

7.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

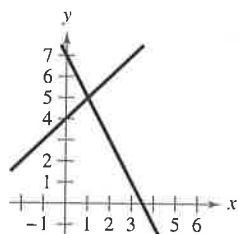
- The first step in solving a system of equations by the method of _____ is to obtain coefficients for x (or y) that differ only in sign.
- Two systems of equations that have the same solution set are _____ systems.
- A system of linear equations that has at least one solution is _____, whereas a system of linear equations that has no solution is _____.
- In business applications, the _____ (x, p) is the price p and the number of units x that satisfy both the demand and supply equations.

Skills and Applications

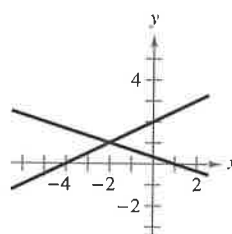


Solving a System by Elimination In Exercises 5–12, solve the system by the method of elimination. Label each line with its equation. To print an enlarged copy of the graph, go to MathGraphs.com.

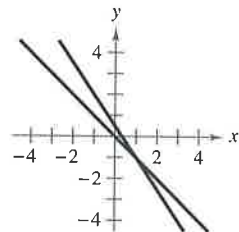
5. $\begin{cases} 2x + y = 7 \\ x - y = -4 \end{cases}$



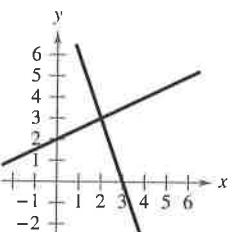
6. $\begin{cases} x + 3y = 1 \\ -x + 2y = 4 \end{cases}$



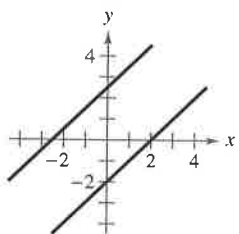
7. $\begin{cases} x + y = 0 \\ 3x + 2y = 1 \end{cases}$



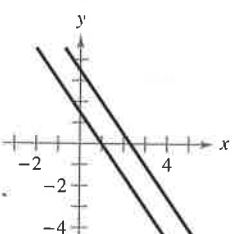
8. $\begin{cases} \frac{1}{2}x - y = -2 \\ x + \frac{1}{3}y = 3 \end{cases}$



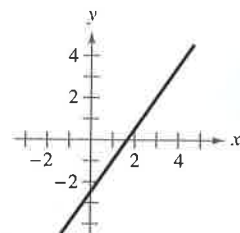
9. $\begin{cases} x - y = 2 \\ -2x + 2y = 5 \end{cases}$



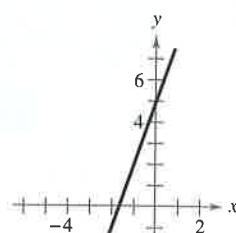
10. $\begin{cases} 3x + 2y = 3 \\ 6x + 4y = 14 \end{cases}$



11. $\begin{cases} 3x - 2y = 5 \\ -6x + 4y = -10 \end{cases}$



12. $\begin{cases} 9x - 3y = -15 \\ -3x + y = 5 \end{cases}$



Solving a System by Elimination In Exercises 13–30, solve the system by the method of elimination and check any solutions algebraically.

13. $\begin{cases} x + 2y = 6 \\ x - 2y = 2 \end{cases}$

14. $\begin{cases} 3x - 5y = 8 \\ 2x + 5y = 22 \end{cases}$

15. $\begin{cases} 5x + 3y = 6 \\ 3x - y = 5 \end{cases}$

16. $\begin{cases} x + 5y = 10 \\ 3x - 10y = -5 \end{cases}$

17. $\begin{cases} 2u + 3v = -1 \\ 7u + 15v = 4 \end{cases}$

18. $\begin{cases} 2r + 4s = 5 \\ 16r + 50s = 55 \end{cases}$

19. $\begin{cases} 3x + 2y = 10 \\ 2x + 5y = 3 \end{cases}$

20. $\begin{cases} 3x + 11y = 4 \\ -2x - 5y = 9 \end{cases}$

21. $\begin{cases} 4b + 3m = 3 \\ 3b + 11m = 13 \end{cases}$

22. $\begin{cases} 2x + 5y = 8 \\ 5x + 8y = 10 \end{cases}$

23. $\begin{cases} 0.2x - 0.5y = -27.8 \\ 0.3x + 0.4y = 68.7 \end{cases}$

24. $\begin{cases} 0.5x - 0.3y = 6.5 \\ 0.7x + 0.2y = 6.0 \end{cases}$

25. $\begin{cases} 3x + 2y = 4 \\ 9x + 6y = 3 \end{cases}$

26. $\begin{cases} -6x + 4y = 7 \\ 15x - 10y = -16 \end{cases}$

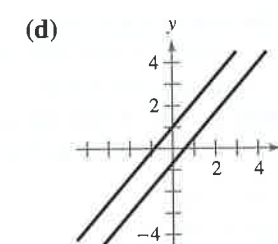
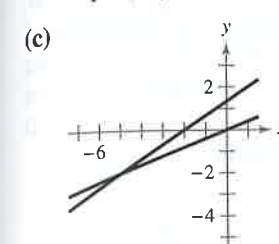
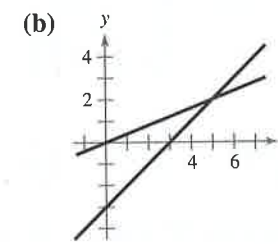
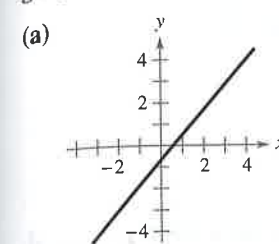
27. $\begin{cases} -5x + 6y = -3 \\ 20x - 24y = 12 \end{cases}$

28. $\begin{cases} 7x + 8y = 6 \\ -14x - 16y = -12 \end{cases}$

29. $\begin{cases} \frac{x+3}{4} + \frac{y-1}{3} = 1 \\ 2x - y = 12 \end{cases}$

30. $\begin{cases} \frac{x-1}{2} + \frac{y+2}{3} = 4 \\ x - 2y = 5 \end{cases}$

Matching a System with Its Graph In Exercises 31–34, match the system of linear equations with its graph. Describe the number of solutions and state whether the system is consistent or inconsistent. [The graphs are labeled (a), (b), (c) and (d).]



31. $\begin{cases} -7x + 6y = -4 \\ 14x - 12y = 8 \end{cases}$

32. $\begin{cases} 2x - 5y = 0 \\ 2x - 3y = -4 \end{cases}$

33. $\begin{cases} 7x - 6y = -6 \\ -7x + 6y = -4 \end{cases}$

34. $\begin{cases} 2x - 5y = 0 \\ x - y = 3 \end{cases}$

Choosing a Solution Method In Exercises 35–40, use any method to solve the system. Explain your choice of method.

35. $\begin{cases} 3x - 5y = 7 \\ 2x + y = 9 \end{cases}$

36. $\begin{cases} -x + 3y = 17 \\ 4x + 3y = 7 \end{cases}$

37. $\begin{cases} -2x + 8y = 20 \\ y = x - 5 \end{cases}$

38. $\begin{cases} -5x + 9y = 13 \\ y = x - 4 \end{cases}$

39. $\begin{cases} y = -2x - 17 \\ y = 2 - 3x \end{cases}$

40. $\begin{cases} y = -3x - 8 \\ y = 15 - 2x \end{cases}$

41. Airplane Speed An airplane flying into a headwind travels the 1800-mile flying distance between Indianapolis, Indiana, and Phoenix, Arizona, in 3 hours. On the return flight, the airplane travels this distance in 2 hours and 30 minutes. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

42. Airplane Speed Two planes start from Los Angeles International Airport and fly in opposite directions. The second plane starts $\frac{1}{2}$ hour after the first plane, but its speed is 80 kilometers per hour faster. Find the speed of each plane when 2 hours after the first plane departs the planes are 3200 kilometers apart.

43. Nutrition Two cheeseburgers and one small order of fries contain a total of 1420 calories. Three cheeseburgers and two small orders of fries contain a total of 2290 calories. Find the caloric content of each item.

44. Nutrition One eight-ounce glass of apple juice and one eight-ounce glass of orange juice contain a total of 179.2 milligrams of vitamin C. Two eight-ounce glasses of apple juice and three eight-ounce glasses of orange juice contain a total of 442.1 milligrams of vitamin C. How much vitamin C is in an eight-ounce glass of each type of juice?



Finding the Equilibrium Point In Exercises 45–48, find the equilibrium point of the demand and supply equations.

Demand

Supply

45. $p = 500 - 0.4x$

$p = 380 + 0.1x$

46. $p = 100 - 0.05x$

$p = 25 + 0.1x$

47. $p = 140 - 0.00002x$

$p = 80 + 0.00001x$

48. $p = 400 - 0.0002x$

$p = 225 + 0.0005x$

49. Chemistry Thirty liters of a 40% acid solution is obtained by mixing a 25% solution with a 50% solution.

- Write a system of equations in which one equation represents the total amount of final mixture required and the other represents the percent of acid in the final mixture. Let x and y represent the amounts of the 25% and 50% solutions, respectively.



- Use a graphing utility to graph the two equations in part (a) in the same viewing window. As the amount of the 25% solution increases, how does the amount of the 50% solution change?

- How much of each solution is required to obtain the specified concentration of the final mixture?

50. Fuel Mixture

Five hundred gallons of 89-octane gasoline is obtained by mixing 87-octane gasoline with 92-octane gasoline.

- Write a system of equations in which one equation represents the total amount of final mixture required and the other represents the amounts of 87- and 92-octane gasoline in the final mixture. Let x and y represent the numbers of gallons of 87- and 92-octane gasoline, respectively.

- Use a graphing utility to graph the two equations in part (a) in the same viewing window. As the amount of 87-octane gasoline increases, how does the amount of 92-octane gasoline change?

- How much of each type of gasoline is required to obtain the 500 gallons of 89-octane gasoline?



51. Investment Portfolio A total of \$24,000 is invested in two corporate bonds that pay 3.5% and 5% simple interest. The investor wants an annual interest income of \$930 from the investments. What amount should be invested in the 3.5% bond?

52. Investment Portfolio A total of \$32,000 is invested in two municipal bonds that pay 5.75% and 6.25% simple interest. The investor wants an annual interest income of \$1900 from the investments. What amount should be invested in the 5.75% bond?

53. Pharmacology The numbers of prescriptions P (in thousands) filled at two pharmacies from 2012 through 2016 are shown in the table.

Year	Pharmacy A	Pharmacy B
2012	19.2	20.4
2013	19.6	20.8
2014	20.0	21.1
2015	20.6	21.5
2016	21.3	22.0

(a) Use a graphing utility to create a scatter plot of the data for pharmacy A and find a linear model. Let t represent the year, with $t = 12$ corresponding to 2012. Repeat the procedure for pharmacy B.

(b) Assume that the models in part (a) can be used to represent future years. Will the number of prescriptions filled at pharmacy A ever exceed the number of prescriptions filled at pharmacy B? If so, when?

54. Daily Sales A store manager wants to know the demand for a product as a function of the price. The table shows the daily sales y for different prices x of the product.

Price, x	Demand, y
\$1.00	45
\$1.20	37
\$1.50	23

(a) Find the least squares regression line $y = ax + b$ for the data by solving the system

$$\begin{cases} 3.00b + 3.70a = 105.00 \\ 3.70b + 4.69a = 123.90 \end{cases}$$

for a and b . Use a graphing utility to confirm the result.

(b) Use the linear model from part (a) to predict the demand when the price is \$1.75.

Fitting a Line to Data One way to find the least squares regression line $y = ax + b$ for a set of points

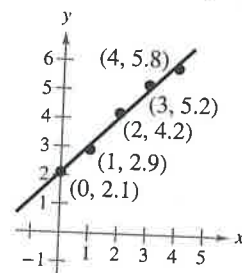
$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$$

is by solving the system below for a and b .

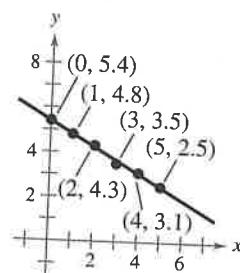
$$\begin{cases} nb + \left(\sum_{i=1}^n x_i\right)a = \left(\sum_{i=1}^n y_i\right) \\ \left(\sum_{i=1}^n x_i\right)b + \left(\sum_{i=1}^n x_i^2\right)a = \left(\sum_{i=1}^n x_i y_i\right) \end{cases}$$

In Exercises 55 and 56, the sums have been evaluated. Solve the simplified system for a and b to find the least squares regression line for the points. Use a graphing utility to confirm the result. (Note: The symbol Σ is used to denote a sum of the terms of a sequence. You will learn how to use this notation in Section 9.1.)

$$55. \begin{cases} 5b + 10a = 20.2 \\ 10b + 30a = 50.1 \end{cases}$$



$$56. \begin{cases} 6b + 15a = 23.6 \\ 15b + 55a = 48.8 \end{cases}$$



57. Agriculture An agricultural scientist used four test plots to determine the relationship between wheat yield y (in bushels per acre) and the amount of fertilizer x (in hundreds of pounds per acre). The table shows the results.

Fertilizer, x	1.0	1.5	2.0	2.5
Yield, y	32	41	48	53

(a) Find the least squares regression line $y = ax + b$ for the data by solving the system for a and b .

$$\begin{cases} 4b + 7a = 174 \\ 7b + 13.5a = 322 \end{cases}$$

(b) Use the linear model from part (a) to estimate the yield for a fertilizer application of 160 pounds per acre.

58. Gross Domestic Product The table shows the total gross domestic products y (in billions of dollars) of the United States for the years 2009 through 2015. (Source: U.S. Office of Management and Budget)

Year	GDP, y
2009	14,414.6
2010	14,798.5
2011	15,379.2
2012	16,027.2
2013	16,498.1
2014	17,183.5
2015	17,803.4

(a) Find the least squares regression line $y = at + b$ for the data, where t represents the year with $t = 9$ corresponding to 2009, by solving the system

$$\begin{cases} 7b + 84a = 112,104.5 \\ 84b + 1036a = 1,361,309.3 \end{cases}$$

for a and b . Use the regression feature of a graphing utility to confirm the result.

(b) Use the linear model to create a table of estimated values of y . Compare the estimated values with the actual data.

(c) Use the linear model to estimate the gross domestic product for 2016.

(d) Use the Internet, your school's library, or some other reference source to find the total national outlay for 2016. How does this value compare with your answer in part (c)?

(e) Is the linear model valid for long-term predictions of gross domestic products? Explain.

Exploration

True or False? In Exercises 59 and 60, determine whether the statement is true or false. Justify your answer.

59. If two lines do not have exactly one point of intersection, then they must be parallel.

60. Solving a system of equations graphically will always give an exact solution.

Finding the Value of a Constant In Exercises 61 and 62, find the value of k such that the system of linear equations is inconsistent.

$$61. \begin{cases} 4x - 8y = -3 \\ 2x + ky = 16 \end{cases} \quad 62. \begin{cases} 15x + 3y = 6 \\ -10x + ky = 9 \end{cases}$$

63. **Writing** Briefly explain whether it is possible for a consistent system of linear equations to have exactly two solutions.

64. Think About It Give examples of systems of linear equations that have (a) no solution and (b) infinitely many solutions.

65. Comparing Methods Use the method of substitution to solve the system in Example 1. Do you prefer the method of substitution or the method of elimination? Explain.

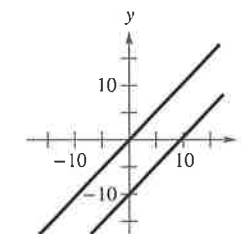
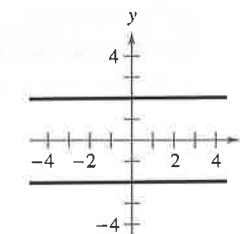
66. HOW DO YOU SEE IT? Use the graphs of the two equations shown below.

(a) Describe the graphs of the two equations.

(b) Can you conclude that the system of equations whose graphs are shown is inconsistent? Explain.

Think About It In Exercises 67 and 68, the graphs of the two equations appear to be parallel. Yet, when you solve the system algebraically, you find that the system does have a solution. Find the solution and explain why it does not appear on the portion of the graph shown.

$$67. \begin{cases} 100y - x = 200 \\ 99y - x = -198 \end{cases} \quad 68. \begin{cases} 21x - 20y = 0 \\ 13x - 12y = 120 \end{cases}$$



Advanced Applications In Exercises 69 and 70, solve the system of equations for u and v . While solving for these variables, consider the trigonometric functions as constants. (Systems of this type appear in a course in differential equations.)

$$69. \begin{cases} u \sin x + v \cos x = 0 \\ u \cos x - v \sin x = \sec x \end{cases}$$

$$70. \begin{cases} u \cos 2x + v \sin 2x = 0 \\ u(-2 \sin 2x) + v(2 \cos 2x) = \csc 2x \end{cases}$$

Project: College Expenses To work an extended application analyzing the average undergraduate tuition, room, and board charges at private degree-granting institutions in the United States from 1993 through 2013, visit this text's website at LarsonPrecalculus.com. (Source: U.S. Department of Education)