8.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercised

Vocabulary: Fill in the blanks.

- 1. If there exists an $n \times n$ matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$, then A^{-1} is the _____ of A.
- 2. A matrix that has an inverse is invertible or _____. A matrix that does not have an inverse is
- 3. A 2×2 matrix is invertible if and only if its _____ is not zero.
- **4.** If A is an invertible matrix, then the system of linear equations represented by AX = B has a unique solution given by X =

Skills and Applications



The Inverse of a Matrix In Exercises 5-12, show that B is the inverse of A.

5.
$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

6.
$$A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$
, $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

7.
$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}, B = \frac{1}{10} \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

8.
$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, B = \frac{1}{5} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\mathbf{9.} \ A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

10.
$$A = \begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix}$$

$$B = \frac{1}{4} \begin{bmatrix} -2 & 4 & 6\\ 1 & -4 & -11\\ -1 & 4 & 7 \end{bmatrix}$$

11.
$$A = \begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

$$B = \frac{1}{3} \begin{bmatrix} -1 & 3 & -2 & -2 \\ -2 & 9 & -7 & -10 \\ 1 & 0 & -1 & -1 \\ 3 & -6 & 6 & 6 \end{bmatrix}$$

$$\mathbf{12.} \ A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix},$$

$$B = \frac{1}{3} \begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}$$



Finding the Inverse of a Matrix In Exercises 13-24, find the inverse of the matrix, if possible.

13.
$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

14.
$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

15.
$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

16.
$$\begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$$

17.
$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

18.
$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$$

$$\mathbf{19.} \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$$

20.
$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

$$\mathbf{21.} \begin{bmatrix}
-5 & 0 & 0 \\
2 & 0 & 0 \\
-1 & 5 & 7
\end{bmatrix}$$

$$22. \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 5 & 5 \end{bmatrix}$$

23.
$$\begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$
 24.
$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$24. \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Finding the Inverse of a Matrix In Exercises 25–32, use the matrix capabilities of a graphing utility to find the inverse of the matrix, if possible.

25.
$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$$

$$\mathbf{26.} \begin{bmatrix}
10 & 5 & -7 \\
-5 & 1 & 4 \\
3 & 2 & -2
\end{bmatrix}$$

$$27. \begin{bmatrix}
-\frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\
1 & 0 & -\frac{3}{2} \\
0 & -1 & \frac{1}{2}
\end{bmatrix}$$

28.
$$\begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} \\ 0 & \frac{2}{3} & 2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$$

29.
$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$$

$$\mathbf{30.} \begin{bmatrix} 0.6 & 0 & -0.3 \\ 0.7 & -1 & 0.2 \\ 1 & 0 & -0.9 \end{bmatrix}$$

31.
$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

31.
$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$
 32.
$$\begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$$



Finding the Inverse of a 2 x 2 Matrix In Exercises 33-38, use the formula on page 572 to find the inverse of the 2×2 matrix, if possible.

33.
$$\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$$

$$34. \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$$

35.
$$\begin{bmatrix} -4 & -6 \\ 2 & 3 \end{bmatrix}$$

36.
$$\begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}$$

37.
$$\begin{bmatrix} 0.5 & 0.3 \\ 1.5 & 0.6 \end{bmatrix}$$

38.
$$\begin{bmatrix} -1.25 & 0.625 \\ 0.16 & 0.32 \end{bmatrix}$$



Solving a System Using an Inverse Matrix In Exercises 39–42, use the inverse matrix found in Exercise 15 to solve the system of linear equations.

39.
$$\begin{cases} x - 2y = 5 \\ 2x - 3y = 10 \end{cases}$$

40.
$$\begin{cases} x - 2y = 0 \\ 2x - 3y = 3 \end{cases}$$

41.
$$\begin{cases} x - 2y = 4 \\ 2x - 3y = 2 \end{cases}$$

42.
$$\begin{cases} x - 2y = 1 \\ 2x - 3y = -2 \end{cases}$$

Solving a System Using an Inverse Matrix In Exercises 43 and 44, use the inverse matrix found in Exercise 19 to solve the system of linear equations.

43.
$$\begin{cases} x + y + z = 0 \\ 3x + 5y + 4z = 5 \\ 3x + 6y + 5z = 2 \end{cases}$$

43.
$$\begin{cases} x + y + z = 0 \\ 3x + 5y + 4z = 5 \\ 3x + 6y + 5z = 2 \end{cases}$$
44.
$$\begin{cases} x + y + z = -1 \\ 3x + 5y + 4z = 2 \\ 3x + 6y + 5z = 0 \end{cases}$$

Solving a System Using an Inverse Matrix In Exercises 45 and 46, use the inverse matrix found in Exercise 32 to solve the system of linear equations.

45.
$$\begin{cases} x_1 - 2x_2 - x_3 - 2x_4 = 0 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 = 1 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 = -1 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 = 2 \end{cases}$$

46.
$$\begin{cases} x_1 - 2x_2 - x_3 - 2x_4 = 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 = -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 = 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 = -3 \end{cases}$$

Solving a System Using an Inverse Matrix In Exercises 47-54, use an inverse matrix to solve the system of linear equations, if possible.

$$\begin{cases} 5x + 4y = -1 \\ 2x + 5y = 3 \end{cases}$$

48.
$$\begin{cases} 18x + 12y = 13 \\ 30x + 24y = 23 \end{cases}$$

$$\begin{cases}
-0.4x + 0.8y = 1.6 \\
2x - 4y = 5
\end{cases}$$

49.
$$\begin{cases} -0.4x + 0.8y = 1.6 \\ 2x - 4y = 5 \end{cases}$$
 50.
$$\begin{cases} 0.2x - 0.6y = 2.4 \\ -x + 1.4y = -8.8 \end{cases}$$

51.
$$\begin{cases} 2.3x - 1.9y = 6 \\ 1.5x + 0.75y = -12 \end{cases}$$
 52.
$$\begin{cases} 5.1x - 3.4y = -20 \\ 0.9x - 0.6y = -51 \end{cases}$$

$$\begin{cases} 2.3x - 1.9y = 6 \\ 1.5x + 0.75y = -12 \end{cases} \begin{cases} 3.1x - 3.4y - -20 \\ 0.9x - 0.6y = -51 \end{cases}$$

$$\begin{cases} 4x - y + z = -3 \\ 2x + 2y + 3z = 10 \\ 5x - 2y + 6z = 10 \end{cases}$$

53.
$$\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10 \\ 5x - 2y + 6z = 1 \end{cases}$$
 54.
$$\begin{cases} 4x - 2y + 3z = -2 \\ 2x + 2y + 5z = 16 \\ 8x - 5y - 2z = 4 \end{cases}$$

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Using a Graphing Utility In Exercises 55 and 56, use the matrix capabilities of a graphing utility to solve the system of linear equations, if possible.

55.
$$\begin{cases} 5x - 3y + 2z = 2 \\ 2x + 2y - 3z = 3 \\ x - 7y + 7z = -4 \end{cases}$$
 56.
$$\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 16z = 13 \end{cases}$$

Investment Portfolio In Exercises 57 and 58, you invest in AAA-rated bonds, A-rated bonds, and B-rated bonds. The average yields are 4.5% on AAA bonds, 5% on A bonds, and 9% on B bonds. You invest twice as much in B bonds as in A bonds. Let x, y, and z represent the amounts invested in AAA, A, and B bonds, respectively.

$$\begin{cases} x + y + z = \text{(total investment)} \\ 0.045x + 0.05y + 0.09z = \text{(annual return)} \\ 2y - z = 0 \end{cases}$$

Use the inverse of the coefficient matrix of this system to find the amount invested in each type of bond for the given total investment and annual return.

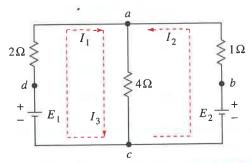
	Total Investment	Annual Return
57.	\$10,000	\$650
58.	\$12,000	\$835

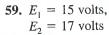
Circuit Analysis

In Exercises 59–62, consider the circuit shown in the figure. The currents I_1 , I_2 , and I_3 (in amperes) are the solution of the system

$$\begin{cases} 2I_1 & +4I_3 = E_1 \\ I_2 + 4I_3 = E_2 \\ I_1 + I_2 - I_3 = 0 \end{cases}$$

where E_1 and E_2 are voltages. Use the inverse of the coefficient matrix of this system to find the unknown currents for the given voltages.





60.
$$E_1 = 10$$
 volts, $E_2 = 10$ volts

61.
$$E_1 = 28$$
 volts, $E_2 = 21$ volts

62.
$$E_1 = 24$$
 volts, $E_2 = 23$ volts



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	Sand	Loam	Peat Moss
Seedlings	2 units	1 unit	1 unit
General	1 unit	2 units	1 unit
Hardwoods	2 units	2 units	2 units

- **63.** 500 units of sand 500 units of loam
- **64.** 500 units of sand 750 units of loam
- 400 units of peat moss
- 450 units of peat moss
- **65. Floral Design** A florist is creating 10 centerpieces. Roses cost \$2.50 each, lilies cost \$4 each, and irises cost \$2 each. The customer has a budget of \$300 allocated for the centerpieces and wants each centerpiece to contain 12 flowers, with twice as many roses as the number of irises and lilies combined.
 - (a) Write a system of linear equations that represents the situation. Then write a matrix equation that corresponds to your system.
 - (b) Solve your system of linear equations using an inverse matrix. Find the number of flowers of each type that the florist can use to create the 10 centerpieces.
- 66. International Travel The table shows the numbers of visitors y (in thousands) to the United States from China from 2012 through 2014. (Source: U.S. Department of Commerce)

Year	Visitors, y (in thousands)
2012	1474
2013	1807
2014	2188

- (a) The data can be modeled by the quadratic function $y = at^2 + bt + c$. Write a system of linear equations for the data. Let t represent the year, with t = 12 corresponding to 2012.
- (b) Use the matrix capabilities of a graphing utility to find the inverse of the coefficient matrix of the system from part (a).
- (c) Use the result of part (b) to solve the system and write the model $y = at^2 + bt + c$.
- (d) Use the graphing utility to graph the model with the data.

Exploration

True or False? In Exercises 67 and 68, determine whether the statement is true or false. Justify your answer

- 67. Multiplication of an invertible matrix and its inverse is commutative.
- 68. When the product of two square matrices is the identity matrix, the matrices are inverses of one another.
- 69. Writing Explain how to determine whether the inverse of a 2 × 2 matrix exists, as well as how to find the inverse when it exists.
- 70. Writing Explain how to write a system of three linear equations in three variables as a matrix equation AX = B, as well as how to solve the system using an inverse matrix.

Think About It In Exercises 71 and 72, find the value of k that makes the matrix singular.

71.
$$\begin{bmatrix} 4 & 3 \\ -2 & k \end{bmatrix}$$

71.
$$\begin{bmatrix} 4 & 3 \\ -2 & k \end{bmatrix}$$
 72. $\begin{bmatrix} 2k+1 & 3 \\ -7 & 1 \end{bmatrix}$

73. Conjecture Consider matrices of the form

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & 0 & \dots & 0 \\ 0 & 0 & a_{33} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix}.$$

- (a) Write a 2×2 matrix and a 3×3 matrix in the form of A. Find the inverse of each.
- (b) Use the result of part (a) to make a conjecture about the inverses of matrices in the form of A.



74. HOW DO YOU SEE IT? Consider the matrix

$$A = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}.$$

Use the determinant of A to state the conditions for which (a) A^{-1} exists and (b) $A^{-1} = A$.

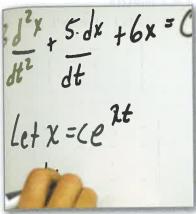
75. Verifying a Formula Verify that the inverse of an invertible 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is given by
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
.

Project: Consumer Credit To work an extended application analyzing the outstanding consumer credit in the United States, visit this text's website at LarsonPrecalculus.com. (Source: Board of Governors of the Federal Reserve System)

8.4 The Determinant of a Square Matrix



Determinants are often used in other branches of mathematics. For example, the types of determinants in Exercises 87-92 on page 584 occur when changes of variables are made in calculus.

- Find the determinants of 2 × 2 matrices.
- Find minors and cofactors of square matrices.
- Find the determinants of square matrices.

The Determinant of a 2 × 2 Matrix

Every square matrix can be associated with a real number called its determinant. Determinants have many uses, and several will be discussed in this section and the next section. Historically, the use of determinants arose from special number patterns that occur when systems of linear equations are solved. For example, the system

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

has a solution

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}$$
 and $y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$

provided that $a_1b_2 - a_2b_1 \neq 0$. Note that the denominators of the two fractions are the same. This denominator is called the determinant of the coefficient matrix of the system.

Coefficient Matrix Determinant

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \qquad \det(A) = a_1 b_2 - a_2 b_1$$

The determinant of matrix A can also be denoted by vertical bars on both sides of the matrix, as shown in the definition below.

Definition of the Determinant of a 2 × 2 Matrix

The determinant of the matrix

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

is given by

$$\det(A) = |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1.$$

In this text, det(A) and |A| are used interchangeably to represent the determinant of A. Although vertical bars are also used to denote the absolute value of a real number, the context will show which use is intended.

A convenient method for remembering the formula for the determinant of a 2×2 matrix is shown below.

$$\det(A) = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

Note that the determinant is the difference of the products of the two diagonals of the matrix.

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