5.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. When solving a trigonometric equation, the preliminary goal is to ________ the trigonometric function on one side of the equation.
2. The ________ solution of the equation $2\sin\theta + 1 = 0$ is $\theta = \frac{7\pi}{6} + 2n\pi$ and $\theta = \frac{11\pi}{6} + 2n\pi$, where n is an integer.
3. The equation $2\tan^2 x - 3\tan x + 1 = 0$ is a trigonometric equation of ________ type.
4. A solution of an equation that does not satisfy the original equation is an ________ solution.

Skills and Applications

Verifying Solutions In Exercises 5–10, verify that each x-value is a solution of the equation.

5. $\tan x - \sqrt{3} = 0$
   (a) $x = \frac{\pi}{3}$
   (b) $x = \frac{4\pi}{3}$

6. $\sec x - 2 = 0$
   (a) $x = \frac{\pi}{3}$
   (b) $x = \frac{5\pi}{3}$

7. $3\tan^2 2x - 1 = 0$
   (a) $x = \frac{\pi}{12}$
   (b) $x = \frac{5\pi}{12}$

8. $2\cos^2 4x - 1 = 0$
   (a) $x = \frac{\pi}{16}$
   (b) $x = \frac{3\pi}{16}$

9. $2\sin^2 x - \sin x - 1 = 0$
   (a) $x = \frac{\pi}{2}$
   (b) $x = \frac{7\pi}{6}$

10. $\csc^4 x - 4\csc^2 x = 0$
   (a) $x = \frac{\pi}{6}$
   (b) $x = \frac{5\pi}{6}$

Solving a Trigonometric Equation In Exercises 11–28, solve the equation.

11. $\sqrt{3}\csc x - 2 = 0$
12. $\tan x + \sqrt{3} = 0$
13. $\cos x + 1 = -\cos x$
14. $3\sin x + 1 = \sin x$
15. $3\sec^2 x - 4 = 0$
16. $3\cot^2 x - 1 = 0$
17. $4\cos^2 x - 1 = 0$
18. $2 - 4\sin^2 x = 0$
19. $\sin x(\sin x + 1) = 0$
20. $(2\sin^2 x - 1)(\tan^2 x - 3) = 0$
21. $\cos^3 x - \cos x = 0$
22. $\sec^2 x - 1 = 0$
23. $3\tan^3 x = \tan x$
24. $\sec x \csc x = 2\csc x$
25. $2\cos^2 x + \cos x - 1 = 0$
26. $2\sin^2 x + 3\sin x + 1 = 0$
27. $\sec^2 x - \sec x = 2$
28. $\csc^2 x + \csc x = 2$

Solving a Trigonometric Equation In Exercises 29–38, find all solutions of the equation in the interval $[0, 2\pi)$.

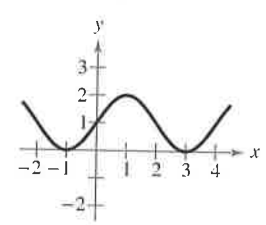
29. $\sin x - 2 = \cos x - 2$
30. $\cos x + \sin x \tan x = 2$
31. $2\sin^2 x = 2 + \cos x$
32. $\tan^2 x = \sec x - 1$
33. $\sin^2 x = 3\cos^2 x$
34. $2\sec^2 x + \tan^2 x - 3 = 0$
35. $2\sin x + \csc x = 0$
36. $3\sec x - 4\cos x = 0$
37. $\csc x + \cot x = 1$
38. $\sec x + \tan x = 1$

Solving a Multiple-Angle Equation In Exercises 39–46, solve the multiple-angle equation.

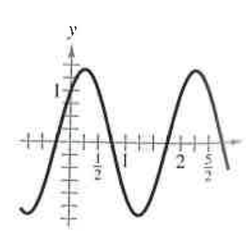
39. $2\cos 2x - 1 = 0$
40. $2\sin 2x + \sqrt{3} = 0$
41. $\tan 3x - 1 = 0$
42. $\sec 4x - 2 = 0$
43. $2\cos\frac{x}{2} - \sqrt{2} = 0$
44. $2\sin\frac{x}{2} + \sqrt{3} = 0$
45. $3\tan\frac{x}{2} - \sqrt{3} = 0$
46. $\tan\frac{x}{2} + \sqrt{3} = 0$

Finding x-Intercepts In Exercises 47 and 48, find the x-intercepts of the graph.

47. $y = \sin\frac{\pi x}{2} + 1$



48. $y = \sin \pi x + \cos \pi x$



Approximating Solutions In Exercises 49–58, use a graphing utility to approximate (to three decimal places) the solutions of the equation in the interval $[0, 2\pi)$.

49. $5\sin x + 2 = 0$
50. $2\tan x + 7 = 0$
51. $\sin x - 3\cos x = 0$
52. $\sin x + 4\cos x = 0$
53. $\cos x = x$
54. $\tan x = \csc x$
55. $\sec^2 x - 3 = 0$
56. $\csc^2 x - 5 = 0$
57. $2\tan^2 x = 15$
58. $6\sin^2 x = 5$

Using Inverse Functions In Exercises 59–70, solve the equation.

59. $\tan^2 x + \tan x - 12 = 0$
60. $\tan^2 x - \tan x - 2 = 0$
61. $\sec^2 x - 6\tan x = -4$
62. $\sec^2 x + \tan x = 3$
63. $2\sin^2 x + 5\cos x = 4$
64. $2\cos^2 x + 7\sin x = 5$
65. $\cot^2 x - 9 = 0$
66. $\cot^2 x - 6\cot x + 5 = 0$
67. $\sec^2 x - 4\sec x = 0$
68. $\sec^2 x + 2\sec x - 8 = 0$
69. $\csc^2 x + 3\csc x - 4 = 0$
70. $\csc^2 x - 5\csc x = 0$

Using the Quadratic Formula In Exercises 71–74, use the Quadratic Formula to find all solutions of the equation in the interval $[0, 2\pi)$. Round your result to four decimal places.

71. $12\sin^2 x - 13\sin x + 3 = 0$
72. $3\tan^2 x + 4\tan x - 4 = 0$
73. $\tan^2 x + 3\tan x + 1 = 0$
74. $4\cos^2 x - 4\cos x - 1 = 0$

Approximating Solutions In Exercises 75–78, use a graphing utility to approximate (to three decimal places) the solutions of the equation in the given interval.

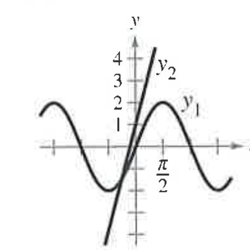
75. $3\tan^2 x + 5\tan x - 4 = 0, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
76. $\cos^2 x - 2\cos x - 1 = 0, \quad [0, \pi]$
77. $4\cos^2 x - 2\sin x + 1 = 0, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
78. $2\sec^2 x + \tan x - 6 = 0, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Approximating Maximum and Minimum Points In Exercises 79–84, (a) use a graphing utility to graph the function and approximate the maximum and minimum points on the graph in the interval $[0, 2\pi)$, and (b) solve the trigonometric equation and verify that its solutions are the x-coordinates of the maximum and minimum points of f. (Calculus is required to find the trigonometric equation.)

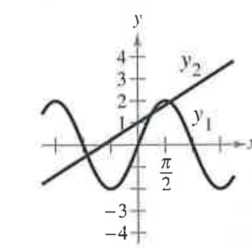
	Function	Trigonometric Equation
79.	$f(x) = \sin^2 x + \cos x$	$2\sin x\cos x - \sin x = 0$
80.	$f(x) = \cos^2 x - \sin x$	$-2\sin x\cos x - \cos x = 0$
81.	$f(x) = \sin x + \cos x$	$\cos x - \sin x = 0$
82.	$f(x) = 2\sin x + \cos 2x$	$2\cos x - 4\sin x\cos x = 0$
83.	$f(x) = \sin x\cos x$	$-\sin^2 x + \cos^2 x = 0$
84.	$f(x) = \sec x + \tan x - x$	$\sec x\tan x + \sec^2 x = 1$

Number of Points of Intersection In Exercises 85 and 86, use the graph to approximate the number of points of intersection of the graphs of y_1 and y_2.

85. $y_1 = 2\sin x$
   $y_2 = 3x + 1$



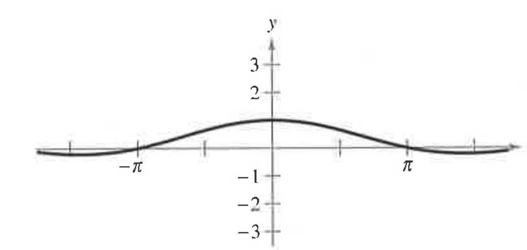
86. $y_1 = 2\sin x$
   $y_2 = \frac{1}{2}x + 1$



87. **Graphical Reasoning** Consider the function

$$f(x) = \frac{\sin x}{x}$$

and its graph, shown in the figure below.



(a) What is the domain of the function?
(b) Identify any symmetry and any asymptotes of the graph.
(c) Describe the behavior of the function as $x \to 0$.
(d) How many solutions does the equation

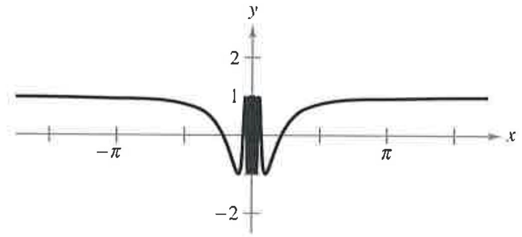
$$\frac{\sin x}{x} = 0$$

have in the interval $[-8, 8]$? Find the solutions.

88. Graphical Reasoning Consider the function

$$f(x) = \cos \frac{1}{x}$$

and its graph, shown in the figure below.



(a) What is the domain of the function?

(b) Identify any symmetry and any asymptotes of the graph.

(c) Describe the behavior of the function as $x \to 0$.

(d) How many solutions does the equation

$$\cos \frac{1}{x} = 0$$

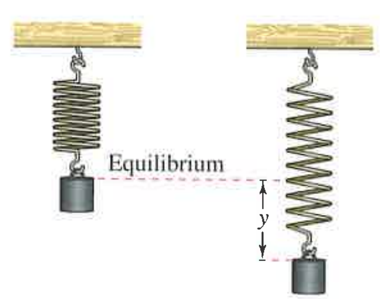
have in the interval $[-1, 1]$? Find the solutions.

(e) Does the equation $\cos(1/x) = 0$ have a greatest solution? If so, then approximate the solution. If not, then explain why.

89. Harmonic Motion A weight is oscillating on the end of a spring (see figure). The displacement from equilibrium of the weight relative to the point of equilibrium is given by

$$y = \tfrac{1}{12}(\cos 8t - 3 \sin 8t)$$

where y is the displacement (in meters) and t is the time (in seconds). Find the times when the weight is at the point of equilibrium ($y = 0$) for $0 \le t \le 1$.



90. Damped Harmonic Motion The displacement from equilibrium of a weight oscillating on the end of a spring is given by

$$y = 1.56e^{-0.22t} \cos 4.9t$$

where y is the displacement (in feet) and t is the time (in seconds). Use a graphing utility to graph the displacement function for $0 \le t \le 10$. Find the time beyond which the distance between the weight and equilibrium does not exceed 1 foot.

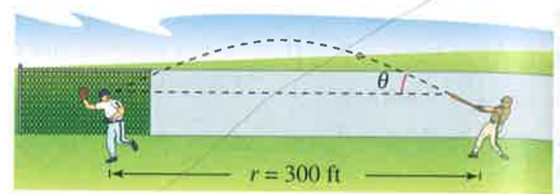
91. Equipment Sales The monthly sales S (in hundreds of units) of skiing equipment at a sports store are approximated by

$$S = 58.3 + 32.5 \cos \frac{\pi t}{6}$$

where t is the time (in months), with $t = 1$ corresponding to January. Determine the months in which sales exceed 7500 units.

92. Projectile Motion A baseball is hit at an angle of θ with the horizontal and with an initial velocity of $v_0 = 100$ feet per second. An outfielder catches the ball 300 feet from home plate (see figure). Find θ when the range r of a projectile is given by

$$r = \frac{1}{32} v_0^2 \sin 2\theta.$$



Not drawn to scale

93. Meteorology The table shows the normal daily high temperatures C in Chicago (in degrees Fahrenheit) for month t, with $t = 1$ corresponding to January. *(Source: NOAA)*

DATA

Month, t	Chicago, C
1	31.0
2	35.3
3	46.6
4	59.0
5	70.0
6	79.7
7	84.1
8	81.9
9	74.8
10	62.3
11	48.2
12	34.8

Spreadsheet at LarsonPrecalculus.com

(a) Use a graphing utility to create a scatter plot of the data.

(b) Find a cosine model for the temperatures.

(c) Graph the model and the scatter plot in the same viewing window. How well does the model fit the data?

(d) What is the overall normal daily high temperature?

(e) Use the graphing utility to determine the months during which the normal daily high temperature is above 72°F and below 72°F.

94. Ferris Wheel

The height h (in feet) above ground of a seat on a Ferris wheel at time t (in minutes) can be modeled by

$$h(t) = 53 + 50 \sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right).$$

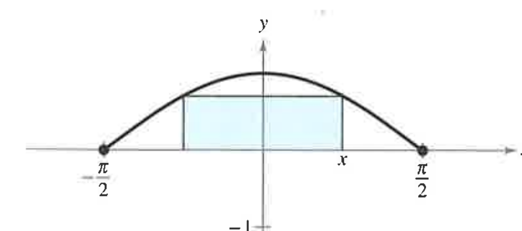
The wheel makes one revolution every 32 seconds. The ride begins when $t = 0$.

(a) During the first 32 seconds of the ride, when will a person's seat on the Ferris wheel be 53 feet above ground?

(b) When will a person's seat be at the top of the Ferris wheel for the first time during the ride? For a ride that lasts 160 seconds, how many times will a person's seat be at the top of the ride, and at what times?

95. Geometry The area of a rectangle inscribed in one arc of the graph of $y = \cos x$ (see figure) is given by

$$A = 2x \cos x,\ 0 < x < \pi/2.$$



(a) Use a graphing utility to graph the area function, and approximate the area of the largest inscribed rectangle.

(b) Determine the values of x for which $A \ge 1$.

96. Quadratic Approximation Consider the function

$$f(x) = 3 \sin(0.6x - 2).$$

(a) Approximate the zero of the function in the interval $[0, 6]$.

(b) A quadratic approximation agreeing with f at $x = 5$ is

$$g(x) = -0.45x^2 + 5.52x - 13.70.$$

Use a graphing utility to graph f and g in the same viewing window. Describe the result.

(c) Use the Quadratic Formula to find the zeros of g. Compare the zero of g in the interval $[0, 6]$ with the result of part (a).

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Fixed Point In Exercises 97 and 98, find the least positive fixed point of the function f. [A *fixed point* of a function f is a real number c such that $f(c) = c$.]

97. $f(x) = \tan(\pi x/4)$

98. $f(x) = \cos x$

Exploration

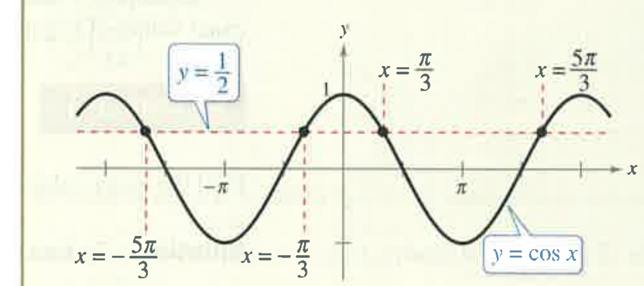
True or False? In Exercises 99 and 100, determine whether the statement is true or false. Justify your answer.

99. The equation $2 \sin 4t - 1 = 0$ has four times the number of solutions in the interval $[0, 2\pi)$ as the equation $2 \sin t - 1 = 0$.

100. The trigonometric equation $\sin x = 3.4$ can be solved using an inverse trigonometric function.

101. Think About It Explain what happens when you divide each side of the equation $\cot x \cos^2 x = 2 \cot x$ by $\cot x$. Is this a correct method to use when solving equations?

102. HOW DO YOU SEE IT? Explain how to use the figure to solve the equation $2 \cos x - 1 = 0$.



103. Graphical Reasoning Use a graphing utility to confirm the solutions found in Example 6 in two different ways.

(a) Graph both sides of the equation and find the x-coordinates of the points at which the graphs intersect.

Left side: $y = \cos x + 1$

Right side: $y = \sin x$

(b) Graph the equation $y = \cos x + 1 - \sin x$ and find the x-intercepts of the graph.

(c) Do both methods produce the same x-values? Which method do you prefer? Explain.

Project: Meteorology To work an extended application analyzing the normal daily high temperatures in Phoenix, Arizona, and in Seattle, Washington, visit this text's website at *LarsonPrecalculus.com.* *(Source: NOAA)*