4.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

	Function	Alternative Notation	Domain	Range
1.	$y = \arcsin x$	________	________	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2.	________	$y = \cos^{-1} x$	$-1 \le x \le 1$	________
3.	$y = \arctan x$	________	________	________

4. A trigonometric function has an ________ function only when its domain is restricted.

Skills and Applications

Evaluating an Inverse Trigonometric Function In Exercises 5–18, find the exact value of the expression, if possible.

5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. $\arccos \frac{1}{2}$
8. $\arccos 0$
9. $\arctan \frac{\sqrt{3}}{3}$
10. $\arctan 1$
11. $\arcsin 3$
12. $\arctan \sqrt{3}$
13. $\tan^{-1}(-\sqrt{3})$
14. $\cos^{-1}(-2)$
15. $\arccos\left(-\frac{1}{2}\right)$
16. $\arcsin \frac{\sqrt{2}}{2}$
17. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
18. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

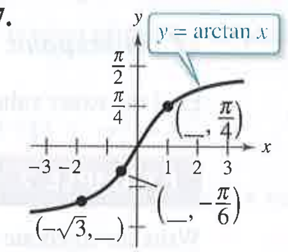
Graphing an Inverse Trigonometric Function In Exercises 19 and 20, use a graphing utility to graph f, g, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)

19. $f(x) = \cos x$, $g(x) = \arccos x$
20. $f(x) = \tan x$, $g(x) = \arctan x$

Calculators and Inverse Trigonometric Functions In Exercises 21–36, use a calculator to approximate the value of the expression, if possible. Round your result to two decimal places.

21. $\arccos 0.37$
22. $\arcsin 0.65$
23. $\arcsin(-0.75)$
24. $\arccos(-0.7)$
25. $\arctan(-3)$
26. $\arctan 25$
27. $\sin^{-1} 1.36$
28. $\cos^{-1} 0.26$
29. $\arccos(-0.41)$
30. $\arcsin(-0.125)$
31. $\arctan 0.92$
32. $\arctan 2.8$
33. $\arcsin \frac{7}{8}$
34. $\arccos\left(-\frac{4}{3}\right)$
35. $\tan^{-1}\left(-\frac{95}{7}\right)$
36. $\tan^{-1}(-\sqrt{372})$

Finding Missing Coordinates In Exercises 37 and 38, determine the missing coordinates of the points on the graph of the function.

37.


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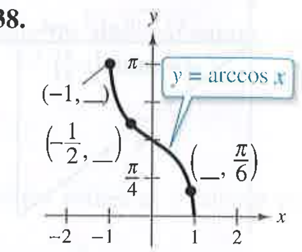
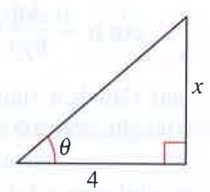


Using an Inverse Trigonometric Function In Exercises 39–44, use an inverse trigonometric function to write θ as a function of x.

39.


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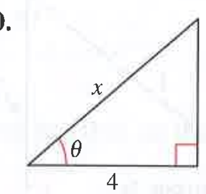

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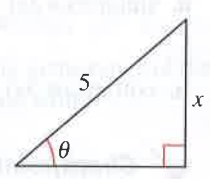

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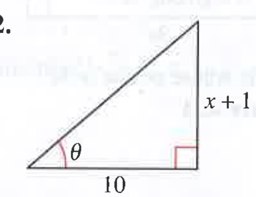
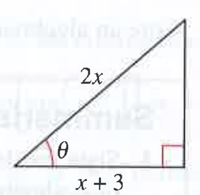


43.


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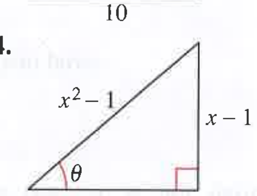


Using Inverse Properties In Exercises 45–50, find the exact value of the expression, if possible.

45. $\sin(\arcsin 0.3)$
46. $\tan(\arctan 45)$
47. $\cos[\arccos(-\sqrt{3})]$
48. $\sin[\arcsin(-0.2)]$
49. $\arcsin[\sin(9\pi/4)]$
50. $\arccos[\cos(-3\pi/2)]$

Evaluating a Composition of Functions In Exercises 51–62, find the exact value of the expression, if possible.

51. $\sin\left(\arctan \frac{3}{4}\right)$
52. $\cos\left(\arcsin \frac{4}{5}\right)$
53. $\cos(\tan^{-1} 2)$
54. $\sin(\cos^{-1} \sqrt{5})$
55. $\sec\left(\arcsin \frac{5}{13}\right)$
56. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$
57. $\cot\left[\arctan\left(-\frac{3}{5}\right)\right]$
58. $\sec\left[\arccos\left(-\frac{3}{4}\right)\right]$
59. $\tan\left[\arccos\left(-\frac{2}{3}\right)\right]$
60. $\cot\left(\arctan \frac{5}{8}\right)$
61. $\csc\left(\cos^{-1} \frac{\sqrt{3}}{2}\right)$
62. $\tan\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

Writing an Expression In Exercises 63–72, write an algebraic expression that is equivalent to the given expression.

63. $\cos(\arcsin 2x)$
64. $\sin(\arctan x)$
65. $\cot(\arctan x)$
66. $\sec(\arctan 3x)$
67. $\sin(\arccos x)$
68. $\csc[\arccos(x - 1)]$
69. $\tan\left(\arccos \frac{x}{3}\right)$
70. $\cot\left(\arctan \frac{1}{x}\right)$
71. $\csc\left(\arctan \frac{x}{a}\right)$
72. $\cos\left(\arcsin \frac{x - h}{r}\right)$

Using Technology In Exercises 73 and 74, use a graphing utility to graph f and g in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.

73. $f(x) = \sin(\arctan 2x)$, $g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

74. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right)$, $g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

Completing an Equation In Exercises 75–78, complete the equation.

75. $\arctan \dfrac{9}{x} = \arcsin(\ \blacksquare\ )$, $x > 0$

76. $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos(\ \blacksquare\ )$, $0 \le x \le 6$

77. $\arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \arcsin(\ \blacksquare\ )$

78. $\arccos \dfrac{x - 2}{2} = \arctan(\ \blacksquare\ )$, $2 < x < 4$

Sketching the Graph of a Function In Exercises 79–84, sketch the graph of the function and compare the graph to the graph of the parent inverse trigonometric function.

79. $y = 2 \arcsin x$
80. $f(x) = \arctan 2x$
81. $f(x) = \dfrac{\pi}{2} + \arctan x$
82. $g(t) = \arccos(t + 2)$
83. $h(v) = \arccos \dfrac{v}{2}$
84. $f(x) = \arcsin \dfrac{x}{4}$

Graphing an Inverse Trigonometric Function In Exercises 85–90, use a graphing utility to graph the function.

85. $f(x) = 2 \arccos 2x$
86. $f(x) = \pi \arcsin 4x$
87. $f(x) = \arctan(2x - 3)$
88. $f(x) = -3 + \arctan \pi x$
89. $f(x) = \pi - \sin^{-1} \dfrac{2}{3}$
90. $f(x) = \dfrac{\pi}{2} + \cos^{-1} \dfrac{1}{\pi}$

Using a Trigonometric Identity In Exercises 91 and 92, write the function in terms of the sine function by using the identity

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right).$$

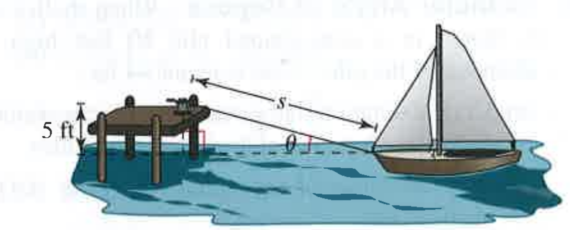
Use a graphing utility to graph both forms of the function. What does the graph imply?

91. $f(t) = 3 \cos 2t + 3 \sin 2t$
92. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

Behavior of an Inverse Trigonometric Function In Exercises 93–98, fill in the blank. If not possible, state the reason. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)

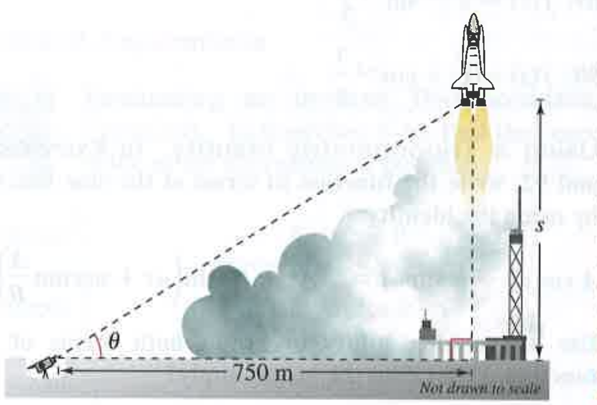
93. As $x \to 1^-$, the value of $\arcsin x \to \blacksquare$.
94. As $x \to 1^-$, the value of $\arccos x \to \blacksquare$.
95. As $x \to \infty$, the value of $\arctan x \to \blacksquare$.
96. As $x \to -1^+$, the value of $\arcsin x \to \blacksquare$.
97. As $x \to -1^+$, the value of $\arccos x \to \blacksquare$.
98. As $x \to -\infty$, the value of $\arctan x \to \blacksquare$.

99. **Docking a Boat** A boat is pulled in by means of a winch located on a dock 5 feet above the deck of the boat (see figure). Let θ be the angle of elevation from the boat to the winch and let s be the length of the rope from the winch to the boat.



(a) Write θ as a function of s.
(b) Find θ when $s = 40$ feet and $s = 20$ feet.

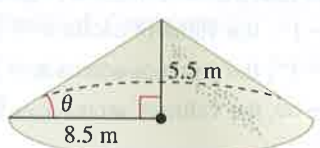
100. Videography A television camera at ground level films the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let θ be the angle of elevation to the shuttle and let s be the height of the shuttle.



(a) Write θ as a function of s.

(b) Find θ when $s = 300$ meters and $s = 1200$ meters.

101. Granular Angle of Repose Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle θ is called the *angle of repose* (see figure). When rock salt is stored in a cone-shaped pile 5.5 meters high, the diameter of the pile's base is about 17 meters.



(a) Find the angle of repose for rock salt.

(b) How tall is a pile of rock salt that has a base diameter of 20 meters?

102. Granular Angle of Repose When shelled corn is stored in a cone-shaped pile 20 feet high, the diameter of the pile's base is about 94 feet.

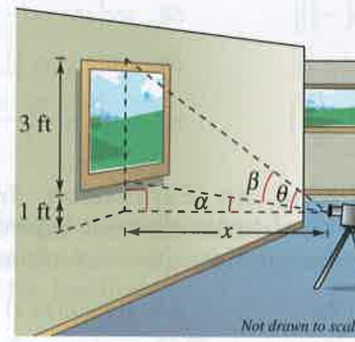
(a) Draw a diagram that gives a visual representation of the problem. Label the known quantities.

(b) Find the angle of repose (see Exercise 101) for shelled corn.

(c) How tall is a pile of shelled corn that has a base diameter of 60 feet?

103. Photography A photographer takes a picture of a three-foot-tall painting hanging in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle β subtended by the camera lens x feet from the painting is given by

$$\beta = \arctan \frac{3x}{x^2 + 4}, \quad x > 0.$$

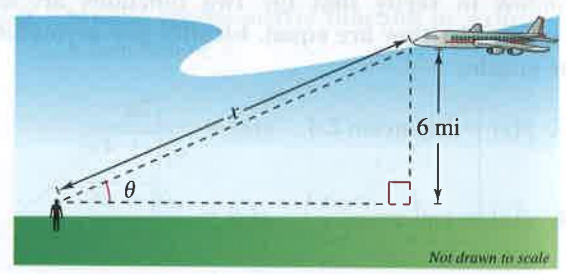


(a) Use a graphing utility to graph β as a function of x.

(b) Use the graph to approximate the distance from the picture when β is maximum.

(c) Identify the asymptote of the graph and interpret its meaning in the context of the problem.

104. Angle of Elevation An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 12$ miles and $x = 7$ miles.

105. Police Patrol A police car with its spotlight on is parked 20 meters from a warehouse. Consider θ and x as shown in the figure.

(a) Write θ as a function of x.

(b) Find θ when $x = 5$ meters and $x = 12$ meters.

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Exploration

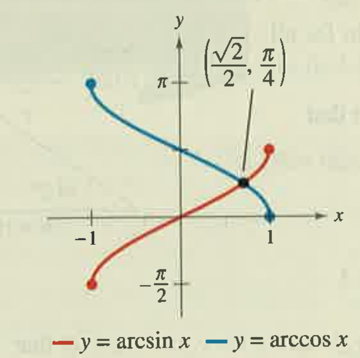
True or False? **In Exercises 106–109, determine whether the statement is true or false. Justify your answer.**

106. $\sin \frac{5\pi}{6} = \frac{1}{2} \Rightarrow \arcsin \frac{1}{2} = \frac{5\pi}{6}$

107. $\tan\left(-\frac{\pi}{4}\right) = -1 \Rightarrow \arctan(-1) = -\frac{\pi}{4}$

108. $\arctan x = \dfrac{\arcsin x}{\arccos x}$ **109.** $\sin^{-1} x = \dfrac{1}{\sin x}$

110. HOW DO YOU SEE IT? Use the figure below to determine the value(s) of x for which each statement is true.



(a) $\arcsin x < \arccos x$

(b) $\arcsin x = \arccos x$

(c) $\arcsin x > \arccos x$

111. Inverse Cotangent Function Define the inverse cotangent function by restricting the domain of the cotangent function to the interval $(0, \pi)$, and sketch the graph of the inverse trigonometric function.

112. Inverse Secant Function Define the inverse secant function by restricting the domain of the secant function to the intervals $[0, \pi/2)$ and $(\pi/2, \pi]$, and sketch the graph of the inverse trigonometric function.

113. Inverse Cosecant Function Define the inverse cosecant function by restricting the domain of the cosecant function to the intervals $[-\pi/2, 0)$ and $(0, \pi/2]$, and sketch the graph of the inverse trigonometric function.

114. Writing Use the results of Exercises 111–113 to explain how to graph (a) the inverse cotangent function, (b) the inverse secant function, and (c) the inverse cosecant function on a graphing utility.

Evaluating an Inverse Trigonometric Function **In Exercises 115–120, use the results of Exercises 111–113 to find the exact value of the expression.**

115. $\operatorname{arcsec} \sqrt{2}$ **116.** $\operatorname{arcsec} 1$

117. $\operatorname{arccot}(-1)$ **118.** $\operatorname{arccot}(-\sqrt{3})$

119. $\operatorname{arccsc}(-1)$ **120.** $\operatorname{arccsc} \frac{2\sqrt{3}}{3}$

Calculators and Inverse Trigonometric Functions **In Exercises 121–126, use the results of Exercises 111–113 and a calculator to approximate the value of the expression. Round your result to two decimal places.**

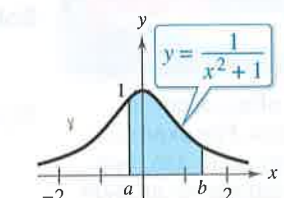
121. $\operatorname{arcsec} 2.54$ **122.** $\operatorname{arcsec}(-1.52)$

123. $\operatorname{arccsc}\left(-\frac{25}{3}\right)$ **124.** $\operatorname{arccsc}(-12)$

125. $\operatorname{arccot} 5.25$ **126.** $\operatorname{arccot}\left(-\frac{16}{7}\right)$

127. Area In calculus, it is shown that the area of the region bounded by the graphs of $y = 0$, $y = 1/(x^2 + 1)$, $x = a$, and $x = b$ (see figure) is given by

Area $= \arctan b - \arctan a$.



Find the area for each value of a and b.

(a) $a = 0, b = 1$ (b) $a = -1, b = 1$

(c) $a = 0, b = 3$ (d) $a = -1, b = 3$

128. Think About It Use a graphing utility to graph the functions $f(x) = \sqrt{x}$ and $g(x) = 6 \arctan x$. For $x > 0$, it appears that $g > f$. Explain how you know that there exists a positive real number a such that $g < f$ for $x > a$. Approximate the number a.

129. Think About It Consider the functions

$$f(x) = \sin x \quad \text{and} \quad f^{-1}(x) = \arcsin x.$$

(a) Use a graphing utility to graph the composite functions $f \circ f^{-1}$ and $f^{-1} \circ f$.

(b) Explain why the graphs in part (a) are not the graph of the line $y = x$. Why do the graphs of $f \circ f^{-1}$ and $f^{-1} \circ f$ differ?

130. Proof Prove each identity.

(a) $\arcsin(-x) = -\arcsin x$

(b) $\arctan(-x) = -\arctan x$

(c) $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad x > 0$

(d) $\arcsin x + \arccos x = \frac{\pi}{2}$

(e) $\arcsin x = \arctan \dfrac{x}{\sqrt{1 - x^2}}$