EXAMPLE 7 Movie Ticket Sales

Two new movies, a comedy and a drama, are released in the same week. In the first six weeks, the weekly ticket sales S (in millions of dollars) decrease for the comedy and increase for the drama according to the models

$$\begin{cases} S = 60 - 8x & \text{Comedy (Equation 1)} \\ S = 10 + 4.5x & \text{Drama (Equation 2)} \end{cases}$$

where x represents the time (in weeks), with $x = 1$ corresponding to the first week of release. According to the models, in what week are the ticket sales of the two movies equal?

Algebraic Solution

Both equations are already solved for S in terms of x, so substitute the expression for S from Equation 2 into Equation 1 and solve for x.

$10 + 4.5x = 60 - 8x$ Substitute for S in Equation 1.

$4.5x + 8x = 60 - 10$ Add $8x$ and -10 to each side.

$12.5x = 50$ Combine like terms.

$x = 4$ Divide each side by 12.5.

According to the models, the weekly ticket sales for the two movies are equal in the fourth week.

Numerical Solution

Create a table of values for each model.

Number of Weeks, x	1	2	3	4	5	6
Sales, S (comedy)	52	44	36	28	20	12
Sales, S (drama)	14.5	19	23.5	28	32.5	37

According to the table, the weekly ticket sales for the two movies are equal in the fourth week.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Two new movies, an animated movie and a horror movie, are released in the same week. In the first eight weeks, the weekly ticket sales S (in millions of dollars) decrease for the animated movie and increase for the horror movie according to the models

$$\begin{cases} S = 108 - 9.4x & \text{Animated} \\ S = 16 + 9x & \text{Horror} \end{cases}$$

where x represents the time (in weeks), with $x = 1$ corresponding to the first week of release. According to the models, in what week are the ticket sales of the two movies equal?

Summarize (Section 7.1)

1. Explain how to use the method of substitution to solve a system of linear equations in two variables *(page 468)*. For examples of using the method of substitution to solve systems of linear equations in two variables, see Examples 1 and 2.
2. Explain how to use the method of substitution to solve a system of nonlinear equations in two variables *(page 471)*. For examples of using the method of substitution to solve systems of nonlinear equations in two variables, see Examples 3 and 4.
3. Explain how to use a graphical method to solve a system of equations in two variables *(page 472)*. For an example of using a graphical approach to solve a system of equations in two variables, see Example 5.
4. Describe examples of how to use systems of equations to model and solve real-life problems *(pages 473 and 474, Examples 6 and 7)*.

7.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A ________ of a system of equations is an ordered pair that satisfies each equation in the system.
2. The first step in solving a system of equations by the method of ________ is to solve one of the equations for one variable in terms of the other.
3. Graphically, solutions of a system of two equations correspond to the ________ of ________ of the graphs of the two equations.
4. In business applications, the total revenue equals the total cost at the ________ point.

Skills and Applications

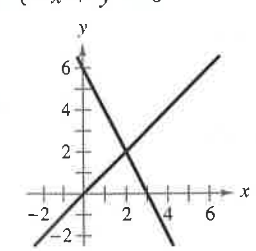
Checking Solutions In Exercises 5 and 6, determine whether each ordered pair is a solution of the system.

5. $\begin{cases} 2x - y = 4 \\ 8x + y = -9 \end{cases}$ (a) $(0, -4)$ (b) $(3, -1)$ (c) $\left(\frac{3}{2}, -1\right)$ (d) $\left(-\frac{1}{2}, -5\right)$

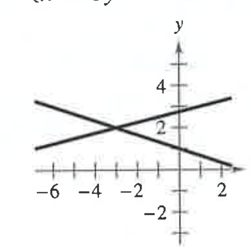
6. $\begin{cases} 4x^2 + y = 3 \\ -x - y = 11 \end{cases}$ (a) $(2, -13)$ (b) $(1, -2)$ (c) $\left(-\frac{3}{2}, -\frac{31}{3}\right)$ (d) $\left(-\frac{7}{4}, -\frac{37}{4}\right)$

Solving a System by Substitution In Exercises 7–14, solve the system by the method of substitution. Check your solution(s) graphically.

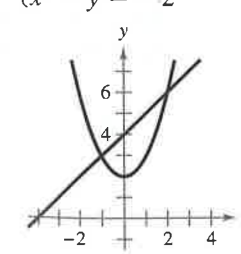
7. $\begin{cases} 2x + y = 6 \\ -x + y = 0 \end{cases}$



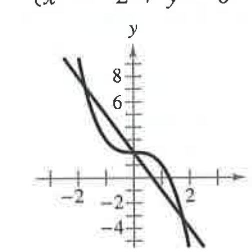
8. $\begin{cases} x - 4y = -11 \\ x + 3y = 3 \end{cases}$



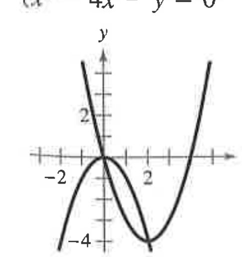
9. $\begin{cases} x - y = -4 \\ x^2 - y = -2 \end{cases}$



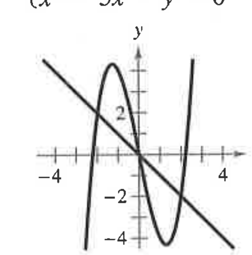
10. $\begin{cases} 3x + y = 2 \\ x^3 - 2 + y = 0 \end{cases}$



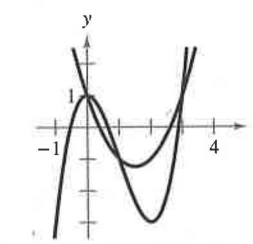
11. $\begin{cases} x^2 + y = 0 \\ x^2 - 4x - y = 0 \end{cases}$



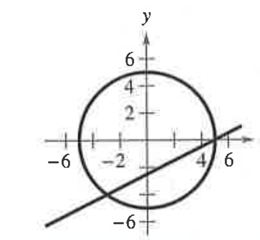
12. $\begin{cases} x + y = 0 \\ x^3 - 5x - y = 0 \end{cases}$



13. $\begin{cases} y = x^3 - 3x^2 + 1 \\ y = x^2 - 3x + 1 \end{cases}$



14. $\begin{cases} -\frac{1}{2}x + y = -\frac{5}{2} \\ x^2 + y^2 = 25 \end{cases}$



Solving a System by Substitution In Exercises 15–24, solve the system by the method of substitution.

15. $\begin{cases} x - y = 2 \\ 6x - 5y = 16 \end{cases}$

16. $\begin{cases} 2x + y = 9 \\ 3x - 5y = 20 \end{cases}$

17. $\begin{cases} 2x - y + 2 = 0 \\ 4x + y - 5 = 0 \end{cases}$

18. $\begin{cases} 6x - 3y - 4 = 0 \\ x + 2y - 4 = 0 \end{cases}$

19. $\begin{cases} 1.5x + 0.8y = 2.3 \\ 0.3x - 0.2y = 0.1 \end{cases}$

20. $\begin{cases} 0.5x + y = -3.5 \\ x - 3.2y = 3.4 \end{cases}$

21. $\begin{cases} \frac{1}{5}x + \frac{1}{2}y = 8 \\ x + y = 20 \end{cases}$

22. $\begin{cases} \frac{1}{2}x + \frac{3}{4}y = 10 \\ \frac{3}{4}x - y = 4 \end{cases}$

23. $\begin{cases} 6x + 5y = -3 \\ -x - \frac{5}{6}y = -7 \end{cases}$

24. $\begin{cases} -\frac{2}{3}x + y = 2 \\ 2x - 3y = 6 \end{cases}$

Solving a System by Substitution In Exercises 25–28, the given amount of annual interest is earned from a total of \$12,000 invested in two funds paying simple interest. Write and solve a system of equations to find the amount invested at each given rate.

	Annual Interest	Rate 1	Rate 2
25.	\$500	2%	6%
26.	\$630	4%	7%
27.	\$396	2.8%	3.8%
28.	\$254	1.75%	2.25%

Solving a System with a Nonlinear Equation In Exercises 29–32, solve the system by the method of substitution.

29. $\begin{cases} x^2 - y = 0 \\ 2x + y = 0 \end{cases}$ 30. $\begin{cases} x - 2y = 0 \\ 3x - y^2 = 0 \end{cases}$

31. $\begin{cases} x - y = -1 \\ x^2 - y = -4 \end{cases}$ 32. $\begin{cases} y = -x \\ y = x^3 + 3x^2 + 2x \end{cases}$

Solving a System of Equations Graphically In Exercises 33–42, solve the system graphically.

33. $\begin{cases} -x + 2y = -2 \\ 3x + y = 20 \end{cases}$ 34. $\begin{cases} x + y = 0 \\ 2x - 7y = -18 \end{cases}$

35. $\begin{cases} x - 3y = -3 \\ 5x + 3y = -6 \end{cases}$ 36. $\begin{cases} -x + 2y = -7 \\ x - y = 2 \end{cases}$

37. $\begin{cases} x + y = 4 \\ x^2 + y^2 - 4x = 0 \end{cases}$ 38. $\begin{cases} -x + y = 3 \\ x^2 - 6x - 27 + y^2 = 0 \end{cases}$

39. $\begin{cases} 3x - 2y = 0 \\ x^2 - y^2 = 4 \end{cases}$ 40. $\begin{cases} 2x - y + 3 = 0 \\ x^2 + y^2 - 4x = 0 \end{cases}$

41. $\begin{cases} x^2 + y^2 = 25 \\ 3x^2 - 16y = 0 \end{cases}$ 42. $\begin{cases} x^2 + y^2 = 25 \\ (x - 8)^2 + y^2 = 25 \end{cases}$

Using Technology In Exercises 43–46, use a graphing utility to solve the system of equations graphically. Round your solution(s) to two decimal places, if necessary.

43. $\begin{cases} y = e^x \\ x - y + 1 = 0 \end{cases}$ 44. $\begin{cases} y = -4e^{-x} \\ y + 3x + 8 = 0 \end{cases}$

45. $\begin{cases} y + 2 = \ln(x - 1) \\ 3y + 2x = 9 \end{cases}$ 46. $\begin{cases} x^2 + y^2 = 4 \\ 2x^2 - y = 2 \end{cases}$

Choosing a Solution Method In Exercises 47–54, solve the system graphically or algebraically. Explain your choice of method.

47. $\begin{cases} y = 2x \\ y = x^2 + 1 \end{cases}$ 48. $\begin{cases} x^2 + y^2 = 9 \\ x - y = -3 \end{cases}$

49. $\begin{cases} x - 2y = 4 \\ x^2 - y = 0 \end{cases}$ 50. $\begin{cases} y = x^3 - 2x^2 + x - 1 \\ y = -x^2 + 3x - 1 \end{cases}$

51. $\begin{cases} y - e^{-x} = 1 \\ y - \ln x = 3 \end{cases}$ 52. $\begin{cases} x^2 + y = 4 \\ e^x - y = 0 \end{cases}$

53. $\begin{cases} xy - 1 = 0 \\ 2x - 4y + 7 = 0 \end{cases}$ 54. $\begin{cases} x - 2y = 1 \\ y = \sqrt{x - 1} \end{cases}$

Break-Even Analysis In Exercises 55 and 56, use the equations for the total cost C and total revenue R to find the number x of units a company must sell to break even. (Round to the nearest whole unit.)

55. $C = 8650x + 250{,}000$, $R = 9502x$

56. $C = 5.5\sqrt{x} + 10{,}000$, $R = 4.22x$

57. **Break-Even Analysis** A small software development company invests \$16,000 to produce a software package that will sell for \$55.95. Each unit costs \$9.45 to produce.
 (a) How many units must the company sell to break even?
 (b) How many units must the company sell to make a profit of \$100,000?

58. **Choice of Two Jobs** You receive two sales job offers. One company offers a straight commission of 6% of sales. The other company offers a salary of \$500 per week plus 3% of sales. How much would you have to sell per week in order to make the straight commission job offer better?

59. **DVD Rentals** Two new DVDs, a horror film and a comedy film, are released in the same week. The weekly number N of rentals decreases for the horror film and increases for the comedy film according to the models

$$\begin{cases} N = 360 - 24x & \text{Horror film} \\ N = 24 + 18x & \text{Comedy film} \end{cases}$$

where x represents the time (in weeks), with $x = 1$ corresponding to the first week of release.
 (a) After how many weeks will the numbers of DVDs rented for the two films be equal?
 (b) Use a table to solve the system of equations numerically. Compare your result with that of part (a).

60. **Supply and Demand** The supply and demand curves for a business dealing with wheat are

Supply: $p = 1.45 + 0.00014x^2$

Demand: $p = (2.388 - 0.07x)^2$

where p is the price (in dollars) per bushel and x is the quantity in bushels per day. Use a graphing utility to graph the supply and demand equations and find the market equilibrium. (The market equilibrium is the point of intersection of the graphs for $x > 0$.)

61. **Error Analysis** Describe the error in solving the system of equations.

$$\begin{cases} x^2 + 2x - y = 3 \\ 2x - y = 2 \end{cases}$$

$$\begin{aligned} x^2 + 2x - (-2x + 2) &= 3 \\ x^2 + 4x - 2 &= 3 \\ x^2 + 4x - 5 &= 0 \\ (x + 5)(x - 1) &= 0 \\ x &= -5, 1 \end{aligned}$$

When $x = -5$, $y = -2(-5) + 2 = -8$, and when $x = 1$, $y = -2(1) + 2 = 0$. So, the solutions are $(-5, -8)$ and $(1, 0)$.

62. **Environmental Science** The table shows the consumption C (in trillions of Btus) of geothermal energy and wind energy in the United States from 2004 through 2014. *(Source: U.S. Energy Information Administration)*

DATA

Year	Geothermal, C	Wind, C
2004	178	142
2005	181	178
2006	181	264
2007	186	341
2008	192	546
2009	200	721
2010	208	923
2011	212	1168
2012	212	1340
2013	214	1601
2014	215	1733

Spreadsheet at LarsonPrecalculus.com

 (a) Use a graphing utility to find a cubic model for the geothermal energy consumption data and a cubic model for the wind energy consumption data. Let t represent the year, with $t = 4$ corresponding to 2004.
 (b) Use the graphing utility to graph the data and the two models in the same viewing window.

 (c) Use the graph from part (b) to approximate the point of intersection of the graphs of the models. Interpret your answer in the context of the problem.
 (d) Describe the behavior of each model. Do you think the models can accurately predict the consumption of geothermal energy and wind energy in the United States for future years? Explain.
 (e) Use your school's library, the Internet, or some other reference source to research the advantages and disadvantages of using renewable energy.

Geometry In Exercises 63 and 64, use a system of equations to find the dimensions of the rectangle meeting the specified conditions.

63. The perimeter is 56 meters and the length is 4 meters greater than the width.

64. The perimeter is 42 inches and the width is three-fourths the length.

ssuaphotos/Shutterstock.com

65. **Geometry** What are the dimensions of a rectangular tract of land when its perimeter is 44 kilometers and its area is 120 square kilometers?

66. **Geometry** What are the dimensions of a right triangle with a two-inch hypotenuse and an area of 1 square inch?

Exploration

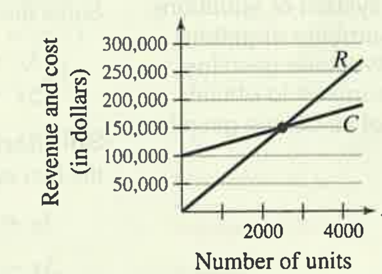
True or False? In Exercises 67 and 68, determine whether the statement is true or false. Justify your answer.

67. In order to solve a system of equations by substitution, you must always solve for y in one of the two equations and then back-substitute.

68. If the graph of a system consists of a parabola and a circle, then the system can have at most two solutions.

69. **Think About It** When solving a system of equations by substitution, how do you recognize that the system has no solution?

70. **HOW DO YOU SEE IT?** The cost C of producing x units and the revenue R obtained by selling x units are shown in the figure.



 (a) Estimate the point of intersection. What does this point represent?
 (b) Use the figure to identify the x-values that correspond to (i) an overall loss and (ii) a profit. Explain.

71. **Think About It** Consider the system of equations

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

 (a) Find values for a, b, c, d, e, and f so that the system has one distinct solution. (There is more than one correct answer.)
 (b) Explain how to solve the system in part (a) by the method of substitution and graphically.
 (c) Write a brief paragraph describing any advantages of the method of substitution over the graphical method of solving a system of equations.