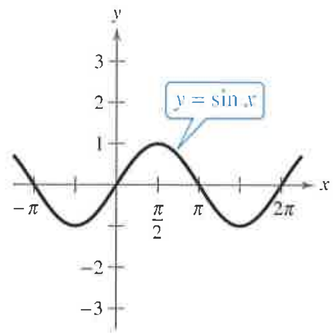
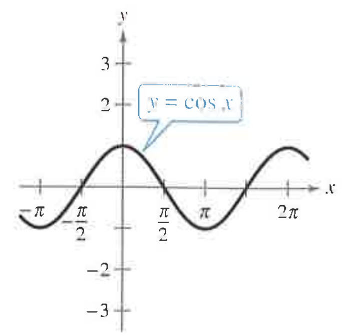
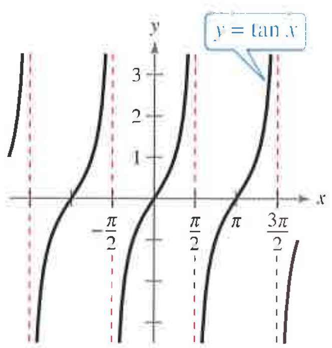
Below is a summary of the characteristics of the six basic trigonometric functions.



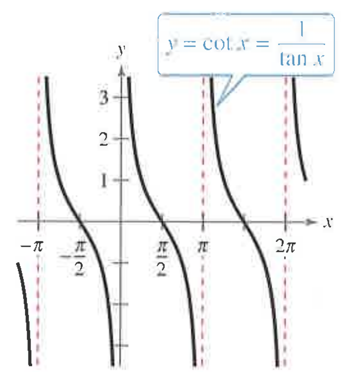
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π



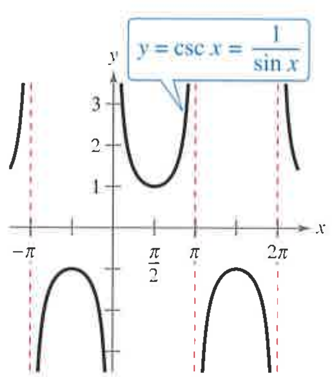
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π



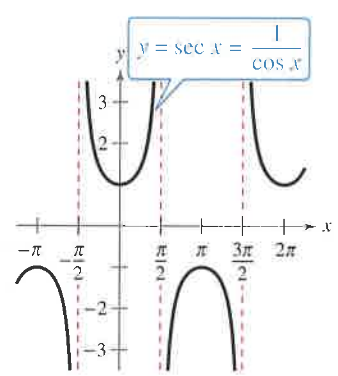
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π



Domain: all $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π



Domain: all $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π



Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

Summarize (Section 4.6)

1. Explain how to sketch the graph of $y = a\tan(bx - c)$ *(page 308)*. For examples of sketching graphs of tangent functions, see Examples 1 and 2.
2. Explain how to sketch the graph of $y = a\cot(bx - c)$ *(page 310)*. For an example of sketching the graph of a cotangent function, see Example 3.
3. Explain how to sketch the graphs of $y = a\csc(bx - c)$ and $y = a\sec(bx - c)$ *(page 311)*. For examples of sketching graphs of cosecant and secant functions, see Examples 4 and 5.
4. Explain how to sketch the graph of a damped trigonometric function *(page 313)*. For an example of sketching the graph of a damped trigonometric function, see Example 6.

4.6 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

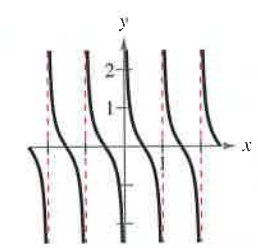
Vocabulary: Fill in the blanks.

1. The tangent, cotangent, and cosecant functions are ________, so the graphs of these functions have symmetry with respect to the ________.
2. The graphs of the tangent, cotangent, secant, and cosecant functions have ________ asymptotes.
3. To sketch the graph of a secant or cosecant function, first make a sketch of its ________ function.
4. For the function $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ________ factor.
5. The period of $y = \tan x$ is ________.
6. The domain of $y = \cot x$ is all real numbers such that ________.
7. The range of $y = \sec x$ is ________.
8. The period of $y = \csc x$ is ________.

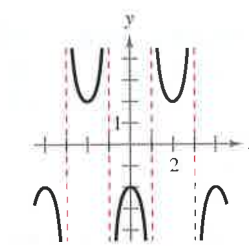
Skills and Applications

Matching In Exercises 9–14, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

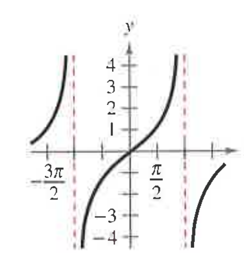
(a)



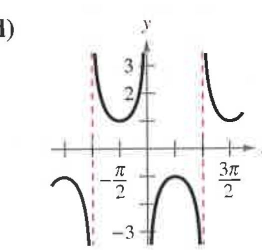
(b)



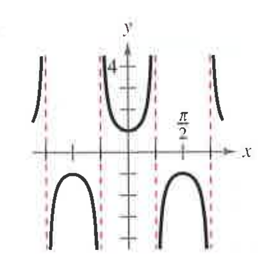
(c)



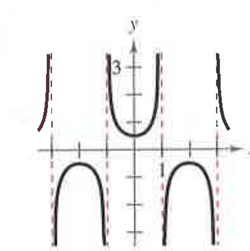
(d)



(e)



(f)



9. $y = \sec 2x$
10. $y = \tan\frac{x}{2}$
11. $y = \frac{1}{2}\cot \pi x$
12. $y = -\csc x$
13. $y = \frac{1}{2}\sec\frac{\pi x}{2}$
14. $y = -2\sec\frac{\pi x}{2}$

Sketching the Graph of a Trigonometric Function In Exercises 15–38, sketch the graph of the function. (Include two full periods.)

15. $y = \frac{1}{3}\tan x$
16. $y = -\frac{1}{2}\tan x$
17. $y = -\frac{1}{2}\sec x$
18. $y = \frac{1}{4}\sec x$
19. $y = -2\tan 3x$
20. $y = -3\tan \pi x$
21. $y = \csc \pi x$
22. $y = 3\csc 4x$
23. $y = \frac{1}{2}\sec \pi x$
24. $y = 2\sec 3x$
25. $y = \csc\frac{x}{2}$
26. $y = \csc\frac{x}{3}$
27. $y = 3\cot 2x$
28. $y = 3\cot\frac{\pi x}{2}$
29. $y = \tan\frac{\pi x}{4}$
30. $y = \tan 4x$
31. $y = 2\csc(x - \pi)$
32. $y = \csc(2x - \pi)$
33. $y = 2\sec(x + \pi)$
34. $y = \tan(x + \pi)$
35. $y = -\sec \pi x + 1$
36. $y = -2\sec 4x + 2$
37. $y = \frac{1}{4}\csc\left(x + \frac{\pi}{4}\right)$
38. $y = 2\cot\left(x + \frac{\pi}{2}\right)$

Graphing a Trigonometric Function In Exercises 39–48, use a graphing utility to graph the function. (Include two full periods.)

39. $y = \tan\frac{x}{3}$
40. $y = -\tan 2x$
41. $y = -2\sec 4x$
42. $y = \sec \pi x$
43. $y = \tan\left(x - \frac{\pi}{4}\right)$
44. $y = \frac{1}{4}\cot\left(x - \frac{\pi}{2}\right)$
45. $y = -\csc(4x - \pi)$
46. $y = 2\sec(2x - \pi)$
47. $y = 0.1\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$
48. $y = \frac{1}{3}\sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)$

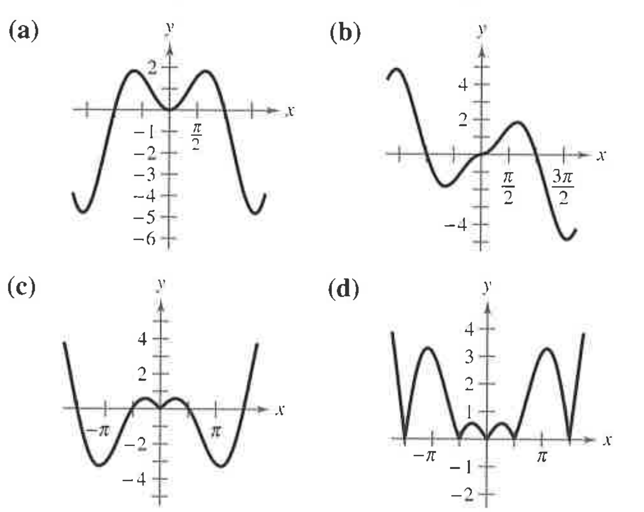
Solving a Trigonometric Equation In Exercises 49–56, find the solutions of the equation in the interval $[-2\pi, 2\pi]$. Use a graphing utility to verify your results.

49. $\tan x = 1$
50. $\tan x = \sqrt{3}$
51. $\cot x = -\sqrt{3}$
52. $\cot x = 1$
53. $\sec x = -2$
54. $\sec x = 2$
55. $\csc x = \sqrt{2}$
56. $\csc x = -2$

Even and Odd Trigonometric Functions In Exercises 57–64, use the graph of the function to determine whether the function is even, odd, or neither. Verify your answer algebraically.

57. $f(x) = \sec x$
58. $f(x) = \tan x$
59. $g(x) = \cot x$
60. $g(x) = \csc x$
61. $f(x) = x + \tan x$
62. $f(x) = x^2 - \sec x$
63. $g(x) = x \csc x$
64. $g(x) = x^2 \cot x$

Identifying Damped Trigonometric Functions In Exercises 65–68, match the function with its graph. Describe the behavior of the function as x approaches zero. [The graphs are labeled (a), (b), (c), and (d).]



65. $f(x) = |x \cos x|$
66. $f(x) = x \sin x$
67. $g(x) = |x| \sin x$
68. $g(x) = |x| \cos x$

Conjecture In Exercises 69–72, graph the functions f and g. Use the graphs to make a conjecture about the relationship between the functions.

69. $f(x) = \sin x + \cos\left(x + \frac{\pi}{2}\right), \quad g(x) = 0$

70. $f(x) = \sin x - \cos\left(x + \frac{\pi}{2}\right), \quad g(x) = 2 \sin x$

71. $f(x) = \sin^2 x, \quad g(x) = \frac{1}{2}(1 - \cos 2x)$

72. $f(x) = \cos^2 \frac{\pi x}{2}, \quad g(x) = \frac{1}{2}(1 + \cos \pi x)$

Analyzing a Damped Trigonometric Graph In Exercises 73–76, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.

73. $g(x) = e^{-x^2/2} \sin x$
74. $f(x) = e^{-x} \cos x$
75. $f(x) = 2^{-x/4} \cos \pi x$
76. $h(x) = 2^{-x^2/4} \sin x$

Analyzing a Trigonometric Graph In Exercises 77–82, use a graphing utility to graph the function. Describe the behavior of the function as x approaches zero.

77. $y = \frac{6}{x} + \cos x, \quad x > 0$

78. $y = \frac{4}{x} + \sin 2x, \quad x > 0$

79. $g(x) = \frac{\sin x}{x}$

80. $f(x) = \frac{1 - \cos x}{x}$

81. $f(x) = \sin \frac{1}{x}$

82. $h(x) = x \sin \frac{1}{x}$

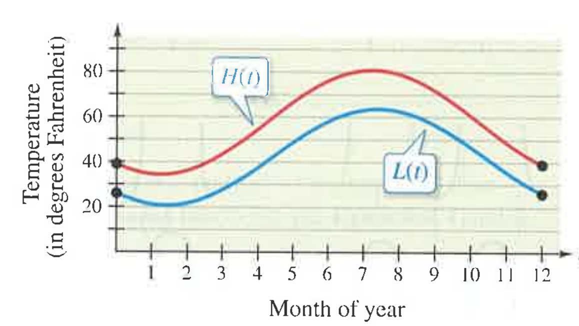
83. **Meteorology** The normal monthly high temperatures H (in degrees Fahrenheit) in Erie, Pennsylvania, are approximated by

$$H(t) = 57.54 - 18.53 \cos \frac{\pi t}{6} - 14.03 \sin \frac{\pi t}{6}$$

and the normal monthly low temperatures L are approximated by

$$L(t) = 42.03 - 15.99 \cos \frac{\pi t}{6} - 14.32 \sin \frac{\pi t}{6}$$

where t is the time (in months), with $t = 1$ corresponding to January (see figure). *(Source: NOAA)*



(a) What is the period of each function?

(b) During what part of the year is the difference between the normal high and normal low temperatures greatest? When is it least?

(c) The sun is northernmost in the sky around June 21, but the graph shows the warmest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

84. **Sales** The projected monthly sales S (in thousands of units) of lawn mowers are modeled by

$$S = 74 + 3t - 40 \cos \frac{\pi t}{6}$$

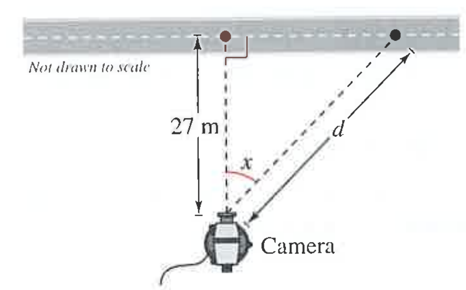
where t is the time (in months), with $t = 1$ corresponding to January.

(a) Graph the sales function over 1 year.

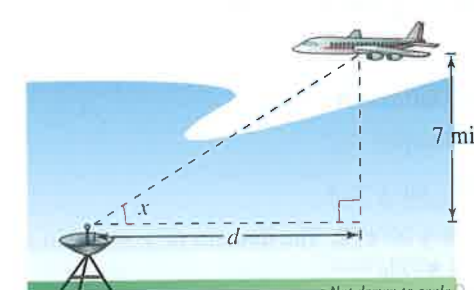
(b) What are the projected sales for June?

85. **Television Coverage**

A television camera is on a reviewing platform 27 meters from the street on which a parade passes from left to right (see figure). Write the distance d from the camera to a unit in the parade as a function of the angle x, and graph the function over the interval $-\pi/2 < x < \pi/2$. (Consider x as negative when a unit in the parade approaches from the left.)



86. **Distance** A plane flying at an altitude of 7 miles above a radar antenna passes directly over the radar antenna (see figure). Let d be the ground distance from the antenna to the point directly under the plane and let x be the angle of elevation to the plane from the antenna. (d is positive as the plane approaches the antenna.) Write d as a function of x and graph the function over the interval $0 < x < \pi$.



lrin-k/Shutterstock.com

Exploration

True or False? In Exercises 87 and 88, determine whether the statement is true or false. Justify your answer.

87. You can obtain the graph of $y = \csc x$ on a calculator by graphing the reciprocal of $y = \sin x$.

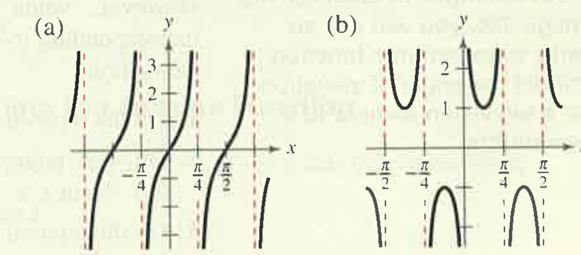
88. You can obtain the graph of $y = \sec x$ on a calculator by graphing a translation of the reciprocal of $y = \sin x$.

89. **Think About It** Consider the function $f(x) = x - \cos x$.

(a) Use a graphing utility to graph the function and verify that there exists a zero between 0 and 1. Use the graph to approximate the zero.

(b) Starting with $x_0 = 1$, generate a sequence $x_1, x_2, x_3, \ldots$, where $x_n = \cos(x_{n-1})$. For example, $x_0 = 1, x_1 = \cos(x_0), x_2 = \cos(x_1), x_3 = \cos(x_2), \ldots$. What value does the sequence approach?

90. **HOW DO YOU SEE IT?** Determine which function each graph represents. Do not use a calculator. Explain.



(a)	(b)
(i) $f(x) = \tan 2x$	(i) $f(x) = \sec 4x$
(ii) $f(x) = \tan(x/2)$	(ii) $f(x) = \csc 4x$
(iii) $f(x) = -\tan 2x$	(iii) $f(x) = \csc(x/4)$
(iv) $f(x) = -\tan(x/2)$	(iv) $f(x) = \sec(x/4)$

Graphical Reasoning In Exercises 91 and 92, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)

(a) $x \to 0^+$ (b) $x \to 0^-$ (c) $x \to \pi^+$ (d) $x \to \pi^-$

91. $f(x) = \cot x$
92. $f(x) = \csc x$

Graphical Reasoning In Exercises 93 and 94, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$.

(a) $x \to (\pi/2)^+$ (b) $x \to (\pi/2)^-$
(c) $x \to (-\pi/2)^+$ (d) $x \to (-\pi/2)^-$

93. $f(x) = \tan x$
94. $f(x) = \sec x$