9.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A sequence is ________ when the differences between consecutive terms are the same. This difference is the ________ difference.
2. The nth term of an arithmetic sequence has the form $a_n =$ ________.
3. When you know the nth term of an arithmetic sequence *and* you know the common difference of the sequence, you can find the $(n + 1)$th term by using the ________ formula $a_{n+1} = a_n + d$.
4. The formula $S_n = \frac{n}{2}(a_1 + a_n)$ gives the sum of a ________ ________ ________ with n terms.

Skills and Applications

Determining Whether a Sequence Is Arithmetic In Exercises 5–12, determine whether the sequence is arithmetic. If so, find the common difference.

5. $1, 2, 4, 8, 16, \ldots$
6. $4, 9, 14, 19, 24, \ldots$
7. $10, 8, 6, 4, 2, \ldots$
8. $80, 40, 20, 10, 5, \ldots$
9. $\frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \ldots$
10. $6.6, 5.9, 5.2, 4.5, 3.8, \ldots$
11. $1^2, 2^2, 3^2, 4^2, 5^2, \ldots$
12. $\ln 1, \ln 2, \ln 4, \ln 8, \ln 16, \ldots$

Writing the Terms of a Sequence In Exercises 13–20, write the first five terms of the sequence. Determine whether the sequence is arithmetic. If so, find the common difference. (Assume that n begins with 1.)

13. $a_n = 5 + 3n$
14. $a_n = 100 - 3n$
15. $a_n = 3 - 4(n - 2)$
16. $a_n = 1 + (n - 1)n$
17. $a_n = (-1)^n$
18. $a_n = n - (-1)^n$
19. $a_n = (2^n)n$
20. $a_n = \frac{3(-1)^n}{n}$

Finding the nth Term In Exercises 21–30, find a formula for a_n for the arithmetic sequence.

21. $a_1 = 1, d = 3$
22. $a_1 = 15, d = 4$
23. $a_1 = 100, d = -8$
24. $a_1 = 0, d = -\frac{2}{3}$
25. $4, \frac{3}{2}, -1, -\frac{7}{2}, \ldots$
26. $10, 5, 0, -5, -10, \ldots$
27. $a_1 = 5, a_4 = 15$
28. $a_1 = -4, a_5 = 16$
29. $a_3 = 94, a_6 = 103$
30. $a_5 = 190, a_{10} = 115$

Writing the Terms of an Arithmetic Sequence In Exercises 31–36, write the first five terms of the arithmetic sequence.

31. $a_1 = 5, d = 6$
32. $a_1 = 5, d = -\frac{3}{4}$
33. $a_1 = 2, a_{12} = -64$
34. $a_4 = 16, a_{10} = 46$
35. $a_8 = 26, a_{12} = 42$
36. $a_3 = 19, a_{15} = -1.7$

Writing the Terms of an Arithmetic Sequence In Exercises 37–40, write the first five terms of the arithmetic sequence defined recursively.

37. $a_1 = 15, a_{n+1} = a_n + 4$
38. $a_1 = 200, a_{n+1} = a_n - 10$
39. $a_5 = 7, a_{n+1} = a_n - 2$
40. $a_3 = 0.5, a_{n+1} = a_n + 0.75$

Using a Recursion Formula In Exercises 41–44, the first two terms of the arithmetic sequence are given. Find the missing term.

41. $a_1 = 5, a_2 = -1, a_{10} = \square$
42. $a_1 = 3, a_2 = 13, a_9 = \square$
43. $a_1 = \frac{1}{8}, a_2 = \frac{3}{4}, a_7 = \square$
44. $a_1 = -0.7, a_2 = -13.8, a_8 = \square$

Sum of a Finite Arithmetic Sequence In Exercises 45–50, find the sum of the finite arithmetic sequence.

45. $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20$
46. $1 + 4 + 7 + 10 + 13 + 16 + 19$
47. $-1 + (-3) + (-5) + (-7) + (-9)$
48. $-5 + (-3) + (-1) + 1 + 3 + 5$
49. Sum of the first 100 positive odd integers
50. Sum of the integers from -100 to 30

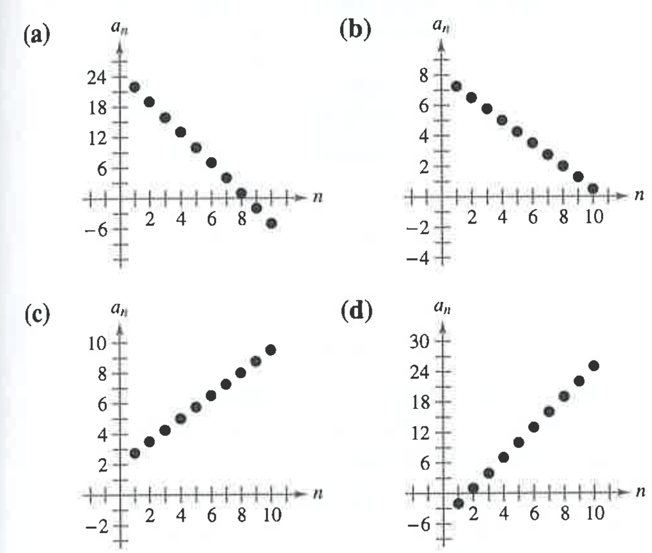
Partial Sum of an Arithmetic Sequence In Exercises 51–54, find the nth partial sum of the arithmetic sequence for the given value of n.

51. $8, 20, 32, 44, \ldots, \quad n = 50$
52. $-6, -2, 2, 6, \ldots, \quad n = 100$
53. $0, -9, -18, -27, \ldots, \quad n = 40$
54. $75, 70, 65, 60, \ldots, \quad n = 25$

Finding a Sum In Exercises 55–60, find the partial sum.

55. $\sum_{n=1}^{50} n$
56. $\sum_{n=51}^{100} 7n$
57. $\sum_{n=1}^{500} (n + 8)$
58. $\sum_{n=1}^{250} (1000 - n)$
59. $\sum_{n=1}^{100} (-6n + 20)$
60. $\sum_{n=1}^{75} (12n - 9)$

Matching an Arithmetic Sequence with Its Graph In Exercises 61–64, match the arithmetic sequence with its graph. [The graphs are labeled (a)–(d).]



61. $a_n = -\frac{3}{4}n + 8$
62. $a_n = 3n - 5$
63. $a_n = 2 + \frac{3}{4}n$
64. $a_n = 25 - 3n$

Graphing the Terms of a Sequence In Exercises 65–68, use a graphing utility to graph the first 10 terms of the sequence. (Assume that n begins with 1.)

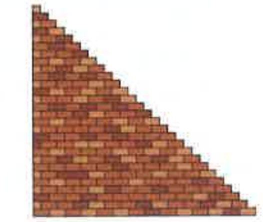
65. $a_n = 15 - \frac{3}{2}n$
66. $a_n = -5 + 2n$
67. $a_n = 0.2n + 3$
68. $a_n = -0.3n + 8$

Job Offer In Exercises 69 and 70, consider a job offer with the given starting salary and annual raise. (a) Determine the salary during the sixth year of employment. (b) Determine the total compensation from the company through six full years of employment.

	Starting Salary	Annual Raise
69.	\$32,500	\$1500
70.	\$36,800	\$1750

71. **Seating Capacity** Determine the seating capacity of an auditorium with 36 rows of seats when there are 15 seats in the first row, 18 seats in the second row, 21 seats in the third row, and so on.

72. **Brick Pattern** A triangular brick wall is made by cutting some bricks in half to use in the first column of every other row (see figure). The wall has 28 rows. The top row is one-half brick wide and the bottom row is 14 bricks wide. How many bricks are in the finished wall?



73. **Falling Object** An object with negligible air resistance is dropped from the top of the Willis Tower in Chicago at a height of 1451 feet. During the first second of fall, the object falls 16 feet; during the second second, it falls 48 feet; during the third second, it falls 80 feet; during the fourth second, it falls 112 feet. Assuming this pattern continues, how many feet does the object fall in the first 7 seconds after it is dropped?

74. **Prize Money** A county fair is holding a baked goods competition in which the top eight bakers receive cash prizes. First place receives \$200, second place receives \$175, third place receives \$150, and so on.
    (a) Write the nth term (a_n) of a sequence that represents the cash prize received in terms of the place n the baked good is awarded.
    (b) Find the total amount of prize money awarded at the competition.

75. **Total Sales** An entrepreneur sells \$15,000 worth of sports memorabilia during one year and sets a goal of increasing annual sales by \$5000 each year for the next 9 years. Assuming that the entrepreneur meets this goal, find the total sales during the first 10 years of this business. What kinds of economic factors could prevent the business from meeting its goals?

76. **Borrowing Money** You borrow \$5000 from your parents to purchase a used car. The arrangements of the loan are such that you make payments of \$250 per month toward the balance plus 1% interest on the unpaid balance from the previous month.
    (a) Find the first year's monthly payments and the unpaid balance after each month.
    (b) Find the total amount of interest paid over the term of the loan.

Eugene Moerman/Shutterstock.com

77. Business The table shows the net numbers of new stores opened by H&M from 2011 through 2015. *(Source: H&M Hennes & Mauritz AB)*

DATA

Year	New Stores
2011	266
2012	304
2013	356
2014	379
2015	413

Spreadsheet at LarsonPrecalculus.com

(a) Construct a bar graph showing the annual net numbers of new stores opened by H&M from 2011 through 2015.

(b) Find the nth term (a_n) of an arithmetic sequence that approximates the data. Let n represent the year, with $n = 1$ corresponding to 2011. (*Hint:* Use the average change per year for d.)

(c) Use a graphing utility to graph the terms of the finite sequence you found in part (b).

(d) Use summation notation to represent the *total* number of new stores opened from 2011 through 2015. Use this sum to approximate the total number of new stores opened during these years.

78. Business In Exercise 77, there are a total number of 2206 stores at the end of 2010. Write the terms of a sequence that represents the total number of stores at the end of each year from 2011 through 2015. Is the sequence approximately arithmetic? Explain.

Exploration

True or False? **In Exercises 79 and 80, determine whether the statement is true or false. Justify your answer.**

79. Given an arithmetic sequence for which only the first two terms are known, it is possible to find the nth term.

80. When the first term, the nth term, and n are known for an arithmetic sequence, you have enough information to find the nth partial sum of the sequence.

81. Comparing Graphs of a Sequence and a Line

(a) Graph the first 10 terms of the arithmetic sequence $a_n = 2 + 3n$.

(b) Graph the equation of the line $y = 3x + 2$.

(c) Discuss any differences between the graph of $a_n = 2 + 3n$ and the graph of $y = 3x + 2$.

(d) Compare the slope of the line in part (b) with the common difference of the sequence in part (a). What can you conclude about the slope of a line and the common difference of an arithmetic sequence?

82. Writing Describe two ways to use the first two terms of an arithmetic sequence to find the 13th term.

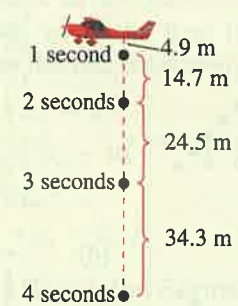
Finding the Terms of a Sequence **In Exercises 83 and 84, find the first 10 terms of the sequence.**

83. $a_1 = x, d = 2x$ **84.** $a_1 = -y, d = 5y$

85. Error Analysis Describe the error in finding the sum of the first 50 odd integers.

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{50}{2}(1 + 101) = 2550 \quad \times$$

86. HOW DO YOU SEE IT? A steel ball with negligible air resistance is dropped from an airplane. The figure shows the distance that the ball falls during each of the first four seconds after it is dropped.



(a) Describe a pattern in the distances shown. Explain why the distances form a finite arithmetic sequence.

(b) Assume the pattern described in part (a) continues. Describe the steps and formulas involved in using the sum of a finite sequence to find the total distance the ball falls in n seconds, where n is a whole number.

87. Pattern Recognition

(a) Compute the following sums of consecutive positive odd integers.

$1 + 3 =$

$1 + 3 + 5 =$

$1 + 3 + 5 + 7 =$

$1 + 3 + 5 + 7 + 9 =$

$1 + 3 + 5 + 7 + 9 + 11 =$

(b) Use the sums in part (a) to make a conjecture about the sums of consecutive positive odd integers. Check your conjecture for the sum

$1 + 3 + 5 + 7 + 9 + 11 + 13 =$.

(c) Verify your conjecture algebraically.

Project: Net Sales To work an extended application analyzing the net sales for Dollar Tree from 2001 through 2014, visit the textbook's website at *LarsonPrecalculus.com*. *(Source: Dollar Tree, Inc.)*

9.3 Geometric Sequences and Series

Geometric sequences can help you model and solve real-life problems. For example, in Exercise 84 on page 636, you will use a geometric sequence to model the population of Argentina from 2009 through 2015.

- **Recognize, write, and find the nth terms of geometric sequences.**
- **Find the sum of a finite geometric sequence.**
- **Find the sum of an infinite geometric series.**
- **Use geometric sequences to model and solve real-life problems.**

Geometric Sequences

In Section 9.2, you learned that a sequence whose consecutive terms have a common *difference* is an arithmetic sequence. In this section, you will study another important type of sequence called a **geometric sequence.** Consecutive terms of a geometric sequence have a common *ratio.*

Definition of Geometric Sequence

A sequence is **geometric** when the ratios of consecutive terms are the same. So, the sequence $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ is geometric when there is a number r such that

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots = r, \quad r \neq 0.$$

The number r is the **common ratio** of the geometric sequence.

REMARK Be sure you understand that a sequence such as $1, 4, 9, 16, \ldots$, whose nth term is n^2, is *not* geometric. The ratio of the second term to the first term is

$$\frac{a_2}{a_1} = \frac{4}{1} = 4$$

but the ratio of the third term to the second term is

$$\frac{a_3}{a_2} = \frac{9}{4}.$$

EXAMPLE 1 Examples of Geometric Sequences

a. The sequence whose nth term is 2^n is geometric. The common ratio of consecutive terms is 2.

$$\underbrace{2, 4}_{\frac{4}{2} = 2}, 8, 16, \ldots, 2^n, \ldots \qquad \text{Begin with } n = 1.$$

b. The sequence whose nth term is $4(3^n)$ is geometric. The common ratio of consecutive terms is 3.

$$\underbrace{12, 36}_{\frac{36}{12} = 3}, 108, 324, \ldots, 4(3^n), \ldots \qquad \text{Begin with } n = 1.$$

c. The sequence whose nth term is $\left(-\frac{1}{3}\right)^n$ is geometric. The common ratio of consecutive terms is $-\frac{1}{3}$.

$$\underbrace{-\frac{1}{3}, \frac{1}{9}}_{\frac{1/9}{-1/3} = -\frac{1}{3}}, -\frac{1}{27}, \frac{1}{81}, \ldots, \left(-\frac{1}{3}\right)^n, \ldots \qquad \text{Begin with } n = 1.$$

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Write the first four terms of the geometric sequence whose nth term is $6(-2)^n$. Then find the common ratio of the consecutive terms.

In Example 1, notice that each of the geometric sequences has an nth term that is of the form ar^n, where the common ratio of the sequence is r. A geometric sequence may be thought of as an exponential function whose domain is the set of natural numbers.

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