8.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. Two matrices are ________ when their corresponding entries are equal.
2. When performing matrix operations, real numbers are usually referred to as ________.
3. A matrix consisting entirely of zeros is called a ________ matrix and is denoted by ________.
4. The $n \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere is called the ________ matrix of dimension $n \times n$.

Skills and Applications

Equality of Matrices In Exercises 5–8, solve for x and y.

5. $\begin{bmatrix} x & -2 \\ 7 & 23 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ 7 & y \end{bmatrix}$

6. $\begin{bmatrix} -5 & x \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 12 & 8 \end{bmatrix}$

7. $\begin{bmatrix} 16 & 4 & x & 4 \\ 0 & 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 2x+1 & 4 \\ 0 & 2 & 3y-5 & 0 \end{bmatrix}$

8. $\begin{bmatrix} x+2 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & y+2 \end{bmatrix} = \begin{bmatrix} 2x+6 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & x \end{bmatrix}$

Operations with Matrices In Exercises 9–16, if possible, find (a) $A + B$, (b) $A - B$, (c) $3A$, and (d) $3A - 2B$.

9. $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix}$

11. $A = \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$

12. $A = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 6 & 2 \end{bmatrix}$

13. $A = \begin{bmatrix} 8 & -1 \\ 2 & 3 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 6 \\ -1 & -5 \\ 1 & 10 \end{bmatrix}$

14. $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & -5 \\ -3 & 4 & -7 \end{bmatrix}$

15. $A = \begin{bmatrix} 4 & 5 & -1 & 3 & 4 \\ 1 & 2 & -2 & -1 & 0 \end{bmatrix}$,
$B = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ -6 & 8 & 2 & -3 & -7 \end{bmatrix}$

16. $A = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$

Evaluating an Expression In Exercises 17–22, evaluate the expression.

17. $\begin{bmatrix} -5 & 0 \\ 3 & -6 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -10 & -8 \\ 14 & 6 \end{bmatrix}$

18. $\begin{bmatrix} 6 & 8 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -11 & -7 \\ 2 & -1 \end{bmatrix}$

19. $4\left(\begin{bmatrix} -4 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -2 \\ 3 & -6 & 0 \end{bmatrix}\right)$

20. $\frac{1}{2}(\begin{bmatrix} 5 & -2 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 6 & -18 & 9 \end{bmatrix})$

21. $-3\left(\begin{bmatrix} 0 & -3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 8 & 1 \end{bmatrix}\right) - 2\begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix}$

22. $-1\begin{bmatrix} 4 & 11 \\ -2 & -1 \\ 9 & 3 \end{bmatrix} + \frac{1}{6}\left(\begin{bmatrix} -5 & -1 \\ 3 & 4 \\ 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -9 & -1 \\ 6 & -1 \end{bmatrix}\right)$

Operations with Matrices In Exercises 23–26, use the matrix capabilities of a graphing utility to evaluate the expression.

23. $\frac{11}{25}\begin{bmatrix} 2 & 5 \\ -1 & -4 \end{bmatrix} + 6\begin{bmatrix} -3 & 0 \\ 2 & 2 \end{bmatrix}$

24. $55\left(\begin{bmatrix} 14 & -11 \\ -22 & 19 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 13 & 6 \end{bmatrix}\right)$

25. $-2\begin{bmatrix} 1.23 & 4.19 & -3.85 \\ 7.21 & -2.60 & 6.54 \end{bmatrix} - \begin{bmatrix} 8.35 & -3.02 & 7.30 \\ -0.38 & -5.49 & 1.68 \end{bmatrix}$

26. $-1\begin{bmatrix} 10 & 15 \\ -20 & 10 \\ 12 & 4 \end{bmatrix} + \frac{1}{8}\left(\begin{bmatrix} -13 & 11 \\ 7 & 0 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 13 \\ -3 & 8 \\ -14 & 15 \end{bmatrix}\right)$

Solving a Matrix Equation In Exercises 27–34, solve for X in the equation, where

$$A = \begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix}.$$

27. $X = 2A + 2B$
28. $X = 3A - 2B$
29. $2X = 2A - B$
30. $2X = A + B$
31. $2X + 3A = B$
32. $3X - 4A = 2B$
33. $4B = -2X - 2A$
34. $5A = 6B - 3X$

Finding the Product of Two Matrices In Exercises 35–40, if possible, find AB and state the dimension of the result.

35. $A = \begin{bmatrix} -1 & 6 \\ -4 & 5 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix}$

36. $A = \begin{bmatrix} 0 & -1 & 2 \\ 6 & 0 & 3 \\ 7 & -1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 4 & -5 \\ 1 & 6 \end{bmatrix}$

37. $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}$

38. $A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 6 & 13 & 8 & -17 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}$

39. $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 7 \end{bmatrix}$, $B = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

40. $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & -3 \\ 0 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 & -11 & 4 \\ 8 & 16 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

Finding the Product of Two Matrices In Exercises 41–44, use the matrix capabilities of a graphing utility to find AB, if possible.

41. $A = \begin{bmatrix} 7 & 5 & -4 \\ -2 & 5 & 1 \\ 10 & -4 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 3 \\ 8 & 1 & 4 \\ -4 & 2 & -8 \end{bmatrix}$

42. $A = \begin{bmatrix} 11 & -12 & 4 \\ 14 & 10 & 12 \\ 6 & -2 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 12 & 10 \\ -5 & 12 \\ 15 & 16 \end{bmatrix}$

43. $A = \begin{bmatrix} -3 & 8 & -6 & 8 \\ -12 & 15 & 9 & 6 \\ 5 & -1 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 6 \\ 24 & 15 & 14 \\ 16 & 10 & 21 \\ 8 & -4 & 10 \end{bmatrix}$

44. $A = \begin{bmatrix} -2 & 4 & 8 \\ 21 & 5 & 6 \\ 13 & 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -7 & 15 \\ 32 & 14 \\ 0.5 & 1.6 \end{bmatrix}$

Operations with Matrices In Exercises 45–52, if possible, find (a) AB, (b) BA, and (c) A^2.

45. $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$

46. $A = \begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

47. $A = \begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

48. $A = \begin{bmatrix} 2 & -2 \\ -3 & 0 \\ 7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

49. $A = \begin{bmatrix} -4 & -1 \\ 2 & 12 \end{bmatrix}$, $B = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$

50. $A = \begin{bmatrix} 1 & 3 & -2 \\ -5 & 10 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

51. $A = \begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$

52. $A = \begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

Operations with Matrices In Exercises 53–56, evaluate the expression. Use the matrix capabilities of a graphing utility to verify your answer.

53. $\begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$

54. $-3\left(\begin{bmatrix} 6 & 5 & -1 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ -1 & -3 \\ 4 & 1 \end{bmatrix}\right)$

55. $\begin{bmatrix} 0 & 2 & -2 \\ 4 & 1 & 2 \end{bmatrix}\left(\begin{bmatrix} 4 & 0 \\ 0 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ -3 & 5 \\ 0 & -3 \end{bmatrix}\right)$

56. $\begin{bmatrix} 3 \\ -1 \\ 5 \\ 7 \end{bmatrix}(\begin{bmatrix} 5 & -6 \end{bmatrix} + \begin{bmatrix} 7 & -1 \end{bmatrix} + \begin{bmatrix} -8 & 9 \end{bmatrix})$

Vector Operations In Exercises 57–60, use matrices to find (a) $\mathbf{u} + \mathbf{v}$, (b) $\mathbf{u} - \mathbf{v}$, and (c) $3\mathbf{v} - \mathbf{u}$.

57. $\mathbf{u} = \langle 1, 5 \rangle$, $\mathbf{v} = \langle 3, 2 \rangle$
58. $\mathbf{u} = \langle 4, 2 \rangle$, $\mathbf{v} = \langle 6, -3 \rangle$
59. $\mathbf{u} = \langle -2, 2 \rangle$, $\mathbf{v} = \langle 5, 4 \rangle$
60. $\mathbf{u} = \langle 7, -4 \rangle$, $\mathbf{v} = \langle 2, 1 \rangle$

Describing a Vector Transformation In Exercises 61–66, find $A\mathbf{v}$, where $\mathbf{v} = \langle 4, 2 \rangle$, and describe the transformation.

61. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
62. $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
63. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
64. $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
65. $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$
66. $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

Solving a System of Linear Equations In Exercises 67–72, (a) write the system of linear equations as a matrix equation, $AX = B$, and (b) use Gauss-Jordan elimination on $[A \,\vdots\, B]$ to solve for the matrix X.

67. $\begin{cases} 2x_1 + 3x_2 = 5 \\ x_1 + 4x_2 = 10 \end{cases}$

68. $\begin{cases} -2x_1 - 3x_2 = -4 \\ 6x_1 + x_2 = -36 \end{cases}$

69. $\begin{cases} x_1 - 2x_2 + 3x_3 = 9 \\ -x_1 + 3x_2 - x_3 = -6 \\ 2x_1 - 5x_2 + 5x_3 = 17 \end{cases}$

70. $\begin{cases} x_1 + x_2 - 3x_3 = -1 \\ -x_1 + 2x_2 = 1 \\ x_1 - x_2 + x_3 = 2 \end{cases}$

71. $\begin{cases} x_1 - 5x_2 + 2x_3 = -20 \\ -3x_1 + x_2 - x_3 = 8 \\ -2x_2 + 5x_3 = -16 \end{cases}$

72. $\begin{cases} x_1 - x_2 + 4x_3 = 17 \\ x_1 + 3x_2 = -11 \\ -6x_2 + 5x_3 = 40 \end{cases}$

73. Manufacturing A corporation has four factories that manufacture sport utility vehicles and pickup trucks. The production levels are represented by A.

$$\begin{array}{cc} & \text{Factory} \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ A = & \begin{bmatrix} 100 & 90 & 70 & 30 \\ 40 & 20 & 60 & 60 \end{bmatrix} \begin{array}{l} \text{SUV} \\ \text{Pickup} \end{array} \Big\} \text{Vehicle Type} \end{array}$$

Find the production levels when production increases by 10%.

74. Vacation Packages A travel agent identifies four resorts with special all-inclusive packages. The current rates for two types of rooms (double and quadruple occupancy) at the four resorts are represented by A.

$$\begin{array}{cc} & \begin{array}{cccc} \text{Resort } w & \text{Resort } x & \text{Resort } y & \text{Resort } z \end{array} \\ A = & \begin{bmatrix} 615 & 670 & 740 & 990 \\ 995 & 1030 & 1180 & 1105 \end{bmatrix} \begin{array}{l} \text{Double} \\ \text{Quadruple} \end{array} \Big\} \text{Occupancy} \end{array}$$

The rates are expected to increase by no more than 12% by next season. Find the maximum rate per package per resort.

75. Agriculture A farmer grows apples and peaches. Each crop is shipped to three different outlets. The shipment levels are represented by A.

$$\begin{array}{cc} & \text{Outlet} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ A = & \begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix} \begin{array}{l} \text{Apples} \\ \text{Peaches} \end{array} \Big\} \text{Crop} \end{array}$$

The profits per unit are represented by the matrix $B = [\$3.50 \quad \$6.00]$. Compute BA and interpret the result.

76. Revenue An electronics manufacturer produces three models of high-definition televisions, which are shipped to two warehouses. The shipment levels are represented by A.

$$\begin{array}{cc} & \text{Warehouse} \\ & \begin{array}{cc} 1 & 2 \end{array} \\ A = & \begin{bmatrix} 5{,}000 & 4{,}000 \\ 6{,}000 & 10{,}000 \\ 8{,}000 & 5{,}000 \end{bmatrix} \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \end{array} \Bigg\} \text{Model} \end{array}$$

The prices per unit are represented by the matrix

$B = [\$699.95 \quad \$899.95 \quad \$1099.95]$.

Compute BA and interpret the result.

77. Labor and Wages A company has two factories that manufacture three sizes of boats. The numbers of hours of labor required to manufacture each size are represented by S.

$$\begin{array}{cc} & \text{Department} \\ & \begin{array}{ccc} \text{Cutting} & \text{Assembly} & \text{Packaging} \end{array} \\ S = & \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 1.6 & 1.0 & 0.2 \\ 2.5 & 2.0 & 1.4 \end{bmatrix} \begin{array}{l} \text{Small} \\ \text{Medium} \\ \text{Large} \end{array} \Bigg\} \text{Boat size} \end{array}$$

The wages of the workers are represented by T.

$$\begin{array}{cc} & \text{Factory} \\ & \begin{array}{cc} A & B \end{array} \\ T = & \begin{bmatrix} \$15 & \$13 \\ \$12 & \$11 \\ \$11 & \$10 \end{bmatrix} \begin{array}{l} \text{Cutting} \\ \text{Assembly} \\ \text{Packaging} \end{array} \Bigg\} \text{Department} \end{array}$$

Compute ST and interpret the result.

78. Profit At a local store, the numbers of gallons of skim milk, 2% milk, and whole milk sold over the weekend are represented by A.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Skim milk} & \text{2\% milk} & \text{Whole milk} \end{array} \\ A = & \begin{bmatrix} 40 & 64 & 52 \\ 60 & 82 & 76 \\ 76 & 96 & 84 \end{bmatrix} \begin{array}{l} \text{Friday} \\ \text{Saturday} \\ \text{Sunday} \end{array} \end{array}$$

The selling prices per gallon and the profits per gallon for the three types of milk are represented by B.

$$\begin{array}{cc} & \begin{array}{cc} \text{Selling price} & \text{Profit} \end{array} \\ B = & \begin{bmatrix} \$3.45 & \$1.20 \\ \$3.65 & \$1.30 \\ \$3.85 & \$1.45 \end{bmatrix} \begin{array}{l} \text{Skim milk} \\ \text{2\% milk} \\ \text{Whole milk} \end{array} \end{array}$$

(a) Compute AB and interpret the result.

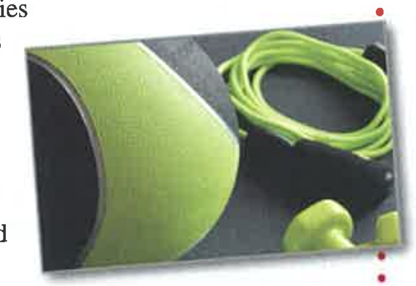
(b) Find the store's total profit from milk sales for the weekend.

79. Voting Preferences The matrix

$$\begin{array}{cc} & \text{From} \\ & \begin{array}{ccc} R & D & I \end{array} \\ P = & \begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix} \begin{array}{l} R \\ D \\ I \end{array} \Bigg\} \text{To} \end{array}$$

is called a *stochastic matrix*. Each entry p_{ij} $(i \neq j)$ represents the proportion of the voting population that changes from party i to party j, and p_{ii} represents the proportion that remains loyal to the party from one election to the next. Compute and interpret P^2.

80. Exercise The numbers of calories burned by individuals of different body weights while performing different types of exercises for a one-hour time period are represented by A.



$$\begin{array}{cc} & \text{Calories burned} \\ & \begin{array}{cc} \text{130-lb person} & \text{155-lb person} \end{array} \\ A = & \begin{bmatrix} 472 & 563 \\ 590 & 704 \\ 177 & 211 \end{bmatrix} \begin{array}{l} \text{Basketball} \\ \text{Jumping rope} \\ \text{Weight lifting} \end{array} \end{array}$$

(a) A 130-pound person and a 155-pound person play basketball for 2 hours, jump rope for 15 minutes, and lift weights for 30 minutes. Organize the times spent exercising in a matrix B.

(b) Compute BA and interpret the result.

Exploration

True or False? In Exercises 81 and 82, determine whether the statement is true or false. Justify your answer.

81. Two matrices can be added only when they have the same dimension.

82. Matrix multiplication is commutative.

Think About It In Exercises 83–86, use the matrices

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}.$$

83. Show that $(A + B)^2 \neq A^2 + 2AB + B^2$.

84. Show that $(A - B)^2 \neq A^2 - 2AB + B^2$.

85. Show that $(A + B)(A - B) \neq A^2 - B^2$.

86. Show that $(A + B)^2 = A^2 + AB + BA + B^2$.

87. Think About It If a, b, and c are real numbers such that $c \neq 0$ and $ac = bc$, then $a = b$. However, if A, B, and C are nonzero matrices such that $AC = BC$, then A is *not necessarily* equal to B. Illustrate this using the following matrices.

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

88. Think About It If a and b are real numbers such that $ab = 0$, then $a = 0$ or $b = 0$. However, if A and B are matrices such that $AB = O$, it is *not necessarily* true that $A = O$ or $B = O$. Illustrate this using the following matrices.

$$A = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

89. Finding Matrices Find two matrices A and B such that $AB = BA$.

90. HOW DO YOU SEE IT? A corporation has three factories that manufacture acoustic guitars and electric guitars. The production levels are represented by A.

$$\begin{array}{cc} & \text{Factory} \\ & \begin{array}{ccc} A & B & C \end{array} \\ A = & \begin{bmatrix} 70 & 50 & 25 \\ 35 & 100 & 70 \end{bmatrix} \begin{array}{l} \text{Acoustic} \\ \text{Electric} \end{array} \Big\} \text{Guitar type} \end{array}$$

(a) Interpret the value of a_{22}.

(b) How could you find the production levels when production increases by 20%?

(c) Each acoustic guitar sells for \$80 and each electric guitar sells for \$120. How could you use matrices to find the total sales value of the guitars produced at each factory?

91. Conjecture Let A and B be unequal diagonal matrices of the same dimension. (A **diagonal matrix** is a square matrix in which each entry not on the main diagonal is zero.) Determine the products AB for several pairs of such matrices. Make a conjecture about a rule that can be used to calculate AB without using row-by-column multiplication.

92. Matrices with Complex Entries Let $i = \sqrt{-1}$ and let

$$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$$

(a) Find A^2, A^3, and A^4. Identify any similarities with i^2, i^3, and i^4.

(b) Find and identify B^2.