7.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. The result of writing a rational expression as the sum of two or more simpler rational expressions is called the _____ _____ _____.
2. If the degree of the numerator of a rational expression is greater than or equal to the degree of the denominator, then the fraction is _____.
3. Each fraction on the right side of the equation $\frac{x-1}{x^2-8x+15} = \frac{-1}{x-3} + \frac{2}{x-5}$ is a _____ _____.
4. You obtain the _____ _____ by multiplying each side of the partial fraction decomposition form by the least common denominator.

Skills and Applications

Matching In Exercises 5–8, match the rational expression with the form of its decomposition. [The decompositions are labeled (a), (b), (c), and (d).]

(a) $\frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$ (b) $\frac{A}{x} + \frac{B}{x-4}$

(c) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-4}$ (d) $\frac{A}{x} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$

5. $\frac{3x-1}{x(x-4)}$
6. $\frac{3x-1}{x^2(x-4)}$
7. $\frac{3x-1}{x(x-4)^2}$
8. $\frac{3x-1}{x(x^2-4)}$

Writing the Form of the Decomposition In Exercises 9–16, write the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.

9. $\frac{3}{x^2-2x}$
10. $\frac{x-2}{x^2+4x+3}$
11. $\frac{6x+5}{(x+2)^4}$
12. $\frac{5x^2+3}{x^2(x-4)^2}$
13. $\frac{2x-3}{x^3+10x}$
14. $\frac{x-1}{x(x^2+1)^2}$
15. $\frac{8x}{x^2(x^2+3)^2}$
16. $\frac{x^2-9}{x^3(x^2+2)^2}$

Writing the Partial Fraction Decomposition In Exercises 17–42, write the partial fraction decomposition of the rational expression. Check your result algebraically.

17. $\frac{1}{x^2+x}$
18. $\frac{3}{x^2-3x}$
19. $\frac{3}{x^2+x-2}$
20. $\frac{x+1}{x^2-x-6}$
21. $\frac{1}{x^2-1}$
22. $\frac{1}{4x^2-9}$
23. $\frac{x^2+12x+12}{x^3-4x}$
24. $\frac{x+2}{x(x^2-9)}$
25. $\frac{3x}{(x-3)^2}$
26. $\frac{2x-3}{(x-1)^2}$
27. $\frac{4x^2+2x-1}{x^2(x+1)}$
28. $\frac{6x^2+1}{x^2(x-1)^2}$
29. $\frac{x^2+2x+3}{x^3+x}$
30. $\frac{2x}{x^3-1}$
31. $\frac{x}{x^3-x^2-2x+2}$
32. $\frac{x+6}{x^3-3x^2-4x+12}$
33. $\frac{x}{16x^4-1}$
34. $\frac{3}{x^4+x}$
35. $\frac{x^2+5}{(x+1)(x^2-2x+3)}$
36. $\frac{x^2-4x+7}{(x+1)(x^2-2x+3)}$
37. $\frac{2x^2+x+8}{(x^2+4)^2}$
38. $\frac{3x^2+1}{(x^2+2)^2}$
39. $\frac{5x^2-2}{(x^2+3)^3}$
40. $\frac{x^2-4x+6}{(x^2+4)^3}$
41. $\frac{8x-12}{x^2(x^2+2)^2}$
42. $\frac{x+1}{x^3(x^2+1)^2}$

Improper Rational Expression Decomposition In Exercises 43–50, write the partial fraction decomposition of the improper rational expression.

43. $\frac{x^2-x}{x^2+x+1}$
44. $\frac{x^2-4x}{x^2+x+6}$
45. $\frac{2x^3-x^2+x+5}{x^2+3x+2}$
46. $\frac{x^3+2x^2-x+1}{x^2+3x-4}$
47. $\frac{x^4}{(x-1)^3}$
48. $\frac{16x^4}{(2x-1)^3}$
49. $\frac{x^4+2x^3+4x^2+8x+2}{x^3+2x^2+x}$
50. $\frac{2x^4+8x^3+7x^2-7x-12}{x^3+4x^2+4x}$

Writing the Partial Fraction Decomposition In Exercises 51–58, write the partial fraction decomposition of the rational expression. Use a graphing utility to check your result.

51. $\frac{5-x}{2x^2+x-1}$
52. $\frac{4x^2-1}{2x(x+1)^2}$
53. $\frac{3x^2-7x-2}{x^3-x}$
54. $\frac{3x+6}{x^3+2x}$
55. $\frac{x^2+x+2}{(x^2+2)^2}$
56. $\frac{x^3}{(x+2)^2(x-2)^2}$
57. $\frac{2x^3-4x^2-15x+5}{x^2-2x-8}$
58. $\frac{x^3-x+3}{x^2+x-2}$

59. **Environmental Science** The predicted cost C (in thousands of dollars) for a company to remove $p\%$ of a chemical from its waste water is given by the model

$$C = \frac{120p}{10{,}000-p^2}, \quad 0 \le p < 100.$$

Write the partial fraction decomposition for the rational function. Verify your result by using a graphing utility to create a table comparing the original function with the partial fractions.

60. **Thermodynamics** The magnitude of the range R of exhaust temperatures (in degrees Fahrenheit) in an experimental diesel engine is approximated by the model

$$R = \frac{5000(4-3x)}{(11-7x)(7-4x)}, \quad 0 < x \le 1$$

where x is the relative load (in foot-pounds).

(a) Write the partial fraction decomposition of the equation.

(b) The decomposition in part (a) is the difference of two fractions. The absolute values of the terms give the expected maximum and minimum temperatures of the exhaust gases for different loads.

Ymax = |1st term| Ymin = |2nd term|

Write the equations for Ymax and Ymin.

(c) Use a graphing utility to graph each equation from part (b) in the same viewing window.

(d) Determine the expected maximum and minimum temperatures for a relative load of 0.5.

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Exploration

True or False? In Exercises 61–63, determine whether the statement is true or false. Justify your answer.

61. For the rational expression $\frac{x}{(x+10)(x-10)^2}$, the partial fraction decomposition is of the form

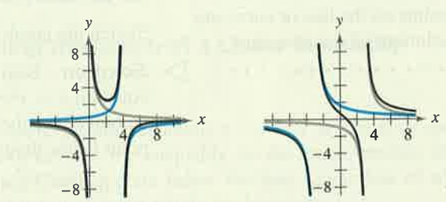
$$\frac{A}{x+10} + \frac{B}{(x-10)^2}.$$

62. When writing the partial fraction decomposition of the expression $\frac{x^3+x-2}{x^2-5x-14}$, the first step is to divide the numerator by the denominator.

63. In the partial fraction decomposition of a rational expression, the denominators of each partial fraction always have a lower degree than the denominator of the original expression.

64. **HOW DO YOU SEE IT?** Identify the graph of the rational function and the graph representing each partial fraction of its partial fraction decomposition. Then state any relationship between the vertical asymptotes of the graph of the rational function and the vertical asymptotes of the graphs representing the partial fractions of the decomposition. To print an enlarged copy of the graph, go to *MathGraphs.com*.

(a) $y = \frac{x-12}{x(x-4)} = \frac{3}{x} - \frac{2}{x-4}$ (b) $y = \frac{2(4x-3)}{x^2-9} = \frac{3}{x-3} + \frac{5}{x+3}$



65. **Error Analysis** Describe the error in writing the basic equation for the partial fraction decomposition of the rational expression.

$$\frac{x^2+1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$x^2+1 = A(x-1) + Bx$$

66. **Writing** Describe two ways of solving for the constants in a partial fraction decomposition.