Application

Sequences have many applications in business and science. Example 10 illustrates one such application.

EXAMPLE 10 Compound Interest

An investor deposits $5000 in an account that earns 3% interest compounded quarterly. The balance in the account after n quarters is given by

$$A_n = 5000\left(1 + \frac{0.03}{4}\right)^n, \quad n = 0, 1, 2, \ldots.$$

a. Write the first three terms of the sequence.

b. Find the balance in the account after 10 years by computing the 40th term of the sequence.

Solution

a. The first three terms of the sequence are as follows.

$$A_0 = 5000\left(1 + \frac{0.03}{4}\right)^0 = \$5000.00 \qquad \text{Original deposit}$$

$$A_1 = 5000\left(1 + \frac{0.03}{4}\right)^1 = \$5037.50 \qquad \text{First-quarter balance}$$

$$A_2 = 5000\left(1 + \frac{0.03}{4}\right)^2 \approx \$5075.28 \qquad \text{Second-quarter balance}$$

b. The 40th term of the sequence is

$$A_{40} = 5000\left(1 + \frac{0.03}{4}\right)^{40} \approx \$6741.74. \qquad \text{Ten-year balance}$$

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com

An investor deposits $1000 in an account that earns 3% interest compounded monthly. The balance in the account after n months is given by

$$A_n = 1000\left(1 + \frac{0.03}{12}\right)^n, \quad n = 0, 1, 2, \ldots.$$

a. Write the first three terms of the sequence.

b. Find the balance in the account after four years by computing the 48th term of the sequence.

Summarize (Section 9.1)

1. State the definition of a sequence *(page 610)*. For examples of writing the terms of sequences, see Examples 1–5.
2. State the definition of a factorial *(page 613)*. For examples of using factorial notation, see Examples 6 and 7.
3. State the definition of summation notation *(page 614)*. For an example of using summation notation, see Example 8.
4. State the definition of a series *(page 615)*. For an example of finding the sum of a series, see Example 9.
5. Describe an example of how to use a sequence to model and solve a real-life problem *(page 616, Example 10)*.

9.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. An ________ ________ is a function whose domain is the set of positive integers.
2. A sequence is a ________ sequence when the domain of the function consists only of the first n positive integers.
3. When you are given one or more of the first few terms of a sequence, and all other terms of the sequence are defined using previous terms, the sequence is defined ________.
4. If n is a positive integer, then n ________ is defined as $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdot \cdot (n-1) \cdot n$.
5. For the sum $\sum_{i=1}^{n} a_i$, i is the ________ of summation, n is the ________ limit of summation, and 1 is the ________ limit of summation.
6. The sum of the terms of a finite or infinite sequence is called a ________.

Skills and Applications

Writing the Terms of a Sequence In Exercises 7–22, write the first five terms of the sequence. (Assume that n begins with 1.)

7. $a_n = 4n - 7$
8. $a_n = -2n + 8$
9. $a_n = (-1)^{n+1} + 4$
10. $a_n = 1 - (-1)^n$
11. $a_n = (-2)^n$
12. $a_n = \left(\frac{1}{2}\right)^n$
13. $a_n = \frac{2}{3}$
14. $a_n = 6(-1)^{n+1}$
15. $a_n = \frac{1}{3}n^3$
16. $a_n = \frac{1}{n^2}$
17. $a_n = \frac{n}{n+2}$
18. $a_n = \frac{6n}{3n^2 - 1}$
19. $a_n = n(n-1)(n-2)$
20. $a_n = n(n^2 - 6)$
21. $a_n = (-1)^n\left(\frac{n}{n+1}\right)$
22. $a_n = \frac{(-1)^{n+1}}{n^2 + 1}$

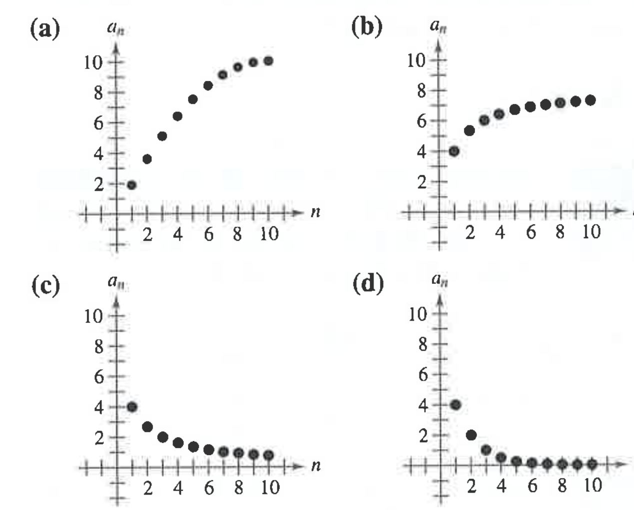
Finding a Term of a Sequence In Exercises 23–26, find the missing term of the sequence.

23. $a_n = (-1)^n(3n - 2)$
 $a_{25} = \square$
24. $a_n = (-1)^{n-1}[n(n-1)]$
 $a_{16} = \square$
25. $a_n = \frac{4n}{2n^2 - 3}$
 $a_{11} = \square$
26. $a_n = \frac{4n^2 - n + 3}{n(n-1)(n+2)}$
 $a_{13} = \square$

Graphing the Terms of a Sequence In Exercises 27–32, use a graphing utility to graph the first 10 terms of the sequence. (Assume that n begins with 1.)

27. $a_n = \frac{2}{3}n$
28. $a_n = 3n + 3(-1)^n$
29. $a_n = 16(-0.5)^{n-1}$
30. $a_n = 8(0.75)^{n-1}$
31. $a_n = \frac{2n}{n+1}$
32. $a_n = \frac{3n^2}{n^2 + 1}$

Matching a Sequence with a Graph In Exercises 33–36, match the sequence with the graph of its first 10 terms. [The graphs are labeled (a), (b), (c), and (d).]



33. $a_n = \frac{8}{n+1}$
34. $a_n = \frac{8n}{n+1}$
35. $a_n = 4(0.5)^{n-1}$
36. $a_n = n\left(2 - \frac{n}{10}\right)$

Finding the nth Term of a Sequence In Exercises 37–50, write an expression for the apparent nth term (a_n) of the sequence. (Assume that n begins with 1.)

37. 3, 7, 11, 15, 19, . . .
38. 0, 3, 8, 15, 24, . . .
39. 3, 10, 29, 66, 127, . . .
40. 91, 82, 73, 64, 55, . . .
41. 1, −1, 1, −1, 1, . . .
42. 1, 3, 1, 3, 1, . . .
43. $-\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}, -\frac{6}{7}, \ldots$
44. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$
45. $\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \ldots$
46. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots$
47. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$
48. 2, 3, 7, 25, 121, . . .
49. $\frac{1}{1}, \frac{3}{1}, \frac{9}{2}, \frac{27}{6}, \frac{81}{24}, \ldots$
50. $\frac{2}{1}, \frac{6}{3}, \frac{24}{7}, \frac{120}{15}, \frac{720}{31}, \ldots$

Writing the Terms of a Recursive Sequence In Exercises 51–56, write the first five terms of the sequence defined recursively.

51. $a_1 = 28, \quad a_{k+1} = a_k - 4$
52. $a_1 = 3, \quad a_{k+1} = 2(a_k - 1)$
53. $a_1 = 81, \quad a_{k+1} = \frac{1}{3}a_k$
54. $a_1 = 14, \quad a_{k+1} = (-2)a_k$
55. $a_0 = 1, \quad a_1 = 2, \quad a_k = a_{k-2} + \frac{1}{2}a_{k-1}$
56. $a_0 = -1, \quad a_1 = 1, \quad a_k = a_{k-2} + a_{k-1}$

Fibonacci Sequence In Exercises 57 and 58, use the Fibonacci sequence. (See Example 5.)

57. Write the first 12 terms of the Fibonacci sequence whose nth term is a_n and the first 10 terms of the sequence given by

$$b_n = \frac{a_{n+1}}{a_n}, \quad n \geq 1.$$

58. Using the definition for b_n in Exercise 57, show that b_n can be defined recursively by

$$b_n = 1 + \frac{1}{b_{n-1}}.$$

Writing the Terms of a Sequence Involving Factorials In Exercises 59–62, write the first five terms of the sequence. (Assume that n begins with 0.)

59. $a_n = \dfrac{5}{n!}$
60. $a_n = \dfrac{1}{(n+1)!}$
61. $a_n = \dfrac{(-1)^n(n+3)!}{n!}$
62. $a_n = \dfrac{(-1)^{2n+1}}{(2n+1)!}$

Simplifying a Factorial Expression In Exercises 63–66, simplify the factorial expression.

63. $\dfrac{4!}{6!}$
64. $\dfrac{12!}{4! \cdot 8!}$
65. $\dfrac{(n+1)!}{n!}$
66. $\dfrac{(2n-1)!}{(2n+1)!}$

Finding a Sum In Exercises 67–74, find the sum.

67. $\displaystyle\sum_{i=0}^{4} 3i^2$
68. $\displaystyle\sum_{k=1}^{4} 10$
69. $\displaystyle\sum_{j=3}^{5} \frac{1}{j^2-3}$
70. $\displaystyle\sum_{i=1}^{5} (2i-1)$
71. $\displaystyle\sum_{k=2}^{5} (k+1)^2(k-3)$
72. $\displaystyle\sum_{i=1}^{4} [(i-1)^2 + (i+1)^3]$
73. $\displaystyle\sum_{i=1}^{4} \frac{i!}{2^i}$
74. $\displaystyle\sum_{j=0}^{5} \frac{(-1)^j}{j!}$

Finding a Sum In Exercises 75–78, use a graphing utility to find the sum.

75. $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k!}$
76. $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k+1}$
77. $\displaystyle\sum_{n=0}^{25} \frac{1}{4^n}$
78. $\displaystyle\sum_{n=0}^{10} \frac{n!}{2^n}$

Using Sigma Notation to Write a Sum In Exercises 79–88, use sigma notation to write the sum.

79. $\dfrac{1}{3(1)} + \dfrac{1}{3(2)} + \dfrac{1}{3(3)} + \cdots + \dfrac{1}{3(9)}$
80. $\dfrac{5}{1+1} + \dfrac{5}{1+2} + \dfrac{5}{1+3} + \cdots + \dfrac{5}{1+15}$
81. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right]$
82. $\left[1 - \left(\frac{1}{6}\right)^2\right] + \left[1 - \left(\frac{2}{6}\right)^2\right] + \cdots + \left[1 - \left(\frac{6}{6}\right)^2\right]$
83. $3 - 9 + 27 - 81 + 243 - 729$
84. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128}$
85. $\dfrac{1^2}{2} + \dfrac{2^2}{6} + \dfrac{3^2}{24} + \dfrac{4^2}{120} + \cdots + \dfrac{7^2}{40{,}320}$
86. $\dfrac{1}{1 \cdot 3} + \dfrac{1}{2 \cdot 4} + \dfrac{1}{3 \cdot 5} + \cdots + \dfrac{1}{10 \cdot 12}$
87. $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$
88. $\frac{1}{2} + \frac{2}{4} + \frac{6}{8} + \frac{24}{16} + \frac{120}{32} + \frac{720}{64}$

Finding a Partial Sum of a Series In Exercises 89–92, find the (a) third, (b) fourth, and (c) fifth partial sums of the series.

89. $\displaystyle\sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i$
90. $\displaystyle\sum_{i=1}^{\infty} 2\left(\frac{1}{3}\right)^i$
91. $\displaystyle\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n$
92. $\displaystyle\sum_{n=1}^{\infty} 5\left(-\frac{1}{4}\right)^n$

Finding the Sum of an Infinite Series In Exercises 93–96, find the sum of the infinite series.

93. $\displaystyle\sum_{i=1}^{\infty} \frac{6}{10^i}$
94. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{10}\right)^k$
95. $\displaystyle\sum_{k=1}^{\infty} 7\left(\frac{1}{10}\right)^k$
96. $\displaystyle\sum_{i=1}^{\infty} \frac{2}{10^i}$

97. **Compound Interest** An investor deposits \$10,000 in an account that earns 3.5% interest compounded quarterly. The balance in the account after n quarters is given by

$$A_n = 10{,}000\left(1 + \frac{0.035}{4}\right)^n, \quad n = 1, 2, 3, \ldots.$$

(a) Write the first eight terms of the sequence.
(b) Find the balance in the account after 10 years by computing the 40th term of the sequence.
(c) Is the balance after 20 years twice the balance after 10 years? Explain.

98. **Physical Activity** The percent p_n of United States adults who met federal physical activity guidelines from 2007 through 2014 can be approximated by

$$p_n = 0.0061n^3 - 0.419n^2 + 7.85n + 4.9,$$
$$n = 7, 8, \ldots, 14$$

where n is the year, with $n = 7$ corresponding to 2007. *(Source: National Center for Health Statistics)*

(a) Write the terms of this finite sequence. Use a graphing utility to construct a bar graph that represents the sequence.
(b) What can you conclude from the bar graph in part (a)?

Exploration

True or False? In Exercises 99 and 100, determine whether the statement is true or false. Justify your answer.

99. $\displaystyle\sum_{i=1}^{4} (i^2 + 2i) = \sum_{i=1}^{4} i^2 + 2\sum_{i=1}^{4} i$
100. $\displaystyle\sum_{j=1}^{4} 2^j = \sum_{j=3}^{6} 2^{j-2}$

Arithmetic Mean In Exercises 101–103, use the following definition of the arithmetic mean $\bar{x}$ of a set of n measurements $x_1, x_2, x_3, \ldots, x_n$.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

101. Find the arithmetic mean of the six checking account balances \$327.15, \$785.69, \$433.04, \$265.38, \$604.12, and \$590.30. Use the statistical capabilities of a graphing utility to verify your result.

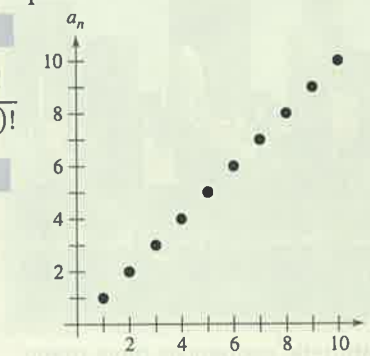
102. **Proof** Prove that $\displaystyle\sum_{i=1}^{n} (x_i - \bar{x}) = 0.$

103. **Proof** Prove that

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2.$$

104. **HOW DO YOU SEE IT?** The graph represents the first 10 terms of a sequence. Complete each expression for the apparent nth term (a_n) of the sequence. Which expressions are appropriate to represent the cost a_n to buy n MP3 songs at a cost of \$1 per song? Explain.

(a) $a_n = 1 \cdot \square$
(b) $a_n = \dfrac{\square\,!}{(n-1)!}$
(c) $a_n = \displaystyle\sum_{k=1}^{n} \square$

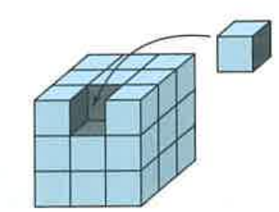


Error Analysis In Exercises 105 and 106, describe the error in finding the sum.

105. $\displaystyle\sum_{k=1}^{4} (3 + 2k^2) = \sum_{k=1}^{4} 3 + \sum_{k=1}^{4} 2k^2$
$= 3 + (2 + 8 + 18 + 32)$
$= 63$

106. $\displaystyle\sum_{n=0}^{3} (-1)^n n! = (-1)(1) + (1)(2) + (-1)(6)$
$= -5$

107. **Cube** A $3 \times 3 \times 3$ cube is made up of 27 unit cubes (a unit cube has a length, width, and height of 1 unit), and only the faces of each cube that are visible are painted blue, as shown in the figure.



(a) Determine how many unit cubes of the $3 \times 3 \times 3$ cube have 0 blue faces, 1 blue face, 2 blue faces, and 3 blue faces.
(b) Repeat part (a) for a $4 \times 4 \times 4$ cube, a $5 \times 5 \times 5$ cube, and a $6 \times 6 \times 6$ cube.
(c) Write formulas you could use to repeat part (a) for an $n \times n \times n$ cube.