


## 7.3 Exercises

See [CalcChat.com](http://CalcChat.com) for tutorial help and worked-out solutions to odd-numbered exercises.**Vocabulary:** Fill in the blanks.

1. A system of equations in \_\_\_\_\_ form has a “stair-step” pattern with leading coefficients of 1.
2. A solution of a system of three linear equations in three variables can be written as an \_\_\_\_\_, which has the form  $(x, y, z)$ .
3. The process used to write a system of linear equations in row-echelon form is called \_\_\_\_\_ elimination.
4. Interchanging two equations of a system of linear equations is a \_\_\_\_\_ that produces an equivalent system.
5. In a \_\_\_\_\_ system, the number of equations differs from the number of variables in the system.
6. The equation  $s = \frac{1}{2}at^2 + v_0t + s_0$  is called the \_\_\_\_\_ equation, and it models the height  $s$  of an object at time  $t$  that is moving in a vertical line with a constant acceleration  $a$ .

**Skills and Applications****Checking Solutions** In Exercises 7–10, determine whether each ordered triple is a solution of the system of equations.

7. 
$$\begin{cases} 6x - y + z = -1 \\ 4x - 3z = -19 \\ 2y + 5z = 25 \end{cases}$$
  - (a)  $(0, 3, 1)$
  - (b)  $(-3, 0, 5)$
  - (c)  $(0, -1, 4)$
  - (d)  $(-1, 0, 5)$
8. 
$$\begin{cases} 3x + 4y - z = 17 \\ 5x - y + 2z = -2 \\ 2x - 3y + 7z = -21 \end{cases}$$
  - (a)  $(3, -1, 2)$
  - (b)  $(1, 3, -2)$
  - (c)  $(1, 5, 6)$
  - (d)  $(1, -2, 2)$
9. 
$$\begin{cases} 4x + y - z = 0 \\ -8x - 6y + z = -\frac{7}{4} \\ 3x - y = -\frac{9}{4} \end{cases}$$
  - (a)  $(\frac{1}{2}, -\frac{3}{4}, -\frac{7}{4})$
  - (b)  $(\frac{3}{2}, -\frac{2}{5}, \frac{3}{5})$
  - (c)  $(-\frac{1}{2}, \frac{3}{4}, -\frac{5}{4})$
  - (d)  $(-\frac{1}{2}, \frac{1}{6}, -\frac{3}{4})$
10. 
$$\begin{cases} -4x - y - 8z = -6 \\ y + z = 0 \\ 4x - 7y = 6 \end{cases}$$
  - (a)  $(-2, -2, 2)$
  - (b)  $(-\frac{33}{2}, -10, 10)$
  - (c)  $(\frac{1}{8}, -\frac{1}{2}, \frac{1}{2})$
  - (d)  $(-\frac{1}{2}, -2, 1)$


**Using Back-Substitution in Row-Echelon Form** In Exercises 11–16, use back-substitution to solve the system of linear equations.

11. 
$$\begin{cases} x - y + 5z = 37 \\ y + 2z = 6 \\ z = 8 \end{cases}$$
12. 
$$\begin{cases} x - 2y + 2z = 20 \\ y - z = 8 \\ z = -1 \end{cases}$$

13. 
$$\begin{cases} x + y - 3z = 7 \\ y + z = 12 \\ z = 2 \end{cases}$$
14. 
$$\begin{cases} x - y + 2z = 22 \\ y - 8z = 13 \\ z = -3 \end{cases}$$
15. 
$$\begin{cases} x - 2y + z = -\frac{1}{4} \\ y - z = -4 \\ z = 11 \end{cases}$$
16. 
$$\begin{cases} x - 8z = \frac{1}{2} \\ y - 5z = 22 \\ z = -4 \end{cases}$$

**Performing Row Operations** In Exercises 17 and 18, perform the row operation and write the equivalent system.

17. Add Equation 1 to Equation 2.


$$\begin{cases} x - 2y + 3z = 5 & \text{Equation 1} \\ -x + 3y - 5z = 4 & \text{Equation 2} \\ 2x - 3z = 0 & \text{Equation 3} \end{cases}$$

What did this operation accomplish?

18. Add
- $-2$
- times Equation 1 to Equation 3.

$$\begin{cases} x - 2y + 3z = 5 & \text{Equation 1} \\ -x + 3y - 5z = 4 & \text{Equation 2} \\ 2x - 3z = 0 & \text{Equation 3} \end{cases}$$

What did this operation accomplish?


**Solving a System of Linear Equations** In Exercises 19–22, solve the system of linear equations and check any solutions algebraically.

19. 
$$\begin{cases} -2x + 3y = 10 \\ x + y = 0 \end{cases}$$
20. 
$$\begin{cases} 2x - y = 0 \\ x - y = 7 \end{cases}$$
21. 
$$\begin{cases} 3x - y = 9 \\ x - 2y = -2 \end{cases}$$
22. 
$$\begin{cases} x + 2y = 1 \\ 5x - 4y = -23 \end{cases}$$

**Solving a System of Linear Equations** In Exercises 23–40, solve the system of linear equations and check any solutions algebraically.

23. 
$$\begin{cases} x + y + z = 7 \\ 2x - y + z = 9 \\ 3x - z = 10 \end{cases}$$
24. 
$$\begin{cases} x + y + z = 5 \\ x - 2y + 4z = 13 \\ 3y + 4z = 13 \end{cases}$$
25. 
$$\begin{cases} 2x + 4y - z = 7 \\ 2x - 4y + 2z = -6 \\ x + 4y + z = 0 \end{cases}$$
26. 
$$\begin{cases} 2x + 4y + z = 1 \\ x - 2y - 3z = 2 \\ x + y - z = -1 \end{cases}$$
27. 
$$\begin{cases} 2x + y - z = 7 \\ x - 2y + 2z = -9 \\ 3x - y + z = 5 \end{cases}$$
28. 
$$\begin{cases} 5x - 3y + 2z = 3 \\ 2x + 4y - z = 7 \\ x - 11y + 4z = 3 \end{cases}$$
29. 
$$\begin{cases} 3x - 5y + 5z = 1 \\ 2x - 2y + 3z = 0 \\ 7x - y + 3z = 0 \end{cases}$$
30. 
$$\begin{cases} 2x + y + 3z = 1 \\ 2x + 6y + 8z = 3 \\ 6x + 8y + 18z = 5 \end{cases}$$
31. 
$$\begin{cases} 2x + 3y = 0 \\ 4x + 3y - z = 0 \\ 8x + 3y + 3z = 0 \end{cases}$$
32. 
$$\begin{cases} 4x + 3y + 17z = 0 \\ 5x + 4y + 22z = 0 \\ 4x + 2y + 19z = 0 \end{cases}$$
33. 
$$\begin{cases} x + 4z = 1 \\ x + y + 10z = 10 \\ 2x - y + 2z = -5 \end{cases}$$
34. 
$$\begin{cases} 2x - 2y - 6z = -4 \\ -3x + 2y + 6z = 1 \\ x - y - 5z = -3 \end{cases}$$
35. 
$$\begin{cases} 3x - 3y + 6z = 6 \\ x + 2y - z = 5 \\ 5x - 8y + 13z = 7 \end{cases}$$
36. 
$$\begin{cases} x + 2z = 5 \\ 3x - y - z = 1 \\ 6x - y + 5z = 16 \end{cases}$$
37. 
$$\begin{cases} x + 2y - 7z = -4 \\ 2x + y + z = 13 \\ 3x + 9y - 36z = -33 \end{cases}$$
38. 
$$\begin{cases} 2x + y - 3z = 4 \\ 4x + 2z = 10 \\ -2x + 3y - 13z = -8 \end{cases}$$
39. 
$$\begin{cases} x + 3w = 4 \\ 2y - z - w = 0 \\ 3y - 2w = 1 \\ 2x - y + 4z = 5 \end{cases}$$
40. 
$$\begin{cases} x + y + z + w = 6 \\ 2x + 3y - w = 0 \\ -3x + 4y + z + 2w = 4 \\ x + 2y - z + w = 0 \end{cases}$$

**Solving a Nonsquare System** In Exercises 41–44, solve the system of linear equations and check any solutions algebraically.

41. 
$$\begin{cases} x - 2y + 5z = 2 \\ 4x - z = 0 \end{cases}$$
42. 
$$\begin{cases} x - 3y + 2z = 18 \\ 5x - 13y + 12z = 80 \end{cases}$$
43. 
$$\begin{cases} 2x - 3y + z = -2 \\ -4x + 9y = 7 \end{cases}$$
44. 
$$\begin{cases} 2x + 3y + 3z = 7 \\ 4x + 18y + 15z = 44 \end{cases}$$

**Modeling Vertical Motion** In Exercises 45 and 46, an object moving vertically is at the given heights at the specified times. Find the position equation  $s = \frac{1}{2}at^2 + v_0t + s_0$  for the object.

45. At  $t = 1$  second,  $s = 128$  feet  
At  $t = 2$  seconds,  $s = 80$  feet  
At  $t = 3$  seconds,  $s = 0$  feet
46. At  $t = 1$  second,  $s = 132$  feet  
At  $t = 2$  seconds,  $s = 100$  feet  
At  $t = 3$  seconds,  $s = 36$  feet

**Finding the Equation of a Parabola** In Exercises 47–52, find the equation

$$y = ax^2 + bx + c$$

of the parabola that passes through the points. To verify your result, use a graphing utility to plot the points and graph the parabola.

47.  $(0, 0), (2, -2), (4, 0)$
48.  $(0, 3), (1, 4), (2, 3)$
49.  $(2, 0), (3, -1), (4, 0)$
50.  $(1, 3), (2, 2), (3, -3)$
51.  $(\frac{1}{2}, 1), (1, 3), (2, 13)$
52.  $(-2, -3), (-1, 0), (\frac{1}{2}, -3)$

**Finding the Equation of a Circle** In Exercises 53–56, find the equation

$$x^2 + y^2 + Dx + Ey + F = 0$$

of the circle that passes through the points. To verify your result, use a graphing utility to plot the points and graph the circle.

53.  $(0, 0), (5, 5), (10, 0)$
54.  $(0, 0), (0, 6), (3, 3)$
55.  $(-3, -1), (2, 4), (-6, 8)$
56.  $(0, 0), (0, -2), (3, 0)$

- 57.
- Error Analysis**
- Describe the error.

The system

$$\begin{cases} x - 2y + 3z = 12 \\ y + 3z = 5 \\ 2z = 4 \end{cases}$$

is in row-echelon form.

58. **Agriculture** A mixture of 5 pounds of fertilizer A, 13 pounds of fertilizer B, and 4 pounds of fertilizer C provides the optimal nutrients for a plant. Commercial brand X contains equal parts of fertilizer B and fertilizer C. Commercial brand Y contains one part of fertilizer A and two parts of fertilizer B. Commercial brand Z contains two parts of fertilizer A, five parts of fertilizer B, and two parts of fertilizer C. How much of each fertilizer brand is needed to obtain the desired mixture?

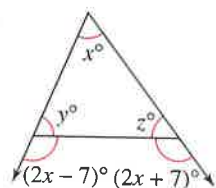


**59. Finance** To expand its clothing line, a small corporation borrowed \$775,000 from three different lenders. The money was borrowed at 8%, 9%, and 10% simple interest. How much was borrowed at each rate when the annual interest owed was \$67,500 and the amount borrowed at 8% was four times the amount borrowed at 10%?

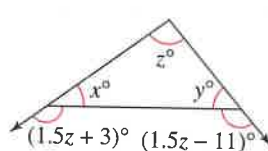
**60. Advertising** A health insurance company advertises on television, on radio, and in the local newspaper. The marketing department has an advertising budget of \$42,000 per month. A television ad costs \$1000, a radio ad costs \$200, and a newspaper ad costs \$500. The department wants to run 60 ads per month and have as many television ads as radio and newspaper ads combined. How many of each type of ad can the department run each month?

**Geometry** In Exercises 61 and 62, find the values of  $x$ ,  $y$ , and  $z$  in the figure.

61.



62.



**63. Geometry** The perimeter of a triangle is 180 feet. The longest side of the triangle is 9 feet shorter than twice the shortest side. The sum of the lengths of the two shorter sides is 30 feet more than the length of the longest side. Find the lengths of the sides of the triangle.

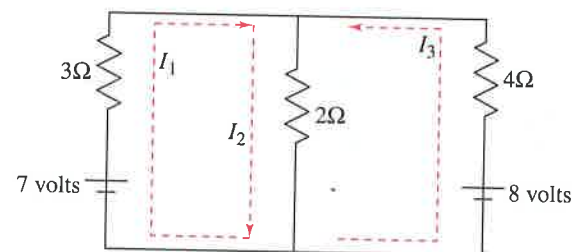
**64. Chemistry** A chemist needs 10 liters of a 25% acid solution. The solution is to be mixed from three solutions whose concentrations are 10%, 20%, and 50%. How many liters of each solution will satisfy each condition?

- Use 2 liters of the 50% solution.
- Use as little as possible of the 50% solution.
- Use as much as possible of the 50% solution.

**65. Electrical Network** Applying Kirchhoff's Laws to the electrical network in the figure, the currents  $I_1$ ,  $I_2$ , and  $I_3$ , are the solution of the system

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 3I_1 + 2I_2 = 7 \\ 2I_2 + 4I_3 = 8 \end{cases}$$

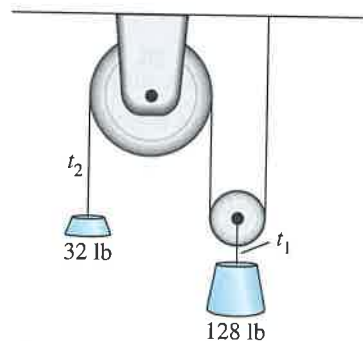
Find the currents.



**66. Pulley System** A system of pulleys is loaded with 128-pound and 32-pound weights (see figure). The tensions  $t_1$  and  $t_2$  in the ropes and the acceleration  $a$  of the 32-pound weight are found by solving the system

$$\begin{cases} t_1 - 2t_2 = 0 \\ t_1 - 2a = 128 \\ t_2 + a = 32 \end{cases}$$

where  $t_1$  and  $t_2$  are in pounds and  $a$  is in feet per second squared. Solve this system.



**Fitting a Parabola** One way to find the least squares regression parabola  $y = ax^2 + bx + c$  for a set of points

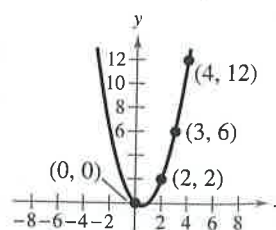
$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$

is by solving the system below for  $a$ ,  $b$ , and  $c$ .

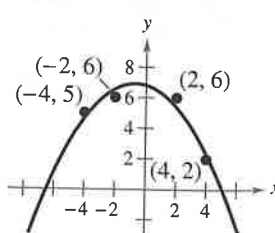
$$\begin{cases} nc + \left(\sum_{i=1}^n x_i\right)b + \left(\sum_{i=1}^n x_i^2\right)a = \sum_{i=1}^n y_i \\ \left(\sum_{i=1}^n x_i\right)c + \left(\sum_{i=1}^n x_i^2\right)b + \left(\sum_{i=1}^n x_i^3\right)a = \sum_{i=1}^n x_i y_i \\ \left(\sum_{i=1}^n x_i^2\right)c + \left(\sum_{i=1}^n x_i^3\right)b + \left(\sum_{i=1}^n x_i^4\right)a = \sum_{i=1}^n x_i^2 y_i \end{cases}$$

In Exercises 67 and 68, the sums have been evaluated. Solve the simplified system for  $a$ ,  $b$ , and  $c$  to find the least squares regression parabola for the points. Use a graphing utility to confirm the result. (Note: The symbol  $\Sigma$  is used to denote a sum of the terms of a sequence. You will learn how to use this notation in Section 9.1.)

67. 
$$\begin{cases} 4c + 9b + 29a = 20 \\ 9c + 29b + 99a = 70 \\ 29c + 99b + 353a = 254 \end{cases}$$



68. 
$$\begin{cases} 4c + 40a = 19 \\ 40b = -12 \\ 40c + 544a = 160 \end{cases}$$



**69. Stopping Distance** In testing a new automobile braking system, engineers recorded the speed  $x$  (in miles per hour) and the stopping distance  $y$  (in feet). The table shows the results.

Speed, $x$	30	40	50	60	70
Stopping Distance, $y$	75	118	175	240	315

(a) Find the least squares regression parabola  $y = ax^2 + bx + c$  for the data by solving the system.

$$\begin{cases} 5c + 250b + 13,500a = 923 \\ 250c + 13,500b + 775,000a = 52,170 \\ 13,500c + 775,000b + 46,590,000a = 3,101,300 \end{cases}$$

(b) Use a graphing utility to graph the model you found in part (a) and the data in the same viewing window. How well does the model fit the data? Explain.

(c) Use the model to estimate the stopping distance when the speed is 75 miles per hour.

**70. Wildlife** . . . . .

A wildlife management team studied the reproductive rates of deer in four tracts of a wildlife preserve. In each tract, the number of females  $x$  and the percent of females  $y$  that had offspring the following year were recorded. The table shows the results.



Number, $x$	100	120	140	160
Percent, $y$	75	68	55	30

(a) Find the least squares regression parabola  $y = ax^2 + bx + c$  for the data by solving the system.

$$\begin{cases} 4c + 520b + 69,600a = 228 \\ 520c + 69,600b + 9,568,000a = 28,160 \\ 69,600c + 9,568,000b + 1,346,880,000a = 3,575,200 \end{cases}$$

(b) Use a graphing utility to graph the model you found in part (a) and the data in the same viewing window. How well does the model fit the data? Explain.

(c) Use the model to estimate the percent of females that had offspring when there were 170 females.

(d) Use the model to estimate the number of females when 40% of the females had offspring.

**Advanced Applications** In Exercises 71 and 72, find values of  $x$ ,  $y$ , and  $\lambda$  that satisfy the system. These systems arise in certain optimization problems in calculus, and  $\lambda$  is called a Lagrange multiplier.

71. 
$$\begin{cases} 2x - 2x\lambda = 0 \\ -2y + \lambda = 0 \\ y - x^2 = 0 \end{cases}$$

72. 
$$\begin{cases} 2 + 2y + 2\lambda = 0 \\ 2x + 1 + \lambda = 0 \\ 2x + y - 100 = 0 \end{cases}$$

### Exploration

**True or False?** In Exercises 73 and 74, determine whether the statement is true or false. Justify your answer.

73. Every nonsquare system of linear equations has a unique solution.

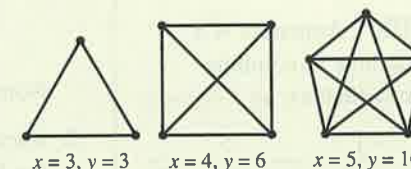
74. If a system of three linear equations is inconsistent, then there are no points common to the graphs of all three equations of the system.

75. **Think About It** Are the following two systems of equations equivalent? Give reasons for your answer.

$$\begin{cases} x + 3y - z = 6 \\ 2x - y + 2z = 1 \\ 3x + 2y - z = 2 \end{cases} \quad \begin{cases} x + 3y - z = 6 \\ -7y + 4z = 1 \\ -7y - 4z = -16 \end{cases}$$



**76. HOW DO YOU SEE IT?** The number of sides  $x$  and the combined number of sides and diagonals  $y$  for each of three regular polygons are shown below. Write a system of linear equations to find an equation of the form  $y = ax^2 + bx + c$  that represents the relationship between  $x$  and  $y$  for the three polygons.



**Finding Systems of Linear Equations** In Exercises 77–80, find two systems of linear equations that have the ordered triple as a solution. (There are many correct answers.)

77.  $(2, 0, -1)$       78.  $(-5, 3, -2)$   
79.  $(\frac{1}{2}, -3, 0)$       80.  $(4, \frac{2}{5}, \frac{1}{2})$

**Project: Earnings per Share** To work an extended application analyzing the earnings per share for Wal-Mart Stores, Inc., from 2001 through 2015, visit this text's website at [LarsonPrecalculus.com](http://LarsonPrecalculus.com). (Source: Wal-Mart Stores, Inc.)