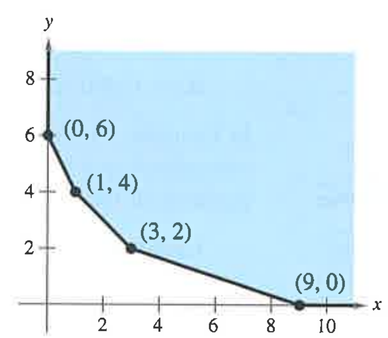
EXAMPLE 9 Nutrition

The liquid portion of a diet is to provide at least 300 calories, 36 units of vitamin A, and 90 units of vitamin C. A cup of dietary drink X provides 60 calories, 12 units of vitamin A, and 10 units of vitamin C. A cup of dietary drink Y provides 60 calories, 6 units of vitamin A, and 30 units of vitamin C. Write a system of linear inequalities that describes how many cups of each drink must be consumed each day to meet or exceed the minimum daily requirements for calories and vitamins.

Solution Begin by letting x represent the number of cups of dietary drink X and y represent the number of cups of dietary drink Y. To meet or exceed the minimum daily requirements, the following inequalities must be satisfied.

$$\begin{cases} 60x + 60y \geq 300 & \text{Calories} \\ 12x + 6y \geq 36 & \text{Vitamin A} \\ 10x + 30y \geq 90 & \text{Vitamin C} \\ x \geq 0 & \\ y \geq 0 & \end{cases}$$

The last two inequalities are included because x and y cannot be negative. The graph of this system of inequalities is shown below. (More is said about this application in Example 6 in Section 7.6.)



✓ Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

A public aquarium is adding coral nutrients to a large reef tank. A bottle of brand X nutrients contains 8 units of nutrient A, 1 unit of nutrient B, and 2 units of nutrient C. A bottle of brand Y nutrients contains 2 units of nutrient A, 1 unit of nutrient B, and 7 units of nutrient C. The minimum amounts of nutrients A, B, and C that need to be added to the tank are 16 units, 5 units, and 20 units, respectively. Set up a system of linear inequalities that describes how many bottles of each brand must be added to meet or exceed the needs.

Summarize (Section 7.5)

1. Explain how to sketch the graph of an inequality in two variables *(page 510)*. For examples of sketching the graphs of inequalities in two variables, see Examples 1–3.
2. Explain how to solve a system of inequalities *(page 512)*. For examples of solving systems of inequalities, see Examples 4–7.
3. Describe examples of how to use systems of inequalities in two variables to model and solve real-life problems *(pages 515 and 516, Examples 8 and 9)*.

7.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. An ordered pair (a, b) is a ________ of an inequality in x and y when the inequality is true after a and b are substituted for x and y, respectively.
2. The ________ of an inequality is the collection of all solutions of the inequality.
3. A ________ of a system of inequalities in x and y is a point (x, y) that satisfies each inequality in the system.
4. The ________ ________ of a system of inequalities in two variables is represented by the region that is common to every graph in the system.

Skills and Applications

Graphing an Inequality In Exercises 5–18, sketch the graph of the inequality.

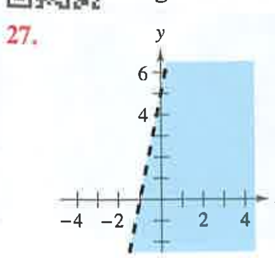
5. $y < 5 - x^2$
6. $y^2 - x < 0$
7. $x \geq 6$
8. $x < -4$
9. $y > -7$
10. $10 \geq y$
11. $y < 2 - x$
12. $y > 4x - 3$
13. $2y - x \geq 4$
14. $5x + 3y \geq -15$
15. $x^2 + (y - 3)^2 < 4$
16. $(x + 2)^2 + y^2 > 9$
17. $y > -\dfrac{2}{x^2 + 1}$
18. $y \leq \dfrac{3}{x^2 + x + 1}$

Graphing an Inequality In Exercises 19–26, use a graphing utility to graph the inequality.

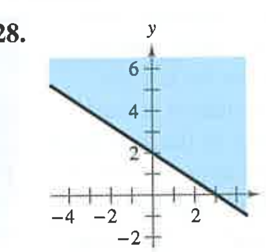
19. $y \geq -\ln(x - 1)$
20. $y < \ln(x + 3) - 1$
21. $y < 2^x$
22. $y \geq 3^{-x} - 2$
23. $y \leq 2 - \frac{1}{5}x$
24. $y > -2.4x + 3.3$
25. $\frac{2}{3}y + 2x^2 - 5 \geq 0$
26. $-\frac{1}{6}x^2 - \frac{2}{7}y < -\frac{1}{3}$

Writing an Inequality In Exercises 27–30, write an inequality for the shaded region shown in the figure.

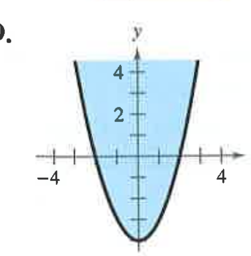
27.



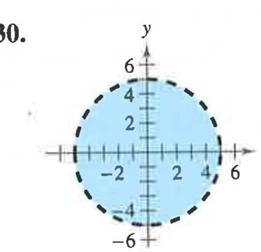
28.



29.



30.



Solving a System of Inequalities In Exercises 31–38, sketch the graph of the solution set of the system of inequalities. Label the vertices of the region.

31. $\begin{cases} x + y \leq 1 \\ -x + y \leq 1 \\ y \geq 0 \end{cases}$
32. $\begin{cases} 3x + 4y < 12 \\ x > 0 \\ y > 0 \end{cases}$
33. $\begin{cases} -3x + 2y < 6 \\ x - 4y > -2 \\ 2x + y < 3 \end{cases}$
34. $\begin{cases} x - 7y > -36 \\ 5x + 2y > 5 \\ 6x - 5y > 6 \end{cases}$
35. $\begin{cases} 2x + y > 2 \\ 6x + 3y < 2 \end{cases}$
36. $\begin{cases} x - 2y < -6 \\ 5x - 3y > -9 \end{cases}$
37. $\begin{cases} 2x - 3y > 7 \\ 5x + y < 9 \end{cases}$
38. $\begin{cases} 4x - 6y > 2 \\ -2x + 3y \geq 5 \end{cases}$

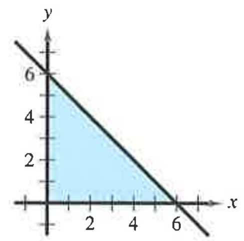
Solving a System of Inequalities In Exercises 39–44, sketch the graph of the solution set of the system.

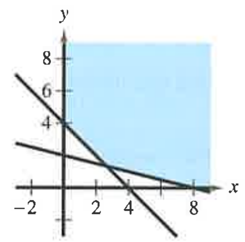
39. $\begin{cases} x^2 + y \leq 7 \\ x \geq -2 \\ y \geq 0 \end{cases}$
40. $\begin{cases} 4x^2 + y \geq 2 \\ x \leq 1 \\ y \leq 1 \end{cases}$
41. $\begin{cases} x - y^2 > 0 \\ x - y > 2 \end{cases}$
42. $\begin{cases} x^2 + y^2 \leq 25 \\ 4x - 3y \leq 0 \end{cases}$
43. $\begin{cases} 3x + 4 \geq y^2 \\ x - y < 0 \end{cases}$
44. $\begin{cases} x < 2y - y^2 \\ 0 < x + y \end{cases}$

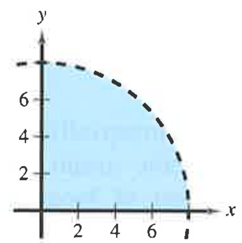
Solving a System of Inequalities In Exercises 45–50, use a graphing utility to graph the solution set of the system of inequalities.

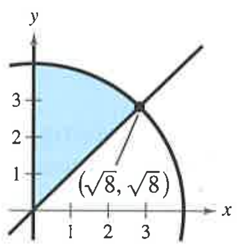
45. $\begin{cases} y \leq \sqrt{3x} + 1 \\ y \geq x^2 + 1 \end{cases}$
46. $\begin{cases} y < 2\sqrt{x} - 1 \\ y \geq x^2 - 1 \end{cases}$
47. $\begin{cases} y < -x^2 + 2x + 3 \\ y > x^2 - 4x + 3 \end{cases}$
48. $\begin{cases} y \geq x^4 - 2x^2 + 1 \\ y \leq 1 - x^2 \end{cases}$
49. $\begin{cases} x^2y \geq 1 \\ 0 < x \leq 4 \\ y \leq 4 \end{cases}$
50. $\begin{cases} y \leq e^{-x^2/2} \\ y \geq 0 \\ -2 \leq x \leq 2 \end{cases}$

Writing a System of Inequalities **In Exercises 51–58, write a system of inequalities that describes the region.**

51. 

52. 

53. 

54. 

55. Rectangle: vertices at (4, 3), (9, 3), (9, 9), (4, 9)

56. Parallelogram: vertices at (0, 0), (4, 0), (1, 4), (5, 4)

57. Triangle: vertices at (0, 0), (6, 0), (1, 5)

58. Triangle: vertices at (−1, 0), (1, 0), (0, 1)

Consumer Surplus and Producer Surplus **In Exercises 59–62, (a) graph the systems of inequalities representing the consumer surplus and producer surplus for the supply and demand equations and (b) find the consumer surplus and producer surplus.**

	Demand	Supply
59.	$p = 50 - 0.5x$	$p = 0.125x$
60.	$p = 100 - 0.05x$	$p = 25 + 0.1x$
61.	$p = 140 - 0.00002x$	$p = 80 + 0.00001x$
62.	$p = 400 - 0.0002x$	$p = 225 + 0.0005x$

63. **Investment Analysis** A person plans to invest up to \$20,000 in two different interest-bearing accounts. Each account must contain at least \$5000. The amount in one account is to be at least twice the amount in the other account. Write and graph a system of inequalities that describes the various amounts that can be deposited in each account.

64. **Ticket Sales** For a concert event, there are \$30 reserved seat tickets and \$20 general admission tickets. There are 2000 reserved seats available, and fire regulations limit the number of paid ticket holders to 3000. The promoter must take in at least \$75,000 in ticket sales. Write and graph a system of inequalities that describes the different numbers of tickets that can be sold.

65. **Production** A furniture company produces tables and chairs. Each table requires 1 hour in the assembly center and $1\frac{1}{3}$ hours in the finishing center. Each chair requires $1\frac{1}{2}$ hours in the assembly center and $1\frac{1}{2}$ hours in the finishing center. The assembly center is available 12 hours per day, and the finishing center is available 15 hours per day. Write and graph a system of inequalities that describes all possible production levels.

66. **Inventory** A store sells two models of laptop computers. The store stocks at least twice as many units of model A as of model B. The costs to the store for the two models are \$800 and \$1200, respectively. The management does not want more than \$20,000 in computer inventory at any one time, and it wants at least four model A laptop computers and two model B laptop computers in inventory at all times. Write and graph a system of inequalities that describes all possible inventory levels.

67. **Nutrition** A dietician prescribes a special dietary plan using two different foods. Each ounce of food X contains 180 milligrams of calcium, 6 milligrams of iron, and 220 milligrams of magnesium. Each ounce of food Y contains 100 milligrams of calcium, 1 milligram of iron, and 40 milligrams of magnesium. The minimum daily requirements of the diet are 1000 milligrams of calcium, 18 milligrams of iron, and 400 milligrams of magnesium.
   (a) Write and graph a system of inequalities that describes the different amounts of food X and food Y that can be prescribed.
   (b) Find two solutions of the system and interpret their meanings in the context of the problem.

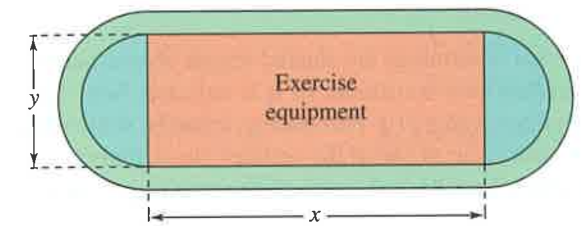
68. **Target Heart Rate** One formula for a person's maximum heart rate is $220 - x$, where x is the person's age in years for $20 \le x \le 70$. The American Heart Association recommends that when a person exercises, the person should strive for a heart rate that is at least 50% of the maximum and at most 85% of the maximum. *(Source: American Heart Association)*

   (a) Write and graph a system of inequalities that describes the exercise target heart rate region.
   (b) Find two solutions of the system and interpret their meanings in the context of the problem.

BLACKDAY/Shutterstock.com

69. **Shipping** A warehouse supervisor has instructions to ship at least 50 bags of gravel that weigh 55 pounds each and at least 40 bags of stone that weigh 70 pounds each. The maximum weight capacity of the truck being used is 7500 pounds.
   (a) Write and graph a system that describes the numbers of bags of stone and gravel that can be shipped.
   (b) Find two solutions of the system and interpret their meanings in the context of the problem.

70. **Physical Fitness Facility** A physical fitness facility is constructing an indoor running track with space for exercise equipment inside the track (see figure). The track must be at least 125 meters long, and the exercise space must have an area of at least 500 square meters.



   (a) Write and graph a system of inequalities that describes the requirements of the facility.
   (b) Find two solutions of the system and interpret their meanings in the context of the problem.

Exploration

True or False? **In Exercises 71 and 72, determine whether the statement is true or false. Justify your answer.**

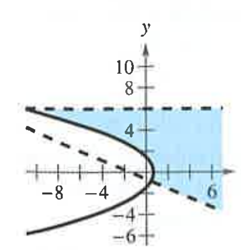
71. The area of the figure described by the system

$$\begin{cases} x \ge -3 \\ x \le 6 \\ y \le 5 \\ y \ge -6 \end{cases}$$

is 99 square units.

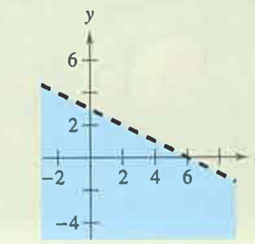
72. The graph shows the solution of the system

$$\begin{cases} y \le 6 \\ -4x - 9y > 6. \\ 3x + y^2 \ge 2 \end{cases}$$

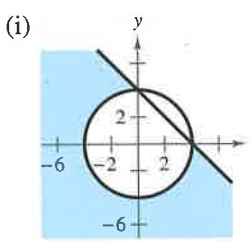


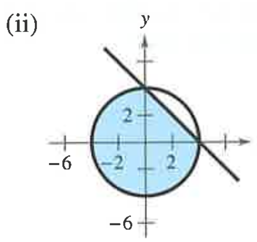
73. **Think About It** After graphing the boundary line of the inequality $x + y < 3$, explain how to determine the region that you need to shade.

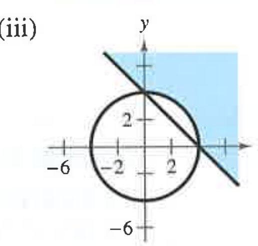
74. **HOW DO YOU SEE IT?** The graph of the solution of the inequality $x + 2y < 6$ is shown in the figure. Describe how the solution set would change for each inequality.
   (a) $x + 2y \le 6$
   (b) $x + 2y > 6$

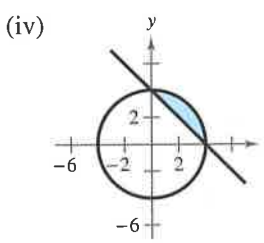


75. **Matching** Match the system of inequalities with the graph of its solution. [The graphs are labeled (i), (ii), (iii), and (iv).]

(i) 

(ii) 

(iii) 

(iv) 

(a) $\begin{cases} x^2 + y^2 \le 16 \\ x + y \ge 4 \end{cases}$ (b) $\begin{cases} x^2 + y^2 \le 16 \\ x + y \le 4 \end{cases}$

(c) $\begin{cases} x^2 + y^2 \ge 16 \\ x + y \ge 4 \end{cases}$ (d) $\begin{cases} x^2 + y^2 \ge 16 \\ x + y \le 4 \end{cases}$

76. **Graphical Reasoning** Two concentric circles have radii x and y, where $y > x$. The area between the circles is at least 10 square units.
   (a) Write a system of inequalities that describes the constraints on the circles.
   (b) Use a graphing utility to graph the system of inequalities in part (a). Graph the line $y = x$ in the same viewing window.
   (c) Identify the graph of the line in relation to the boundary of the inequality. Explain its meaning in the context of the problem.