
39. $y = -\sin\frac{2\pi x}{3}$
40. $y = 10\cos\frac{\pi x}{6}$
41. $y = \cos\left(x - \frac{\pi}{2}\right)$
42. $y = \sin(x - 2\pi)$
43. $y = 3\sin(x + \pi)$
44. $y = -4\cos\left(x + \frac{\pi}{4}\right)$
45. $y = 2 - \sin\frac{2\pi x}{3}$
46. $y = -3 + 5\cos\frac{\pi t}{12}$
47. $y = 2 + 5\cos 6\pi x$
48. $y = 2\sin 3x + 5$
49. $y = 3\sin(x + \pi) - 3$
50. $y = -3\sin(6x + \pi)$
51. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$
52. $y = 4\cos\left(\pi x + \frac{\pi}{2}\right) - 1$

Describing a Transformation In Exercises 53–58, g is related to a parent function $f(x) = \sin(x)$ or $f(x) = \cos(x)$. (a) Describe the sequence of transformations from f to g. (b) Sketch the graph of g. (c) Use function notation to write g in terms of f.

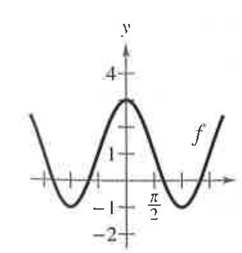
53. $g(x) = \sin(4x - \pi)$
54. $g(x) = \sin(2x + \pi)$
55. $g(x) = \cos\left(x - \frac{\pi}{2}\right) + 2$
56. $g(x) = 1 + \cos(x + \pi)$
57. $g(x) = 2\sin(4x - \pi) - 3$
58. $g(x) = 4 - \sin\left(2x + \frac{\pi}{2}\right)$

Graphing a Sine or Cosine Function In Exercises 59–64, use a graphing utility to graph the function. (Include two full periods.) Be sure to choose an appropriate viewing window.

59. $y = -2\sin(4x + \pi)$
60. $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$
61. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$
62. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$
63. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$
64. $y = \frac{1}{100}\cos 120\pi t$

Graphical Reasoning In Exercises 65–68, find a and d for the function $f(x) = a\cos x + d$ such that the graph of f matches the figure.

65.



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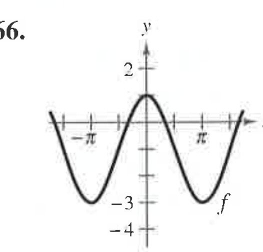

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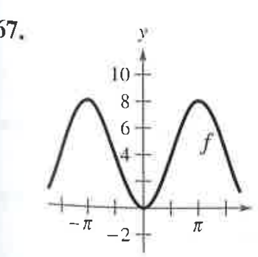

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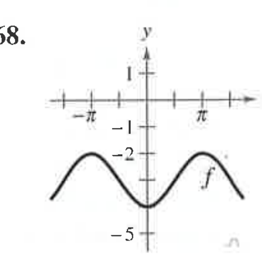
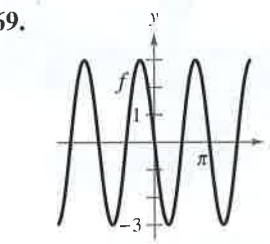


Graphical Reasoning In Exercises 69–72, find a, b, and c for the function $f(x) = a\sin(bx - c)$ such that the graph of f matches the figure.

69.



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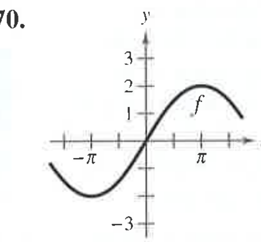

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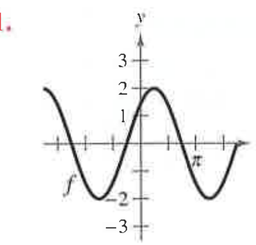

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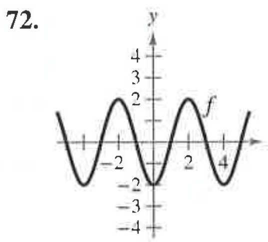


Using Technology In Exercises 73 and 74, use a graphing utility to graph y_1 and y_2 in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers x such that $y_1 = y_2$.

73. $y_1 = \sin x$, $y_2 = -\frac{1}{2}$
74. $y_1 = \cos x$, $y_2 = -1$

Writing an Equation In Exercises 75–78, write an equation for a function with the given characteristics.

75. A sine curve with a period of π, an amplitude of 2, a right phase shift of $\pi/2$, and a vertical translation up 1 unit
76. A sine curve with a period of 4π, an amplitude of 3, a left phase shift of $\pi/4$, and a vertical translation down 1 unit
77. A cosine curve with a period of π, an amplitude of 1, a left phase shift of π, and a vertical translation down $\frac{3}{2}$ units
78. A cosine curve with a period of 4π, an amplitude of 3, a right phase shift of $\pi/2$, and a vertical translation up 2 units

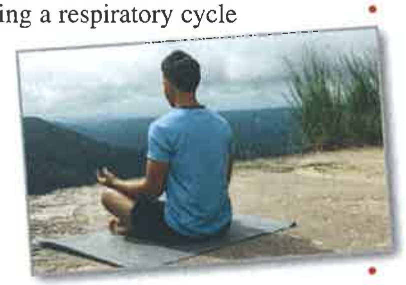
79. **Respiratory Cycle** For a person exercising, the velocity v (in liters per second) of airflow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is modeled by

$$v = 1.75\sin(\pi t/2)$$

where t is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.
(b) Find the number of cycles per minute.
(c) Sketch the graph of the velocity function.

80. Respiratory Cycle For a person at rest, the velocity v (in liters per second) of airflow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is modeled by $v = 0.85 \sin(\pi t/3)$, where t is the time (in seconds).



(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function. Use the graph to confirm your answer in part (a) by finding two times when new breaths begin. (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

81. Biology The function $P = 100 - 20\cos(5\pi t/3)$ approximates the blood pressure P (in millimeters of mercury) at time t (in seconds) for a person at rest.

(a) Find the period of the function.

(b) Find the number of heartbeats per minute.

82. Piano Tuning When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up a wave motion that can be approximated by $y = 0.001 \sin 880\pi t$, where t is the time (in seconds).

(a) What is the period of the function?

(b) The frequency f is given by $f = 1/p$. What is the frequency of the note?

83. Astronomy The table shows the percent y (in decimal form) of the moon's face illuminated on day x in the year 2018, where $x = 1$ corresponds to January 1. *(Source: U.S. Naval Observatory)*

DATA

x	y
1	1.0
8	0.5
16	0.0
24	0.5
31	1.0
38	0.5

Spreadsheet at LarsonPrecalculus.com

(a) Create a scatter plot of the data.

(b) Find a trigonometric model for the data.

(c) Add the graph of your model in part (b) to the scatter plot. How well does the model fit the data?

(d) What is the period of the model?

(e) Estimate the percent of the moon's face illuminated on March 12, 2018.

84. Meteorology The table shows the maximum daily high temperatures (in degrees Fahrenheit) in Las Vegas L and International Falls I for month t, where $t = 1$ corresponds to January. *(Source: National Climatic Data Center)*

DATA

Month, t	Las Vegas, L	International Falls, I
1	57.1	13.8
2	63.0	22.4
3	69.5	34.9
4	78.1	51.5
5	87.8	66.6
6	98.9	74.2
7	104.1	78.6
8	101.8	76.3
9	93.8	64.7
10	80.8	51.7
11	66.0	32.5
12	57.3	18.1

Spreadsheet at LarsonPrecalculus.com

(a) A model for the temperatures in Las Vegas is

$$L(t) = 80.60 + 23.50\cos\left(\frac{\pi t}{6} - 3.67\right).$$

Find a trigonometric model for the temperatures in International Falls.

(b) Use a graphing utility to graph the data points and the model for the temperatures in Las Vegas. How well does the model fit the data?

(c) Use the graphing utility to graph the data points and the model for the temperatures in International Falls. How well does the model fit the data?

(d) Use the models to estimate the average maximum temperature in each city. Which value in each model did you use? Explain.

(e) What is the period of each model? Are the periods what you expected? Explain.

(f) Which city has the greater variability in temperature throughout the year? Which value in each model determines this variability? Explain.

85. Ferris Wheel The height h (in feet) above ground of a seat on a Ferris wheel at time t (in seconds) is modeled by

$$h(t) = 53 + 50\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right).$$

(a) Find the period of the model. What does the period tell you about the ride?

(b) Find the amplitude of the model. What does the amplitude tell you about the ride?

(c) Use a graphing utility to graph one cycle of the model.

86. Fuel Consumption The daily consumption C (in gallons) of diesel fuel on a farm is modeled by

$$C = 30.3 + 21.6\sin\left(\frac{2\pi t}{365} + 10.9\right)$$

where t is the time (in days), with $t = 1$ corresponding to January 1.

(a) What is the period of the model? Is it what you expected? Explain.

(b) What is the average daily fuel consumption? Which value in the model did you use? Explain.

(c) Use a graphing utility to graph the model. Use the graph to approximate the time of the year when consumption exceeds 40 gallons per day.

Exploration

True or False? **In Exercises 87–89, determine whether the statement is true or false. Justify your answer.**

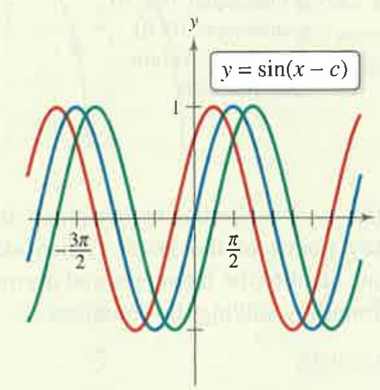
87. The graph of $g(x) = \sin(x + 2\pi)$ is a translation of the graph of $f(x) = \sin x$ exactly one period to the right, and the two graphs look identical.

88. The function $y = \frac{1}{2}\cos 2x$ has an amplitude that is twice that of the function $y = \cos x$.

89. The graph of $y = -\cos x$ is a reflection of the graph of $y = \sin[x + (\pi/2)]$ in the x-axis.

90. HOW DO YOU SEE IT? The figure below shows the graph of $y = \sin(x - c)$ for

$$c = -\frac{\pi}{4},\quad 0,\quad \text{and}\quad \frac{\pi}{4}.$$



(a) How does the value of c affect the graph?

(b) Which graph is equivalent to that of

$$y = -\cos\left(x + \frac{\pi}{4}\right)?$$

Conjecture **In Exercises 91 and 92, graph f and g in the same coordinate plane. (Include two full periods.) Make a conjecture about the functions.**

91. $f(x) = \sin x,\quad g(x) = \cos\left(x - \frac{\pi}{2}\right)$

92. $f(x) = \sin x,\quad g(x) = -\cos\left(x + \frac{\pi}{2}\right)$

93. Writing Sketch the graph of $y = \cos bx$ for $b = \frac{1}{2}$, 2, and 3. How does the value of b affect the graph? How many complete cycles of the graph occur between 0 and 2π for each value of b?

94. Polynomial Approximations Using calculus, it can be shown that the sine and cosine functions can be approximated by the polynomials

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

and

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

where x is in radians.

(a) Use a graphing utility to graph the sine function and its polynomial approximation in the same viewing window. How do the graphs compare?

(b) Use the graphing utility to graph the cosine function and its polynomial approximation in the same viewing window. How do the graphs compare?

(c) Study the patterns in the polynomial approximations of the sine and cosine functions and predict the next term in each. Then repeat parts (a) and (b). How does the accuracy of the approximations change when an additional term is added?

95. Polynomial Approximations Use the polynomial approximations of the sine and cosine functions in Exercise 94 to approximate each function value. Compare the results with those given by a calculator. Is the error in the approximation the same in each case? Explain.

(a) $\sin\frac{1}{2}$ (b) $\sin 1$

(c) $\sin\frac{\pi}{6}$ (d) $\cos(-0.5)$

(e) $\cos 1$ (f) $\cos\frac{\pi}{4}$

Project: Meteorology To work an extended application analyzing the mean monthly temperature and mean monthly precipitation for Honolulu, Hawaii, visit this text's website at *LarsonPrecalculus.com*. *(Source: National Climatic Data Center)*