6.1 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

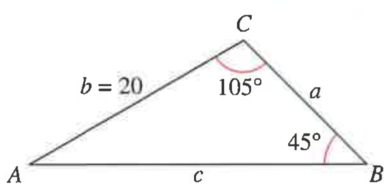
Vocabulary: Fill in the blanks.

1. An ________ triangle is a triangle that has no right angle.
2. For triangle ABC, the Law of Sines is $\dfrac{a}{\sin A} =$ ________ $= \dfrac{c}{\sin C}$.
3. Two ________ and one ________ determine a unique triangle.
4. The area of an oblique triangle ABC is $\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C =$ ________.

Skills and Applications

Using the Law of Sines In Exercises 5–22, use the Law of Sines to solve the triangle. Round your answers to two decimal places.

5.



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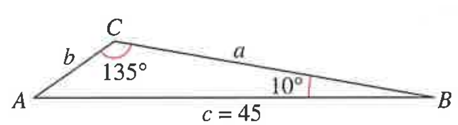

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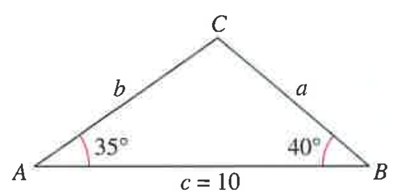

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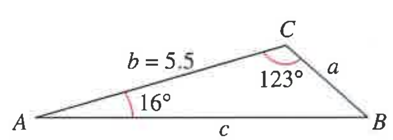


9. $A = 102.4°$, $C = 16.7°$, $a = 21.6$
10. $A = 24.3°$, $C = 54.6°$, $c = 2.68$
11. $A = 83° 20'$, $C = 54.6°$, $c = 18.1$
12. $A = 5° 40'$, $B = 8° 15'$, $b = 4.8$
13. $A = 35°$, $B = 65°$, $c = 10$
14. $A = 120°$, $B = 45°$, $c = 16$
15. $A = 55°$, $B = 42°$, $c = \frac{3}{4}$
16. $B = 28°$, $C = 104°$, $a = 3\frac{5}{8}$
17. $A = 36°$, $a = 8$, $b = 5$
18. $A = 60°$, $a = 9$, $c = 7$
19. $A = 145°$, $a = 14$, $b = 4$
20. $A = 100°$, $a = 125$, $c = 10$
21. $B = 15° 30'$, $a = 4.5$, $b = 6.8$
22. $B = 2° 45'$, $b = 6.2$, $c = 5.8$

Using the Law of Sines In Exercises 23–32, use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

23. $A = 110°$, $a = 125$, $b = 100$
24. $A = 110°$, $a = 125$, $b = 200$
25. $A = 76°$, $a = 18$, $b = 20$
26. $A = 76°$, $a = 34$, $b = 21$
27. $A = 58°$, $a = 11.4$, $b = 12.8$
28. $A = 58°$, $a = 4.5$, $b = 12.8$
29. $A = 120°$, $a = b = 25$
30. $A = 120°$, $a = 25$, $b = 24$
31. $A = 45°$, $a = b = 1$
32. $A = 25° 4'$, $a = 9.5$, $b = 22$

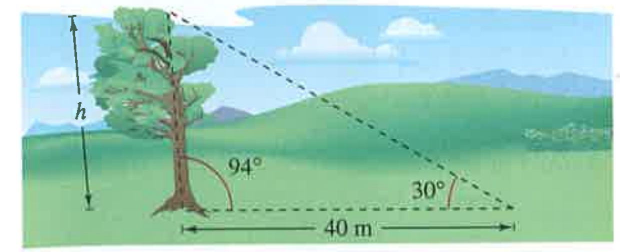
Using the Law of Sines In Exercises 33–36, find values for b such that the triangle has (a) one solution, (b) two solutions (if possible), and (c) no solution.

33. $A = 36°$, $a = 5$
34. $A = 60°$, $a = 10$
35. $A = 105°$, $a = 80$
36. $A = 132°$, $a = 215$

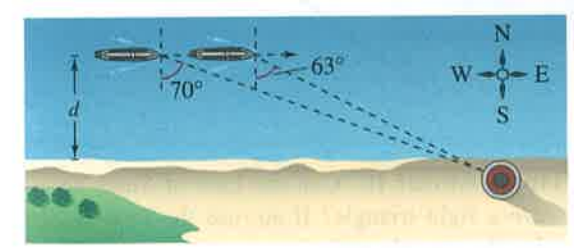
Finding the Area of a Triangle In Exercises 37–44, find the area of the triangle. Round your answers to one decimal place.

37. $A = 125°$, $b = 9$, $c = 6$
38. $C = 150°$, $a = 17$, $b = 10$
39. $B = 39°$, $a = 25$, $c = 12$
40. $A = 72°$, $b = 31$, $c = 44$
41. $C = 103° 15'$, $a = 16$, $b = 28$
42. $B = 54° 30'$, $a = 62$, $c = 35$
43. $A = 67°$, $B = 43°$, $a = 8$
44. $B = 118°$, $C = 29°$, $a = 52$

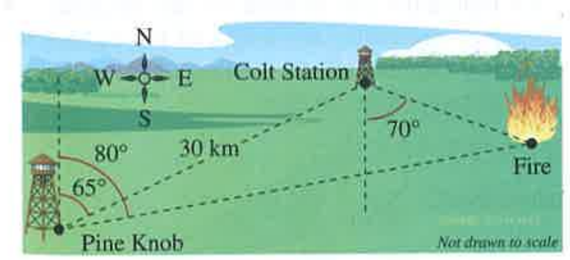
45. **Height** A tree grows at an angle of 4° from the vertical due to prevailing winds. At a point 40 meters from the base of the tree, the angle of elevation to the top of the tree is 30° (see figure).
 (a) Write an equation that you can use to find the height h of the tree.
 (b) Find the height of the tree.



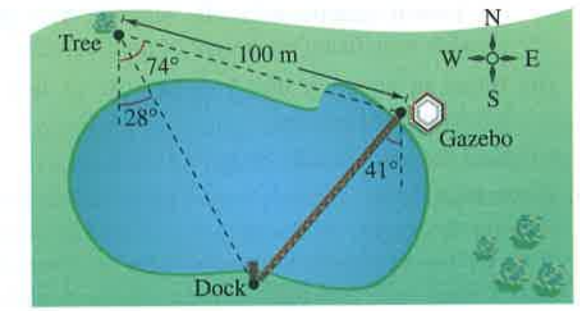
46. **Distance** A boat is traveling due east parallel to the shoreline at a speed of 10 miles per hour. At a given time, the bearing to a lighthouse is S 70° E, and 15 minutes later the bearing is S 63° E (see figure). The lighthouse is located at the shoreline. What is the distance from the boat to the shoreline?



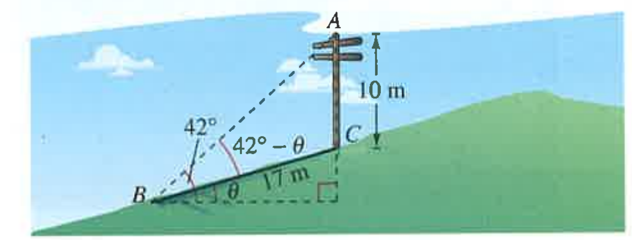
47. **Environmental Science** The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 70° E from Colt Station (see figure). Find the distance of the fire from each tower.



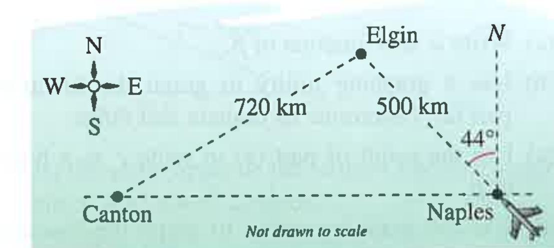
48. **Bridge Design** A bridge is built across a small lake from a gazebo to a dock (see figure). The bearing from the gazebo to the dock is S 41° W. From a tree 100 meters from the gazebo, the bearings to the gazebo and the dock are S 74° E and S 28° E, respectively. Find the distance from the gazebo to the dock.



49. **Angle of Elevation** A 10-meter utility pole casts a 17-meter shadow directly down a slope when the angle of elevation of the sun is 42° (see figure). Find θ, the angle of elevation of the ground.



50. **Flight Path** A plane flies 500 kilometers with a bearing of 316° from Naples to Elgin (see figure). The plane then flies 720 kilometers from Elgin to Canton (Canton is due west of Naples). Find the bearing of the flight from Elgin to Canton.



51. **Altitude** The angles of elevation to an airplane from two points A and B on level ground are 55° and 72°, respectively. The points A and B are 2.2 miles apart, and the airplane is east of both points in the same vertical plane.
 (a) Draw a diagram that represents the problem. Show the known quantities on the diagram.
 (b) Find the distance between the plane and point B.
 (c) Find the altitude of the plane.
 (d) Find the distance the plane must travel before it is directly above point A.

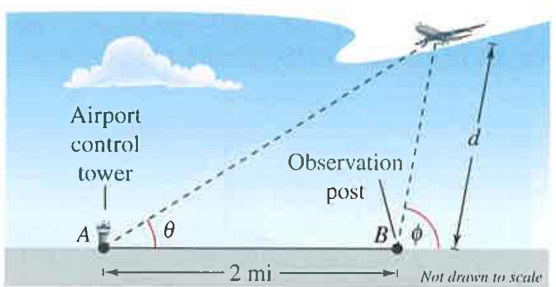
52. Height A flagpole at a right angle to the horizontal is located on a slope that makes an angle of 12° with the horizontal. The flagpole's shadow is 16 meters long and points directly up the slope. The angle of elevation from the tip of the shadow to the sun is 20°.

(a) Draw a diagram that represents the problem. Show the known quantities on the diagram and use a variable to indicate the height of the flagpole.

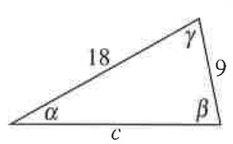
(b) Write an equation that you can use to find the height of the flagpole.

(c) Find the height of the flagpole.

53. Distance Air traffic controllers continuously monitor the angles of elevation θ and ϕ to an airplane from an airport control tower and from an observation post 2 miles away (see figure). Write an equation giving the distance d between the plane and the observation post in terms of θ and ϕ.



54. Numerical Analysis In the figure, α and β are positive angles.



(a) Write α as a function of β.

(b) Use a graphing utility to graph the function in part (a). Determine its domain and range.

(c) Use the result of part (a) to write c as a function of β.

(d) Use the graphing utility to graph the function in part (c). Determine its domain and range.

(e) Complete the table. What can you infer?

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α							
c							

Exploration

True or False? **In Exercises 55–58, determine whether the statement is true or false. Justify your answer.**

55. If a triangle contains an obtuse angle, then it must be oblique.

56. Two angles and one side of a triangle do not necessarily determine a unique triangle.

57. When you know the three angles of an oblique triangle, you can solve the triangle.

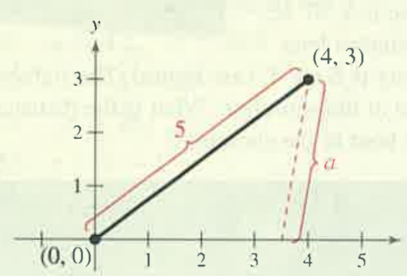
58. The ratio of any two sides of a triangle is equal to the ratio of the sines of the opposite angles of the two sides.

59. Error Analysis Describe the error.

The area of the triangle with $C = 58°$, $b = 11$ feet, and $c = 16$ feet is

$$\text{Area} = \frac{1}{2}(11)(16)(\sin 58°)$$
$$= 88(\sin 58°)$$
$$\approx 74.63 \text{ square feet.}$$

60. HOW DO YOU SEE IT? In the figure, a triangle is to be formed by drawing a line segment of length a from (4, 3) to the positive x-axis. For what value(s) of a can you form (a) one triangle, (b) two triangles, and (c) no triangles? Explain.



61. Think About It Can the Law of Sines be used to solve a right triangle? If so, use the Law of Sines to solve the triangle with

$$B = 50°, \quad C = 90°, \quad \text{and} \quad a = 10.$$

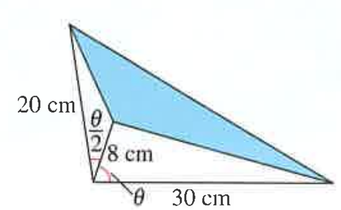
Is there another way to solve the triangle? Explain.

62. Using Technology

(a) Write the area A of the shaded region in the figure as a function of θ.

(b) Use a graphing utility to graph the function.

(c) Determine the domain of the function. Explain how decreasing the length of the eight-centimeter line segment affects the area of the region and the domain of the function.



6.2 Law of Cosines

The Law of Cosines is a useful tool for solving real-life problems involving oblique triangles. For example, in Exercise 56 on page 415, you will use the Law of Cosines to determine the total distance a piston moves in an engine.

- **Use the Law of Cosines to solve oblique triangles (SSS or SAS).**
- **Use the Law of Cosines to model and solve real-life problems.**
- **Use Heron's Area Formula to find areas of triangles.**

Introduction

Two cases remain in the list of conditions needed to solve an oblique triangle—SSS and SAS. When you are given three sides (SSS), or two sides and their included angle (SAS), you cannot solve the triangle using the Law of Sines alone. In such cases, use the **Law of Cosines.**

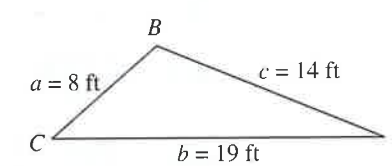
Law of Cosines

Standard Form	Alternative Form
$a^2 = b^2 + c^2 - 2bc \cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac \cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab \cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

For a proof of the Law of Cosines, see Proofs in Mathematics on page 462.

EXAMPLE 1 Given Three Sides—SSS

Find the three angles of the triangle shown below.



Solution It is a good idea to find the angle opposite the longest side first—side b in this case. Using the alternative form of the Law of Cosines,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{8^2 + 14^2 - 19^2}{2(8)(14)} \approx -0.4509.$$

Because $\cos B$ is negative, B is an *obtuse* angle given by $B \approx 116.80°$. At this point, use the Law of Sines to determine A.

$$\sin A = a\left(\frac{\sin B}{b}\right) \approx 8\left(\frac{\sin 116.80°}{19}\right) \approx 0.3758$$

The angle B is obtuse and a triangle can have at most one obtuse angle, so you know that A must be acute. So, $A \approx 22.07°$ and $C \approx 180° - 22.07° - 116.80° = 41.13°$.

✓ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the three angles of the triangle whose sides have lengths $a = 6$ centimeters, $b = 8$ centimeters, and $c = 12$ centimeters.