5.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: **Fill in the blank to complete the trigonometric formula.**

1. $\sin 2u =$ _____
2. $\cos 2u =$ _____
3. $\sin u \cos v =$ _____
4. $\dfrac{1 - \cos 2u}{1 + \cos 2u} =$ _____
5. $\sin \dfrac{u}{2} =$ _____
6. $\cos u - \cos v =$ _____

Skills and Applications

Solving a Multiple-Angle Equation **In Exercises 7–14, solve the equation.**

7. $\sin 2x - \sin x = 0$
8. $\sin 2x \sin x = \cos x$
9. $\cos 2x - \cos x = 0$
10. $\cos 2x + \sin x = 0$
11. $\sin 4x = -2 \sin 2x$
12. $(\sin 2x + \cos 2x)^2 = 1$
13. $\tan 2x - \cot x = 0$
14. $\tan 2x - 2 \cos x = 0$

Using a Double-Angle Formula **In Exercises 15–20, use a double-angle formula to rewrite the expression.**

15. $6 \sin x \cos x$
16. $\sin x \cos x$
17. $6 \cos^2 x - 3$
18. $\cos^2 x - \frac{1}{2}$
19. $4 - 8 \sin^2 x$
20. $10 \sin^2 x - 5$

Evaluating Functions Involving Double Angles **In Exercises 21–24, use the given conditions to find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.**

21. $\sin u = -3/5, \quad 3\pi/2 < u < 2\pi$
22. $\cos u = -4/5, \quad \pi/2 < u < \pi$
23. $\tan u = 3/5, \quad 0 < u < \pi/2$
24. $\sec u = -2, \quad \pi < u < 3\pi/2$

25. **Deriving a Multiple-Angle Formula** Rewrite $\cos 4x$ in terms of $\cos x$.

26. **Deriving a Multiple-Angle Formula** Rewrite $\tan 3x$ in terms of $\tan x$.

Reducing Powers **In Exercises 27–34, use the power-reducing formulas to rewrite the expression in terms of first powers of the cosines of multiple angles.**

27. $\cos^4 x$
28. $\sin^8 x$
29. $\sin^4 2x$
30. $\cos^4 2x$
31. $\tan^4 2x$
32. $\tan^2 2x \cos^4 2x$
33. $\sin^2 2x \cos^2 2x$
34. $\sin^4 x \cos^2 x$

Using Half-Angle Formulas **In Exercises 35–40, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.**

35. $75°$
36. $165°$
37. $112° 30'$
38. $67° 30'$
39. $\pi/8$
40. $7\pi/12$

Using Half-Angle Formulas **In Exercises 41–44, use the given conditions to (a) determine the quadrant in which $u/2$ lies, and (b) find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.**

41. $\cos u = 7/25, \quad 0 < u < \pi/2$
42. $\sin u = 5/13, \quad \pi/2 < u < \pi$
43. $\tan u = -5/12, \quad 3\pi/2 < u < 2\pi$
44. $\cot u = 3, \quad \pi < u < 3\pi/2$

Solving a Trigonometric Equation **In Exercises 45–48, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.**

45. $\sin \dfrac{x}{2} + \cos x = 0$
46. $\sin \dfrac{x}{2} + \cos x - 1 = 0$
47. $\cos \dfrac{x}{2} - \sin x = 0$
48. $\tan \dfrac{x}{2} - \sin x = 0$

Using Product-to-Sum Formulas **In Exercises 49–52, use the product-to-sum formulas to rewrite the product as a sum or difference.**

49. $\sin 5\theta \sin 3\theta$
50. $7 \cos(-5\beta) \sin 3\beta$
51. $\cos 2\theta \cos 4\theta$
52. $\sin(x + y) \cos(x - y)$

Using Sum-to-Product Formulas **In Exercises 53–56, use the sum-to-product formulas to rewrite the sum or difference as a product.**

53. $\sin 5\theta - \sin 3\theta$
54. $\sin 3\theta + \sin \theta$
55. $\cos 6x + \cos 2x$
56. $\cos x + \cos 4x$

Using Sum-to-Product Formulas **In Exercises 57–60, use the sum-to-product formulas to find the exact value of the expression.**

57. $\sin 75° + \sin 15°$
58. $\cos 120° + \cos 60°$
59. $\cos \dfrac{3\pi}{4} - \cos \dfrac{\pi}{4}$
60. $\sin \dfrac{5\pi}{4} - \sin \dfrac{3\pi}{4}$

Solving a Trigonometric Equation **In Exercises 61–64, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.**

61. $\sin 6x + \sin 2x = 0$
62. $\cos 2x - \cos 6x = 0$
63. $\dfrac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$
64. $\sin^2 3x - \sin^2 x = 0$

Verifying a Trigonometric Identity **In Exercises 65–70, verify the identity.**

65. $\csc 2\theta = \dfrac{\csc \theta}{2 \cos \theta}$
66. $\cos^4 x - \sin^4 x = \cos 2x$
67. $(\sin x + \cos x)^2 = 1 + \sin 2x$
68. $\tan \dfrac{u}{2} = \csc u - \cot u$
69. $\dfrac{\sin x \pm \sin y}{\cos x + \cos y} = \tan \dfrac{x \pm y}{2}$
70. $\cos\left(\dfrac{\pi}{3} + x\right) + \cos\left(\dfrac{\pi}{3} - x\right) = \cos x$

71. **Mach Number** The Mach number M of a supersonic airplane is the ratio of its speed to the speed of sound. When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane. The Mach number is related to the apex angle θ of the cone by $\sin(\theta/2) = 1/M$.
   (a) Use a half-angle formula to rewrite the equation in terms of $\cos \theta$.
   (b) Find the angle θ that corresponds to a Mach number of 2.
   (c) Find the angle θ that corresponds to a Mach number of 4.5.
   (d) The speed of sound is about 760 miles per hour. Determine the speed of an object with the Mach numbers from parts (b) and (c).

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72. **Projectile Motion** The range of a projectile fired at an angle θ with the horizontal and with an initial velocity of v_0 feet per second is

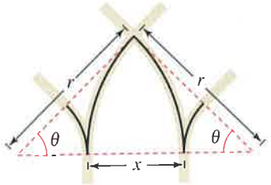
$$r = \frac{1}{32} v_0^2 \sin 2\theta$$

where r is the horizontal distance (in feet) the projectile travels. An athlete throws a javelin at 75 feet per second. At what angle must the athlete throw the javelin so that the javelin travels 130 feet?

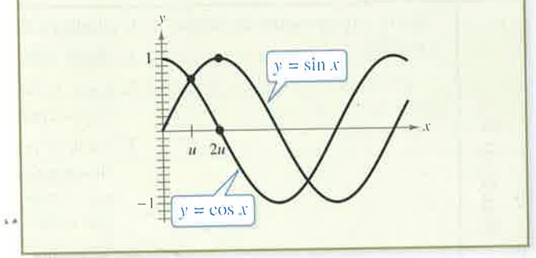
73. **Railroad Track** When two railroad tracks merge, the overlapping portions of the tracks are in the shapes of circular arcs (see figure). The radius r (in feet) of each arc and the angle θ are related by

$$\frac{x}{2} = 2r \sin^2 \frac{\theta}{2}.$$

Write a formula for x in terms of $\cos \theta$.



74. **HOW DO YOU SEE IT?** Explain how to use the figure to verify the double-angle formulas (a) $\sin 2u = 2 \sin u \cos u$ and (b) $\cos 2u = \cos^2 u - \sin^2 u$.



Exploration

True or False? **In Exercises 75 and 76, determine whether the statement is true or false. Justify your answer.**

75. The sine function is an odd function, so

$$\sin(-2x) = -2 \sin x \cos x.$$

76. $\sin \dfrac{u}{2} = -\sqrt{\dfrac{1 - \cos u}{2}}$ when u is in the second quadrant.

77. **Complementary Angles** Verify each identity for complementary angles ϕ and θ.
   (a) $\sin(\phi - \theta) = \cos 2\theta$
   (b) $\cos(\phi - \theta) = \sin 2\theta$