

HISTORICAL NOTE

Heron of Alexandria (10–75 A.D.) was a Greek geometer and inventor. His works describe how to find areas of triangles, quadrilaterals, regular polygons with 3 to 12 sides, and circles, as well as surface areas and volumes of three-dimensional objects.

Heron's Area Formula

The Law of Cosines can be used to establish a formula for the area of a triangle. This formula is called **Heron's Area Formula** after the Greek mathematician Heron (ca. 10–75 A.D.).

Heron's Area Formula

Given any triangle with sides of lengths a , b , and c , the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{a+b+c}{2}$$

For a proof of Heron's Area Formula, see Proofs in Mathematics on page 463.

EXAMPLE 5 Using Heron's Area Formula

Use Heron's Area Formula to find the area of a triangle with sides of lengths $a = 43$ meters, $b = 53$ meters, and $c = 72$ meters.

Solution First, determine that $s = (a + b + c)/2 = 168/2 = 84$. Then Heron's Area Formula yields

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{84(84-43)(84-53)(84-72)} \\ &= \sqrt{84(41)(31)(12)} \\ &\approx 1131.89 \text{ square meters.}\end{aligned}$$

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Use Heron's Area Formula to find the area of a triangle with sides of lengths $a = 5$ inches, $b = 9$ inches, and $c = 8$ inches.

You have now studied three different formulas for the area of a triangle.

Standard Formula: $\text{Area} = \frac{1}{2}bh$

Oblique Triangle: $\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$

Heron's Area Formula: $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

Summarize (Section 6.2)

1. State the Law of Cosines (page 409). For examples of using the Law of Cosines to solve oblique triangles (SSS or SAS), see Examples 1 and 2.
2. Describe real-life applications of the Law of Cosines (page 411, Examples 3 and 4).
3. State Heron's Area Formula (page 412). For an example of using Heron's Area Formula to find the area of a triangle, see Example 5.

6.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. The standard form of the Law of Cosines for $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ is _____.
2. When solving an oblique triangle given three sides, use the _____ form of the Law of Cosines to solve for an angle.
3. When solving an oblique triangle given two sides and their included angle, use the _____ form of the Law of Cosines to solve for the remaining side.
4. The Law of Cosines can be used to establish a formula for the area of a triangle called _____ Formula.

Skills and Applications



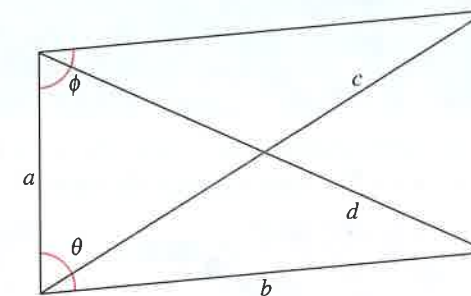
Using the Law of Cosines In Exercises 5–24, use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

- 5.
- 6.
- 7.
- 8.
- 9.
- 10.
- 11.
- 12.

13. $a = 11$, $b = 15$, $c = 21$
14. $a = 55$, $b = 25$, $c = 72$
15. $a = 2.5$, $b = 1.8$, $c = 0.9$
16. $a = 75.4$, $b = 52.5$, $c = 52.5$
17. $A = 120^\circ$, $b = 6$, $c = 7$
18. $A = 48^\circ$, $b = 3$, $c = 14$
19. $B = 10^\circ 35'$, $a = 40$, $c = 30$
20. $B = 75^\circ 20'$, $a = 9$, $c = 6$
21. $B = 125^\circ 40'$, $a = 37$, $c = 37$
22. $C = 15^\circ 15'$, $a = 7.45$, $b = 2.15$
23. $C = 43^\circ$, $a = \frac{4}{9}$, $b = \frac{7}{9}$
24. $C = 101^\circ$, $a = \frac{3}{8}$, $b = \frac{3}{4}$



Finding Measures in a Parallelogram In Exercises 25–30, find the missing values by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by c and d .)



	a	b	c	d	θ	ϕ
25.	5	8			45°	
26.	25	35				120°
27.	10	14	20			
28.	40	60		80		
29.	15		25	20		
30.		25	50	35		



Solving a Triangle In Exercises 31–36, determine whether the Law of Cosines is needed to solve the triangle. Then solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

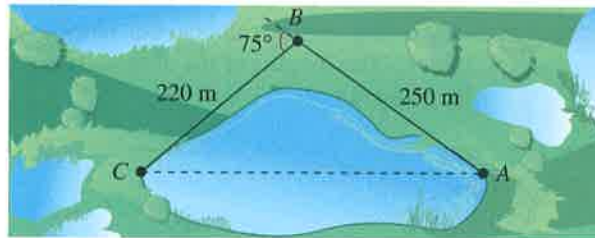
31. $a = 8$, $c = 5$, $B = 40^\circ$
32. $a = 10$, $b = 12$, $C = 70^\circ$
33. $A = 24^\circ$, $a = 4$, $b = 18$
34. $a = 11$, $b = 13$, $c = 7$
35. $A = 42^\circ$, $B = 35^\circ$, $c = 1.2$
36. $B = 12^\circ$, $a = 160$, $b = 63$



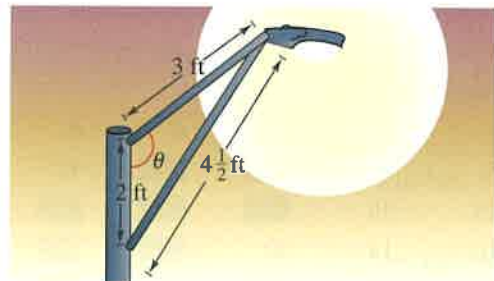
Using Heron's Area Formula In Exercises 37–44, use Heron's Area Formula to find the area of the triangle.

37. $a = 6$, $b = 12$, $c = 17$
 38. $a = 33$, $b = 36$, $c = 21$
 39. $a = 2.5$, $b = 10.2$, $c = 8$
 40. $a = 12.32$, $b = 8.46$, $c = 15.9$
 41. $a = 1$, $b = \frac{1}{2}$, $c = \frac{5}{4}$
 42. $a = \frac{3}{5}$, $b = \frac{4}{3}$, $c = \frac{7}{8}$
 43. $A = 80^\circ$, $b = 75$, $c = 41$
 44. $C = 109^\circ$, $a = 16$, $b = 3.5$

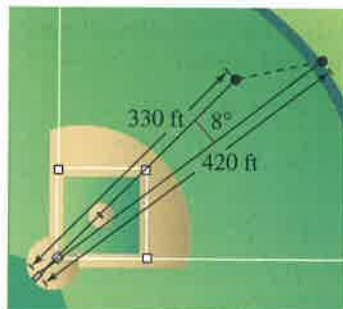
45. **Surveying** To approximate the length of a marsh, a surveyor walks 250 meters from point A to point B, then turns 75° and walks 220 meters to point C (see figure). Approximate the length AC of the marsh.



46. **Streetlight Design** Determine the angle θ in the design of the streetlight shown in the figure.

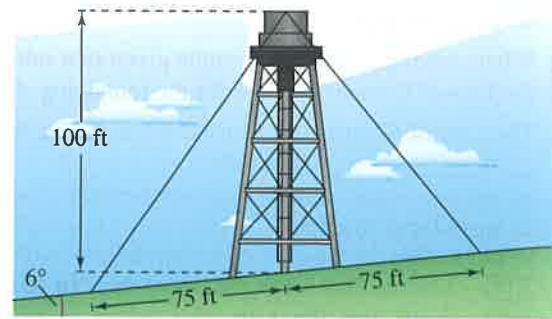


47. **Baseball** A baseball player in center field is approximately 330 feet from a television camera that is behind home plate. A batter hits a fly ball that goes to the wall 420 feet from the camera (see figure). The camera turns 8° to follow the play. Approximately how far does the center fielder have to run to make the catch?



48. **Baseball** On a baseball diamond with 90-foot sides, the pitcher's mound is 60.5 feet from home plate. How far is the pitcher's mound from third base?

49. **Length** A 100-foot vertical tower is built on the side of a hill that makes a 6° angle with the horizontal (see figure). Find the length of each of the two guy wires that are anchored 75 feet uphill and downhill from the base of the tower.



50. **Navigation** On a map, Minneapolis is 165 millimeters due west of Albany, Phoenix is 216 millimeters from Minneapolis, and Phoenix is 368 millimeters from Albany (see figure).



- (a) Find the bearing of Minneapolis from Phoenix.
 (b) Find the bearing of Albany from Phoenix.

51. **Navigation** A boat race runs along a triangular course marked by buoys A, B, and C. The race starts with the boats headed west for 3700 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1700 meters and 3000 meters. Draw a diagram that gives a visual representation of the problem. Then find the bearings for the last two legs of the race.

52. **Air Navigation** A plane flies 810 miles from Franklin to Centerville with a bearing of 75° . Then it flies 648 miles from Centerville to Rosemount with a bearing of 32° . Draw a diagram that gives a visual representation of the problem. Then find the straight-line distance and bearing from Franklin to Rosemount.

53. **Surveying** A triangular parcel of land has 115 meters of frontage, and the other boundaries have lengths of 76 meters and 92 meters. What angles does the frontage make with the two other boundaries?

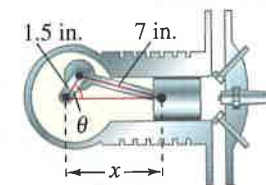
54. **Surveying** A triangular parcel of ground has sides of lengths 725 feet, 650 feet, and 575 feet. Find the measure of the largest angle.

55. **Distance** Two ships leave a port at 9 A.M. One travels at a bearing of N 53° W at 12 miles per hour, and the other travels at a bearing of S 67° W at s miles per hour.

- (a) Use the Law of Cosines to write an equation that relates s and the distance d between the two ships at noon.
 (b) Find the speed s that the second ship must travel so that the ships are 43 miles apart at noon.

56. Mechanical Engineering

An engine has a seven-inch connecting rod fastened to a crank (see figure).

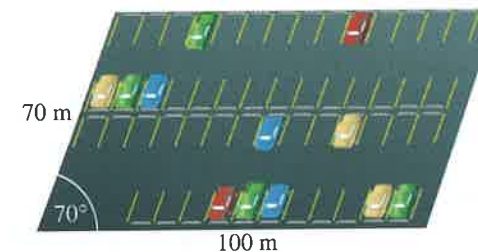


- (a) Use the Law of Cosines to write an equation giving the relationship between x and θ .
 (b) Write x as a function of θ . (Select the sign that yields positive values of x .)
 (c) Use a graphing utility to graph the function in part (b).
 (d) Use the graph in part (c) to determine the total distance the piston moves in one cycle.



57. **Geometry** A triangular parcel of land has sides of lengths 200 feet, 500 feet, and 600 feet. Find the area of the parcel.

58. **Geometry** A parking lot has the shape of a parallelogram (see figure). The lengths of two adjacent sides are 70 meters and 100 meters. The angle between the two sides is 70° . What is the area of the parking lot?



59. **Geometry** You want to buy a triangular lot measuring 510 yards by 840 yards by 1120 yards. The price of the land is \$2000 per acre. How much does the land cost? (Hint: 1 acre = 4840 square yards)

60. **Geometry** You want to buy a triangular lot measuring 1350 feet by 1860 feet by 2490 feet. The price of the land is \$2200 per acre. How much does the land cost? (Hint: 1 acre = 43,560 square feet)

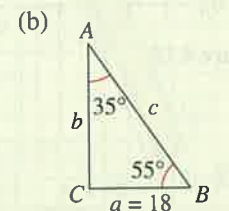
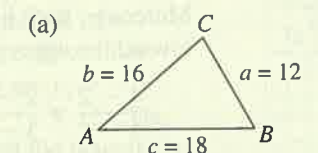
Exploration

True or False? In Exercises 61 and 62, determine whether the statement is true or false. Justify your answer.

61. In Heron's Area Formula, s is the average of the lengths of the three sides of the triangle.
 62. In addition to SSS and SAS, the Law of Cosines can be used to solve triangles with AAS conditions.
 63. **Think About It** What familiar formula do you obtain when you use the standard form of the Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, and you let $C = 90^\circ$? What is the relationship between the Law of Cosines and this formula?
 64. **Writing** Describe how the Law of Cosines can be used to solve the ambiguous case of the oblique triangle ABC, where $a = 12$ feet, $b = 30$ feet, and $A = 20^\circ$. Is the result the same as when the Law of Sines is used to solve the triangle? Describe the advantages and the disadvantages of each method.
 65. **Writing** In Exercise 64, the Law of Cosines was used to solve a triangle in the two-solution case of SSA. Can the Law of Cosines be used to solve the no-solution and single-solution cases of SSA? Explain.



66. HOW DO YOU SEE IT? To solve the triangle, would you begin by using the Law of Sines or the Law of Cosines? Explain.



67. **Proof** Use the Law of Cosines to prove each identity.

$$(a) \frac{1}{2}bc(1 + \cos A) = \frac{a+b+c}{2} \cdot \frac{-a+b+c}{2}$$

$$(b) \frac{1}{2}bc(1 - \cos A) = \frac{a-b+c}{2} \cdot \frac{a+b-c}{2}$$