EXAMPLE 9 **A System with an Infinite Number of Solutions**

Solve the system $\begin{cases} 2x + 4y - 2z = 0 \\ 3x + 5y = 1 \end{cases}$.

Solution

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

$$\tfrac{1}{2}R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

$$-3R_1 + R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 3 & 1 \end{array}\right]$$

$$-R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -3 & -1 \end{array}\right]$$

$$-2R_2 + R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & -3 & -1 \end{array}\right]$$

The corresponding system of equations is

$$\begin{cases} x + 5z = 2 \\ y - 3z = -1 \end{cases}.$$

Solving for x and y in terms of z, you have

$$x = -5z + 2 \quad \text{and} \quad y = 3z - 1.$$

To write a solution of the system that does not use any of the three variables of the system, let a represent any real number and let $z = a$. Substitute a for z in the equations for x and y.

$$x = -5z + 2 = -5a + 2 \quad \text{and} \quad y = 3z - 1 = 3a - 1$$

So, the solution set can be written as an ordered triple of the form

$$(-5a + 2, 3a - 1, a)$$

where a is any real number. Remember that a solution set of this form represents an infinite number of solutions. Substitute values for a to obtain a few solutions. Then check each solution in the original system of equations.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve the system $\begin{cases} 2x - 6y + 6z = 46 \\ 2x - 3y = 31 \end{cases}$.

Summarize (Section 8.1)

1. State the definition of a matrix *(page 540)*. For examples of writing matrices and determining their dimensions, see Examples 1 and 2.
2. List the elementary row operations *(page 542)*. For an example of performing elementary row operations, see Example 3.
3. Explain how to use matrices and Gaussian elimination to solve systems of linear equations *(page 543)*. For examples of using Gaussian elimination, see Examples 4, 6, and 7.
4. Explain how to use matrices and Gauss-Jordan elimination to solve systems of linear equations *(page 547)*. For examples of using Gauss-Jordan elimination, see Examples 8 and 9.

8.1 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A matrix is ________ when the number of rows equals the number of columns.
2. For a square matrix, the entries $a_{11}, a_{22}, a_{33}, \ldots$ are the ________ ________ entries.
3. A matrix derived from a system of linear equations (each written in standard form with the constant term on the right) is the ________ matrix of the system.
4. A matrix derived from the coefficients of a system of linear equations (but not including the constant terms) is the ________ matrix of the system.
5. Two matrices are ________ when one can be obtained from the other by a sequence of elementary row operations.
6. A matrix in row-echelon form is in ________ ________ ________ when every column that has a leading 1 has zeros in every position above and below its leading 1.

Skills and Applications

Dimension of a Matrix **In Exercises 7–14, determine the dimension of the matrix.**

7. $\begin{bmatrix} 7 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 5 & -3 & 8 & 7 \end{bmatrix}$

9. $\begin{bmatrix} 2 \\ 36 \\ 3 \end{bmatrix}$

10. $\begin{bmatrix} -3 & 7 & 15 & 0 \\ 0 & 0 & 3 & 3 \\ 1 & 1 & 6 & 7 \end{bmatrix}$

11. $\begin{bmatrix} 33 & 45 \\ -9 & 20 \end{bmatrix}$

12. $\begin{bmatrix} -7 & 6 & 4 \\ 0 & -5 & 1 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 6 & -1 \\ 8 & 0 & 3 \\ 3 & -9 & 9 \end{bmatrix}$

14. $\begin{bmatrix} 3 & -1 \\ 4 & 1 \\ -5 & 9 \end{bmatrix}$

Writing an Augmented Matrix **In Exercises 15–20, write the augmented matrix for the system of linear equations.**

15. $\begin{cases} 2x - y = 7 \\ x + y = 2 \end{cases}$

16. $\begin{cases} 5x + 2y = 13 \\ -3x + 4y = -24 \end{cases}$

17. $\begin{cases} x - y + 2z = 2 \\ 4x - 3y + z = -1 \\ 2x + y = 0 \end{cases}$

18. $\begin{cases} -2x - 4y + z = 13 \\ 6x - 7z = 22 \\ 3x - y + z = 9 \end{cases}$

19. $\begin{cases} 3x - 5y + 2z = 12 \\ 12x - 7z = 10 \end{cases}$

20. $\begin{cases} 9x + y - 3z = 21 \\ -15y + 13z = -8 \end{cases}$

Writing a System of Equations **In Exercises 21–26, write the system of linear equations represented by the augmented matrix. (Use variables x, y, z, and w, if applicable.)**

21. $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 5 & -3 & -1 \end{array}\right]$

22. $\left[\begin{array}{cc|c} 5 & 2 & 9 \\ 3 & -8 & 0 \end{array}\right]$

23. $\left[\begin{array}{ccc|c} 2 & 0 & 5 & -12 \\ 0 & 1 & -2 & 7 \\ 6 & 3 & 0 & 2 \end{array}\right]$

24. $\left[\begin{array}{ccc|c} 4 & -5 & -1 & 18 \\ -11 & 0 & 6 & 25 \\ 3 & 8 & 0 & -29 \end{array}\right]$

25. $\left[\begin{array}{cccc|c} 9 & 12 & 3 & 0 & 0 \\ -2 & 18 & 5 & 2 & 10 \\ 1 & 7 & -8 & 0 & -4 \\ 3 & 0 & 2 & 0 & -10 \end{array}\right]$

26. $\left[\begin{array}{cccc|c} 6 & 2 & -1 & -5 & -25 \\ -1 & 0 & 7 & 3 & 7 \\ 4 & -1 & -10 & 6 & 23 \\ 0 & 8 & 1 & -11 & -21 \end{array}\right]$

Identifying an Elementary Row Operation **In Exercises 27–30, identify the elementary row operation(s) performed to obtain the new row-equivalent matrix.**

	Original Matrix	New Row-Equivalent Matrix
27.	$\begin{bmatrix} -2 & 5 & 1 \\ 3 & -1 & -8 \end{bmatrix}$	$\begin{bmatrix} 13 & 0 & -39 \\ 3 & -1 & -8 \end{bmatrix}$
28.	$\begin{bmatrix} 3 & -1 & -4 \\ -4 & 3 & 7 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 & -4 \\ 5 & 0 & -5 \end{bmatrix}$
29.	$\begin{bmatrix} 0 & -1 & -5 & 5 \\ -1 & 3 & -7 & 6 \\ 4 & -5 & 1 & 3 \end{bmatrix}$	$\begin{bmatrix} -1 & 3 & -7 & 6 \\ 0 & -1 & -5 & 5 \\ 0 & 7 & -27 & 27 \end{bmatrix}$
30.	$\begin{bmatrix} -1 & -2 & 3 & -2 \\ 2 & -5 & 1 & -7 \\ 5 & 4 & -7 & 6 \end{bmatrix}$	$\begin{bmatrix} -1 & -2 & 3 & -2 \\ 0 & -9 & 7 & -11 \\ 0 & -6 & 8 & -4 \end{bmatrix}$

Elementary Row Operations In Exercises 31–38, fill in the blank(s) using elementary row operations to form a row-equivalent matrix.

31. $\begin{bmatrix} 3 & 6 & 8 \\ 4 & -3 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & \square & \frac{8}{3} \\ 4 & -3 & 6 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 1 & 4 & 3 \\ 0 & \square & -1 \end{bmatrix}$

33. $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -2 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & \square & -1 \end{bmatrix}$

34. $\begin{bmatrix} -3 & 3 & 12 \\ 18 & -8 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & \square \\ 18 & -8 & 4 \end{bmatrix}$

35. $\begin{bmatrix} 1 & 5 & 4 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & \square & \square \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

36. $\begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & -1 & 0 & 7 \\ 0 & 0 & -1 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 0 & \square \\ 0 & 0 & 1 & \square \end{bmatrix}$

37. $\begin{bmatrix} 1 & 1 & 4 & -1 \\ 3 & 8 & 10 & 3 \\ -2 & 1 & 12 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 5 & \square & \square \\ 0 & 3 & \square & \square \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 1 & -\frac{2}{5} & \frac{6}{5} \\ 0 & 3 & \square & \square \end{bmatrix}$

38. $\begin{bmatrix} 2 & 4 & 8 & 3 \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{bmatrix} 1 & \square & \square & \square \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 4 & \frac{3}{2} \\ 0 & \square & -7 & \frac{1}{2} \\ 0 & 2 & \square & \square \end{bmatrix}$

Comparing Linear Systems and Matrix Operations In Exercises 39 and 40, (a) perform the row operations to solve the augmented matrix, (b) write and solve the system of linear equations (in variables x, y, and z, if applicable) represented by the augmented matrix, and (c) compare the two solution methods. Which do you prefer?

39. $\left[\begin{array}{cc|c} -3 & 4 & 22 \\ 6 & -4 & -28 \end{array}\right]$

(i) Add R_2 to R_1.
(ii) Add -2 times R_1 to R_2.
(iii) Multiply R_2 by $-\frac{1}{4}$.
(iv) Multiply R_1 by $\frac{1}{3}$.

40. $\left[\begin{array}{ccc|c} 7 & 13 & 1 & -4 \\ -3 & -5 & -1 & -4 \\ 3 & 6 & 1 & -2 \end{array}\right]$

(i) Add R_2 to R_1.
(ii) Multiply R_1 by $\frac{1}{4}$.
(iii) Add R_3 to R_2.
(iv) Add -3 times R_1 to R_3.
(v) Add -2 times R_2 to R_1.

Row-Echelon Form In Exercises 41–44, determine whether the matrix is in row-echelon form. If it is, determine whether it is in reduced row-echelon form.

41. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

42. $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

43. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

44. $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Writing a Matrix in Row-Echelon Form In Exercises 45–48, write the matrix in row-echelon form. (Remember that the row-echelon form of a matrix is not unique.)

45. $\begin{bmatrix} 1 & 1 & 0 & 5 \\ -2 & -1 & 2 & -10 \\ 3 & 6 & 7 & 14 \end{bmatrix}$

46. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 7 & -5 & 14 \\ -2 & -1 & -3 & 8 \end{bmatrix}$

47. $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 5 & -4 & 1 & 8 \\ -6 & 8 & 18 & 0 \end{bmatrix}$

48. $\begin{bmatrix} 1 & -3 & 0 & -7 \\ -3 & 10 & 1 & 23 \\ 4 & -10 & 2 & -24 \end{bmatrix}$

Using a Graphing Utility In Exercises 49–54, use the matrix capabilities of a graphing utility to write the matrix in reduced row-echelon form.

49. $\begin{bmatrix} -1 & 2 & 1 \\ 3 & 4 & 9 \\ 2 & 1 & -2 \end{bmatrix}$

50. $\begin{bmatrix} 1 & 3 & 2 \\ 5 & 15 & 9 \\ 2 & 6 & 10 \end{bmatrix}$

51. $\begin{bmatrix} 1 & 2 & 3 & -5 \\ 1 & 2 & 4 & -9 \\ -2 & -4 & -4 & 3 \\ 4 & 8 & 11 & -14 \end{bmatrix}$

52. $\begin{bmatrix} -2 & 3 & -1 & -2 \\ 4 & -2 & 5 & 8 \\ 1 & 5 & -2 & 0 \\ 3 & 8 & -10 & -30 \end{bmatrix}$

53. $\begin{bmatrix} -3 & 5 & 1 & 12 \\ 1 & -1 & 1 & 4 \end{bmatrix}$

54. $\begin{bmatrix} 5 & 1 & 2 & 4 \\ -1 & 5 & 10 & -32 \end{bmatrix}$

Using Back-Substitution In Exercises 55–58, write the system of linear equations represented by the augmented matrix. Then use back-substitution to solve the system. (Use variables x, y, and z, if applicable.)

55. $\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -1 \end{array}\right]$

56. $\left[\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 1 & 6 \end{array}\right]$

57. $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$

58. $\left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & -3 \end{array}\right]$

Gaussian Elimination with Back-Substitution In Exercises 59–68, use matrices to solve the system of linear equations, if possible. Use Gaussian elimination with back-substitution.

59. $\begin{cases} x + 2y = 7 \\ -x + y = 8 \end{cases}$

60. $\begin{cases} 2x + 6y = 16 \\ 2x + 3y = 7 \end{cases}$

61. $\begin{cases} 3x - 2y = -27 \\ x + 3y = 13 \end{cases}$

62. $\begin{cases} -x + y = 4 \\ 2x - 4y = -34 \end{cases}$

63. $\begin{cases} x + 2y - 3z = -28 \\ 4y + 2z = 0 \\ -x + y - z = -5 \end{cases}$

64. $\begin{cases} 3x - 2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$

65. $\begin{cases} -3x + 2y = -22 \\ 3x + 4y = 4 \\ 4x - 8y = 32 \end{cases}$

66. $\begin{cases} x + 2y = 0 \\ x + y = 6 \\ 3x - 2y = 8 \end{cases}$

67. $\begin{cases} 3x + 2y - z + w = 0 \\ x - y + 4z + 2w = 25 \\ -2x + y + 2z - w = 2 \\ x + y + z + w = 6 \end{cases}$

68. $\begin{cases} x - 4y + 3z - 2w = 9 \\ 3x - 2y + z - 4w = -13 \\ -4x + 3y - 2z + w = -4 \\ -2x + y - 4z + 3w = -10 \end{cases}$

Interpreting Reduced Row-Echelon Form In Exercises 69 and 70, an augmented matrix that represents a system of linear equations (in variables x, y, and z, if applicable) has been reduced using Gauss-Jordan elimination. Write the solution represented by the augmented matrix.

69. $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -4 \end{array}\right]$

70. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right]$

Gauss-Jordan Elimination In Exercises 71–78, use matrices to solve the system of linear equations, if possible. Use Gauss-Jordan elimination.

71. $\begin{cases} -2x + 6y = -22 \\ x + 2y = -9 \end{cases}$

72. $\begin{cases} 5x - 5y = -5 \\ -2x - 3y = 7 \end{cases}$

73. $\begin{cases} x + 2y + z = 8 \\ 3x + 7y + 6z = 26 \end{cases}$

74. $\begin{cases} x + y + 4z = 5 \\ 2x + y - z = 9 \end{cases}$

75. $\begin{cases} x - 3z = -2 \\ 3x + y - 2z = 5 \\ 2x + 2y + z = 4 \end{cases}$

76. $\begin{cases} 2x - y + 3z = 24 \\ 2y - z = 14 \\ 7x - 5y = 6 \end{cases}$

77. $\begin{cases} -x + y - z = -14 \\ 2x - y + z = 21 \\ 3x + 2y + z = 19 \end{cases}$

78. $\begin{cases} 2x + 2y - z = 2 \\ x - 3y + z = -28 \\ -x + y = 14 \end{cases}$

Using a Graphing Utility In Exercises 79–84, use the matrix capabilities of a graphing utility to write the augmented matrix corresponding to the system of linear equations in reduced row-echelon form. Then solve the system.

79. $\begin{cases} 3x + 3y + 12z = 6 \\ x + y + 4z = 2 \\ 2x + 5y + 20z = 10 \\ -x + 2y + 8z = 4 \end{cases}$

80. $\begin{cases} 2x + 10y + 2z = 6 \\ x + 5y + 2z = 6 \\ x + 5y + z = 3 \\ -3x - 15y - 3z = -9 \end{cases}$

81. $\begin{cases} 2x + y - z + 2w = -6 \\ 3x + 4y + w = 1 \\ x + 5y + 2z + 6w = -3 \\ 5x + 2y - z - w = 3 \end{cases}$

82. $\begin{cases} x + 2y + 2z + 4w = 11 \\ 3x + 6y + 5z + 12w = 30 \\ x + 3y - 3z + 2w = -5 \\ 6x - y - z + w = -9 \end{cases}$

83. $\begin{cases} x + y + z + w = 0 \\ 2x + 3y + z - 2w = 0 \\ 3x + 5y + z = 0 \end{cases}$

84. $\begin{cases} x + 2y + z + 3w = 0 \\ x - y + w = 0 \\ y - z + 2w = 0 \end{cases}$

85. **Error Analysis** Describe the error.

The matrix

$\begin{bmatrix} 3 \\ 0 \\ 8 \\ -1 \end{bmatrix}$

has four rows and one column, so the dimension of the matrix is 1×4.

86. **Error Analysis** Describe the error.

The matrix

$\begin{bmatrix} 1 & 2 & 7 & 2 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$

is in reduced row-echelon form.

Curve Fitting In Exercises 87–92, use a system of linear equations to find the quadratic function $f(x) = ax^2 + bx + c$ that satisfies the given conditions. Solve the system using matrices.

87. $f(1) = 1, f(2) = -1, f(3) = -5$

88. $f(1) = 2, f(2) = 9, f(3) = 20$

89. $f(-2) = -15, f(-1) = 7, f(1) = -3$

90. $f(-2) = -3, f(1) = -3, f(2) = -11$

91. $f(1) = 8, f(2) = 13, f(3) = 20$

92. $f(1) = 9, f(2) = 8, f(3) = 5$

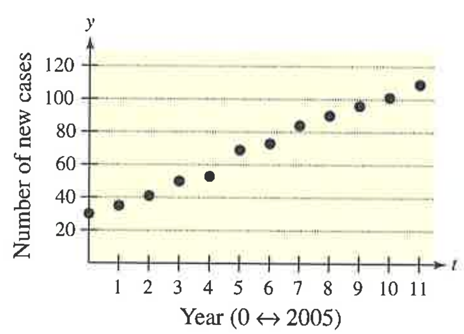
93. Waterborne Disease From 2005 through 2016, the numbers of new cases of a waterborne disease in a small city increased in a pattern that was approximately linear (see figure). Find the least squares regression line

$$y = at + b$$

for the data shown in the figure by solving the system below using matrices. Let t represent the year, with $t = 0$ corresponding to 2005.

$$\begin{cases} 12b + 66a = 831 \\ 66b + 506a = 5643 \end{cases}$$

Use the result to predict the number of new cases of the waterborne disease in 2020. Is the estimate reasonable? Explain.



94. Museum A natural history museum borrows \$2,000,000 at simple annual interest to purchase new exhibits. Some of the money is borrowed at 7%, some at 8.5%, and some at 9.5%. Use a system of linear equations to determine how much is borrowed at each rate given that the total annual interest is \$169,750 and the amount borrowed at 8.5% is four times the amount borrowed at 9.5%. Solve the system of linear equations using matrices.

95. Breeding Facility A city zoo borrows \$2,000,000 at simple annual interest to construct a breeding facility. Some of the money is borrowed at 8%, some at 9%, and some at 12%. Use a system of linear equations to determine how much is borrowed at each rate given that the total annual interest is \$186,000 and the amount borrowed at 8% is twice the amount borrowed at 12%. Solve the system of linear equations using matrices.

96. Mathematical Modeling A video of the path of a ball thrown by a baseball player was analyzed with a grid covering the TV screen. The video was paused three times, and the position of the ball was measured each time. The coordinates obtained are shown in the table. (x and y are measured in feet.)

Horizontal Distance, x	0	15	30
Height, y	5.0	9.6	12.4

(a) Use a system of equations to find the equation of the parabola $y = ax^2 + bx + c$ that passes through the three points. Solve the system using matrices.

(b) Use a graphing utility to graph the parabola.

(c) Graphically approximate the maximum height of the ball and the point at which the ball struck the ground.

(d) Analytically find the maximum height of the ball and the point at which the ball struck the ground.

(e) Compare your results from parts (c) and (d).

Exploration

True or False? In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.

97. $\begin{bmatrix} 5 & 0 & -2 & 7 \\ -1 & 3 & -6 & 0 \end{bmatrix}$ is a 4×2 matrix.

98. The method of Gaussian elimination reduces a matrix until a reduced row-echelon form is obtained.

99. Think About It What is the relationship between the three elementary row operations performed on an augmented matrix and the operations that lead to equivalent systems of equations?

100. HOW DO YOU SEE IT? Determine whether the matrix below is in row-echelon form, reduced row-echelon form, or neither when it satisfies the given conditions.

$$\begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix}$$

(a) $b = 0, c = 0$ (b) $b \neq 0, c = 0$

(c) $b = 0, c \neq 0$ (d) $b \neq 0, c \neq 0$

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8.2 Operations with Matrices

Matrix operations have many practical applications. For example, in Exercise 80 on page 567, you will use matrix multiplication to analyze the calories burned by individuals of different body weights while performing different types of exercises.

- Determine whether two matrices are equal.
- Add and subtract matrices, and multiply matrices by scalars.
- Multiply two matrices.
- Use matrices to transform vectors.
- Use matrix operations to model and solve real-life problems.

Equality of Matrices

In Section 8.1, you used matrices to solve systems of linear equations. There is a rich mathematical theory of matrices, and its applications are numerous. This section and the next two sections introduce some fundamental concepts of matrix theory. It is standard mathematical convention to represent matrices in any of the three ways listed below.

Representation of Matrices

1. A matrix can be denoted by an uppercase letter such as A, B, or C.
2. A matrix can be denoted by a representative element enclosed in brackets, such as $[a_{ij}]$, $[b_{ij}]$, or $[c_{ij}]$.
3. A matrix can be denoted by a rectangular array of numbers such as

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}.$$

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are **equal** when they have the same dimension ($m \times n$) and $a_{ij} = b_{ij}$ for $1 \le i \le m$ and $1 \le j \le n$. In other words, two matrices are equal when their corresponding entries are equal.

EXAMPLE 1 Equality of Matrices

Solve for a_{11}, a_{12}, a_{21}, and a_{22} in the matrix equation $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$.

Solution Two matrices are equal when their corresponding entries are equal, so $a_{11} = 2$, $a_{12} = -1$, $a_{21} = -3$, and $a_{22} = 0$.

✓ ***Checkpoint*** Audio-video solution in English & Spanish at LarsonPrecalculus.com

Solve for a_{11}, a_{12}, a_{21}, and a_{22} in the matrix equation $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -2 & 4 \end{bmatrix}$.

Be sure you see that for two matrices to be equal, they must have the same dimension *and* their corresponding entries must be equal. For example,

$$\begin{bmatrix} 2 & -1 \\ \sqrt{4} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 2 & 0.5 \end{bmatrix} \quad \text{but} \quad \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 0 \end{bmatrix} \neq \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}.$$

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