5.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary

In Exercises 1 and 2, fill in the blanks.

1. An equation that is true for all real values in the domain of the variable is an _____.
2. An equation that is true for only some values in the domain of the variable is a _____ _____.

In Exercises 3–8, fill in the blank to complete the fundamental trigonometric identity.

3. $\dfrac{1}{\cot u} =$ _____
4. $\dfrac{\cos u}{\sin u} =$ _____
5. $\cos\left(\dfrac{\pi}{2} - u\right) =$ _____
6. $1 +$ _____ $= \csc^2 u$
7. $\csc(-u) =$ _____
8. $\sec(-u) =$ _____

Skills and Applications

Verifying a Trigonometric Identity **In Exercises 9–18, verify the identity.**

9. $\tan t \cot t = 1$
10. $\dfrac{\tan x \cot x}{\cos x} = \sec x$
11. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$
12. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$
13. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$
14. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$
15. $\tan\left(\dfrac{\pi}{2} - \theta\right)\tan \theta = 1$
16. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$
17. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \tan t$
18. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$

Verifying a Trigonometric Identity **In Exercises 19–24, verify the identity algebraically. Use the *table* feature of a graphing utility to check your result numerically.**

19. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$
20. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$
21. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2\sec \theta$
22. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$
23. $\dfrac{1}{\cos x + 1} + \dfrac{1}{\cos x - 1} = -2\csc x \cot x$
24. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$

Verifying a Trigonometric Identity **In Exercises 25–30, verify the identity algebraically. Use a graphing utility to check your result graphically.**

25. $\sec y \cos y = 1$
26. $\cot^2 y(\sec^2 y - 1) = 1$
27. $\dfrac{\tan^2 \theta}{\sec \theta} = \sin \theta \tan \theta$
28. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$
29. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$
30. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$

Converting to Sines and Cosines **In Exercises 31–36, verify the identity by converting the left side into sines and cosines.**

31. $\dfrac{\cot^2 t}{\csc t} = \dfrac{1 - \sin^2 t}{\sin t}$
32. $\cos x + \sin x \tan x = \sec x$
33. $\sec x - \cos x = \sin x \tan x$
34. $\cot x - \tan x = \sec x(\csc x - 2\sin x)$
35. $\dfrac{\cot x}{\sec x} = \csc x - \sin x$
36. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$

Verifying a Trigonometric Identity **In Exercises 37–42, verify the identity.**

37. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x\sqrt{\sin x}$
38. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$
39. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$
40. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$
41. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$
42. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

Error Analysis **In Exercises 43 and 44, describe the error(s).**

43. $\dfrac{1}{\tan x} + \cot(-x) = \cot x + \cot x = 2\cot x$
44. $\dfrac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} = \dfrac{1 - \sec \theta}{\sin \theta - \tan \theta}$
$= \dfrac{1 - \sec \theta}{(\sin \theta)[1 - (1/\cos \theta)]}$
$= \dfrac{1 - \sec \theta}{\sin \theta(1 - \sec \theta)}$
$= \dfrac{1}{\sin \theta}$
$= \csc \theta$

Determining Trigonometric Identities **In Exercises 45–50, (a) use a graphing utility to graph each side of the equation to determine whether the equation is an identity, (b) use the *table* feature of the graphing utility to determine whether the equation is an identity, and (c) confirm the results of parts (a) and (b) algebraically.**

45. $(1 + \cot^2 x)(\cos^2 x) = \cot^2 x$
46. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$
47. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(3 + 2\cos^2 x)$
48. $\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$
49. $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$
50. $\dfrac{\cot \alpha}{\csc \alpha + 1} = \dfrac{\csc \alpha + 1}{\cot \alpha}$

Verifying a Trigonometric Identity **In Exercises 51–54, verify the identity.**

51. $\tan^5 x = \tan^3 x \sec^2 x - \tan^3 x$
52. $\sec^4 x \tan^2 x = (\tan^2 x + \tan^4 x)\sec^2 x$
53. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$
54. $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$

Using Cofunction Identities **In Exercises 55 and 56, use the cofunction identities to evaluate the expression without using a calculator.**

55. $\sin^2 25° + \sin^2 65°$
56. $\tan^2 63° + \cot^2 16° - \sec^2 74° - \csc^2 27°$

Verifying a Trigonometric Identity **In Exercises 57–60, verify the identity.**

57. $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1 - x^2}}$
58. $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$
59. $\tan\left(\sin^{-1} \dfrac{x - 1}{4}\right) = \dfrac{x - 1}{\sqrt{16 - (x - 1)^2}}$
60. $\tan\left(\cos^{-1} \dfrac{x + 1}{2}\right) = \dfrac{\sqrt{4 - (x + 1)^2}}{x + 1}$

61. **Rate of Change** The rate of change of the function $f(x) = \sin x + \csc x$ is given by the expression $\cos x - \csc x \cot x$. Show that the expression for the rate of change can also be written as $-\cos x \cot^2 x$.

62. **Shadow Length** The length s of a shadow cast by a vertical gnomon (a device used to tell time) of height h when the angle of the sun above the horizon is θ can be modeled by the equation

$$s = \frac{h\sin(90° - \theta)}{\sin \theta},$$

$0° < \theta \le 90°$.

(a) Verify that the expression for s is equal to $h \cot \theta$.

(b) Use a graphing utility to create a table of the lengths s for different values of θ. Let $h = 5$ feet.

(c) Use your table from part (b) to determine the angle of the sun that results in the minimum length of the shadow.

(d) Based on your results from part (c), what time of day do you think it is when the angle of the sun above the horizon is 90°?

Exploration

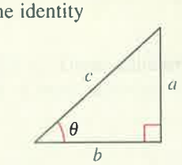
True or False? **In Exercises 63–65, determine whether the statement is true or false. Justify your answer.**

63. $\tan x^2 = \tan^2 x$
64. $\cos\left(\theta - \dfrac{\pi}{2}\right) = \sin \theta$
65. The equation $\sin^2 \theta + \cos^2 \theta = 1 + \tan^2 \theta$ is an identity because $\sin^2(0) + \cos^2(0) = 1$ and $1 + \tan^2(0) = 1$.

66. **HOW DO YOU SEE IT?** Explain how to use the figure to derive the identity

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

given in Example 1.



Think About It **In Exercises 67–70, explain why the equation is not an identity and find one value of the variable for which the equation is not true.**

67. $\sin \theta = \sqrt{1 - \cos^2 \theta}$
68. $\tan \theta = \sqrt{\sec^2 \theta - 1}$
69. $1 - \cos \theta = \sin \theta$
70. $1 + \tan \theta = \sec \theta$

Leonard Zhukovsky/Shutterstock.com