## Application

**EXAMPLE 8** **Increasing Annuity**

*See LarsonPrecalculus.com for an interactive version of this type of example.*

An investor deposits \$50 on the first day of each month in an account that pays 3% interest, compounded monthly. What is the balance at the end of 2 years? (This type of investment plan is called an **increasing annuity.**)

**Solution** To find the balance in the account after 24 months, consider each of the 24 deposits separately. The first deposit will gain interest for 24 months, and its balance will be

$$A_{24} = 50\left(1 + \frac{0.03}{12}\right)^{24}$$
$$= 50(1.0025)^{24}.$$

The second deposit will gain interest for 23 months, and its balance will be

$$A_{23} = 50\left(1 + \frac{0.03}{12}\right)^{23}$$
$$= 50(1.0025)^{23}.$$

The last deposit will gain interest for only 1 month, and its balance will be

$$A_1 = 50\left(1 + \frac{0.03}{12}\right)^{1}$$
$$= 50(1.0025).$$

The total balance in the annuity will be the sum of the balances of the 24 deposits. Using the formula for the sum of a finite geometric sequence, with $A_1 = 50(1.0025)$, $r = 1.0025$, and $n = 24$, you have

$$S_n = A_1\left(\frac{1 - r^n}{1 - r}\right) \qquad \text{Sum of a finite geometric sequence}$$

$$S_{24} = 50(1.0025)\left[\frac{1 - (1.0025)^{24}}{1 - 1.0025}\right] \qquad \text{Substitute } 50(1.0025) \text{ for } A_1, 1.0025 \text{ for } r, \text{ and } 24 \text{ for } n.$$

$$\approx \$1238.23. \qquad \text{Use a calculator.}$$

**REMARK** Recall from Section 3.1 that the formula for compound interest (for $n$ compoundings per year) is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

So, in Example 8, \$50 is the principal $P$, 0.03 is the annual interest rate $r$, 12 is the number $n$ of compoundings per year, and 2 is the time $t$ in years. When you substitute these values into the formula, you obtain

$$A = 50\left(1 + \frac{0.03}{12}\right)^{12(2)}$$
$$= 50\left(1 + \frac{0.03}{12}\right)^{24}.$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

An investor deposits \$70 on the first day of each month in an account that pays 2% interest, compounded monthly. What is the balance at the end of 4 years?

### Summarize (Section 9.3)

1. State the definition of a geometric sequence *(page 629)* and state the formula for the $n$th term of a geometric sequence *(page 630)*. For examples of recognizing, writing, and finding the $n$th terms of geometric sequences, see Examples 1–5.
2. State the formula for the sum of a finite geometric sequence *(page 632)*. For an example of finding the sum of a finite geometric sequence, see Example 6.
3. State the formula for the sum of an infinite geometric series *(page 633)*. For an example of finding the sums of infinite geometric series, see Example 7.
4. Describe an example of how to use a geometric sequence to model and solve a real-life problem *(page 634, Example 8)*.



## 9.3 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

### Vocabulary: Fill in the blanks.

1. A sequence is ________ when the ratios of consecutive terms are the same. This ratio is the ________ ratio.
2. The term of a geometric sequence has the form $a_n =$ ________.
3. The sum of a finite geometric sequence with common ratio $r \neq 1$ is given by $S_n =$ ________.
4. The sum of the terms of an infinite geometric sequence is called a ________ ________.

### Skills and Applications

**Determining Whether a Sequence Is Geometric** In Exercises 5–12, determine whether the sequence is geometric. If so, find the common ratio.

5. $3, 6, 12, 24, \ldots$
6. $5, 10, 15, 20, \ldots$
7. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, \ldots$
8. $27, -9, 3, -1, \ldots$
9. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$
10. $5, 1, 0.2, 0.04, \ldots$
11. $1, -\sqrt{7}, 7, -7\sqrt{7}, \ldots$
12. $2, \frac{4}{\sqrt{3}}, \frac{8}{3}, \frac{16}{3\sqrt{3}}, \ldots$

**Writing the Terms of a Geometric Sequence** In Exercises 13–22, write the first five terms of the geometric sequence.

13. $a_1 = 4, r = 3$
14. $a_1 = 7, r = 4$
15. $a_1 = 1, r = \frac{1}{2}$
16. $a_1 = 6, r = -\frac{1}{4}$
17. $a_1 = 1, r = e$
18. $a_1 = 2, r = \pi$
19. $a_1 = 3, r = \sqrt{5}$
20. $a_1 = 4, r = -1/\sqrt{2}$
21. $a_1 = 2, r = 3x$
22. $a_1 = 4, r = x/5$

**Finding a Term of a Geometric Sequence** In Exercises 23–32, write an expression for the $n$th term of the geometric sequence. Then find the missing term.

23. $a_1 = 4, r = \frac{1}{2}, a_{10} =$
24. $a_1 = 5, r = \frac{7}{2}, a_8 =$
25. $a_1 = 6, r = -\frac{1}{3}, a_{12} =$
26. $a_1 = 64, r = -\frac{1}{4}, a_{10} =$
27. $a_1 = 100, r = e^x, a_9 =$
28. $a_1 = 1, r = e^{-x}, a_4 =$
29. $a_1 = 1, r = \sqrt{2}, a_{12} =$
30. $a_1 = 1, r = \sqrt{3}, a_8 =$
31. $a_1 = 500, r = 1.02, a_{40} =$
32. $a_1 = 1000, r = 1.005, a_{60} =$

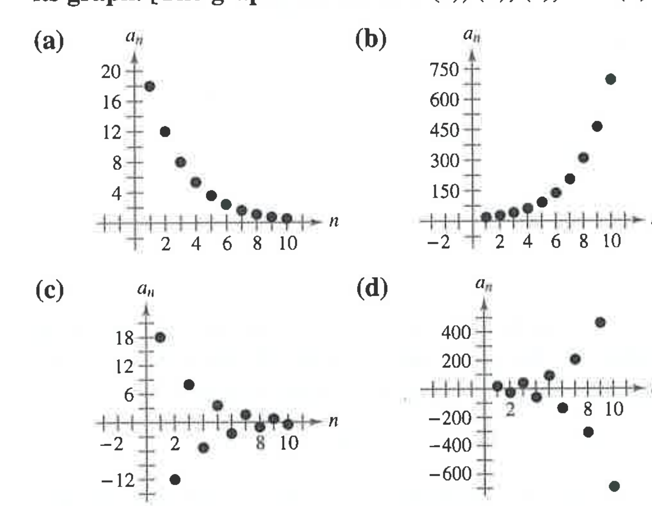
**Writing the $n$th Term of a Geometric Sequence** In Exercises 33–38, find a formula for the $n$th term of the sequence.

33. $64, 32, 16, \ldots$
34. $81, 27, 9, \ldots$
35. $9, 18, 36, \ldots$
36. $5, -10, 20, \ldots$
37. $6, -9, \frac{27}{2}, \ldots$
38. $80, -40, 20, \ldots$

**Finding a Term of a Geometric Sequence** In Exercises 39–46, find the specified term of the geometric sequence.

39. 8th term: $6, 18, 54, \ldots$
40. 7th term: $5, 20, 80, \ldots$
41. 9th term: $\frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, \ldots$
42. 8th term: $\frac{3}{2}, -1, \frac{2}{3}, \ldots$
43. $a_3$: $a_1 = 16, a_4 = \frac{27}{4}$
44. $a_1$: $a_2 = 3, a_5 = \frac{3}{64}$
45. $a_6$: $a_4 = -18, a_7 = \frac{2}{3}$
46. $a_5$: $a_2 = 2, a_3 = -\sqrt{2}$

**Matching a Geometric Sequence with Its Graph** In Exercises 47–50, match the geometric sequence with its graph. [The graphs are labeled (a), (b), (c), and (d).]




47. $a_n = 18\left(\frac{2}{3}\right)^{n-1}$
48. $a_n = 18\left(-\frac{2}{3}\right)^{n-1}$
49. $a_n = 18\left(\frac{3}{2}\right)^{n-1}$
50. $a_n = 18\left(-\frac{3}{2}\right)^{n-1}$

**Graphing the Terms of a Sequence** In Exercises 51–54, use a graphing utility to graph the first 10 terms of the sequence.

51. $a_n = 14(1.4)^{n-1}$
52. $a_n = 18(0.7)^{n-1}$
53. $a_n = 8(-0.3)^{n-1}$
54. $a_n = 11(-1.9)^{n-1}$

**Sum of a Finite Geometric Sequence** **In Exercises 55–64, find the sum of the finite geometric sequence.**

55. $\sum_{n=1}^{7} 4^{n-1}$

56. $\sum_{n=1}^{10} \left(\frac{3}{2}\right)^{n-1}$

57. $\sum_{n=1}^{6} (-7)^{n-1}$

58. $\sum_{n=1}^{8} 5\left(-\frac{5}{2}\right)^{n-1}$

59. $\sum_{n=0}^{20} 3\left(\frac{3}{2}\right)^{n}$

60. $\sum_{n=0}^{40} 5\left(\frac{3}{5}\right)^{n}$

61. $\sum_{n=0}^{5} 200(1.05)^{n}$

62. $\sum_{n=0}^{6} 500(1.04)^{n}$

63. $\sum_{n=0}^{40} 2\left(-\frac{1}{4}\right)^{n}$

64. $\sum_{n=0}^{50} 10\left(\frac{2}{3}\right)^{n-1}$

**Using Summation Notation** **In Exercises 65–68, use summation notation to write the sum.**

65. $10 + 30 + 90 + \cdots + 7290$

66. $15 - 3 + \frac{3}{5} - \cdots - \frac{3}{625}$

67. $0.1 + 0.4 + 1.6 + \cdots + 102.4$

68. $32 + 24 + 18 + 13.5 + 10.125$

**Sum of an Infinite Geometric Series** **In Exercises 69–78, find the sum of the infinite geometric series.**

69. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n}$

70. $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^{n}$

71. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^{n}$

72. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^{n}$

73. $\sum_{n=0}^{\infty} (0.8)^{n}$

74. $\sum_{n=0}^{\infty} 4(0.2)^{n}$

75. $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdots$

76. $9 + 6 + 4 + \frac{8}{3} + \cdots$

77. $\frac{1}{9} - \frac{1}{3} + 1 - 3 + \cdots$

78. $-\frac{125}{36} + \frac{25}{6} - 5 + 6 - \cdots$

**Writing a Repeating Decimal as a Rational Number** **In Exercises 79 and 80, find the rational number representation of the repeating decimal.**

79. $0.\overline{36}$

80. $0.3\overline{18}$

**Graphical Reasoning** **In Exercises 81 and 82, use a graphing utility to graph the function. Identify the horizontal asymptote of the graph and determine its relationship to the sum.**

81. $f(x) = 6\left[\frac{1 - (0.5)^x}{1 - (0.5)}\right]$, $\sum_{n=0}^{\infty} 6\left(\frac{1}{2}\right)^n$

82. $f(x) = 2\left[\frac{1 - (0.8)^x}{1 - (0.8)}\right]$, $\sum_{n=0}^{\infty} 2\left(\frac{4}{5}\right)^n$

83. **Depreciation** A tool and die company buys a machine for $175,000 and it depreciates at a rate of 30% per year. (In other words, at the end of each year the depreciated value is 70% of what it was at the beginning of the year.) Find the depreciated value of the machine after 5 full years.

84. **Population** The table shows the mid-year populations of Argentina (in millions) from 2009 through 2015. *(Source: U.S. Census Bureau)*

DATA

| Year | Population |
|---|---|
| 2009 | 40.9 |
| 2010 | 41.3 |
| 2011 | 41.8 |
| 2012 | 42.2 |
| 2013 | 42.6 |
| 2014 | 43.0 |
| 2015 | 43.4 |

Spreadsheet at LarsonPrecalculus.com

(a) Use the *exponential regression* feature of a graphing utility to find the $n$th term $(a_n)$ of a geometric sequence that models the data. Let $n$ represent the year, with $n = 9$ corresponding to 2009.

(b) Use the sequence from part (a) to describe the rate at which the population of Argentina is growing.

(c) Use the sequence from part (a) to predict the population of Argentina in 2025. The U.S. Census Bureau predicts the population of Argentina will be 47.2 million in 2025. How does this value compare with your prediction?

(d) Use the sequence from part (a) to predict when the population of Argentina will reach 50.0 million.

85. **Annuity** An investor deposits $P$ dollars on the first day of each month in an account with an annual interest rate $r$, compounded monthly. The balance $A$ after $t$ years is

$$A = P\left(1 + \frac{r}{12}\right) + \cdots + P\left(1 + \frac{r}{12}\right)^{12t}.$$

Show that the balance is

$$A = P\left[\left(1 + \frac{r}{12}\right)^{12t} - 1\right]\left(1 + \frac{12}{r}\right).$$

iStockphoto.com/Holger Mette



86. **Annuity** An investor deposits $100 on the first day of each month in an account that pays 2% interest, compounded monthly. The balance $A$ in the account at the end of 5 years is

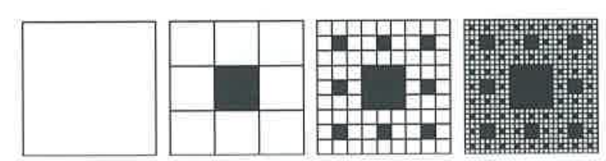
$$A = 100\left(1 + \frac{0.02}{12}\right)^1 + \cdots + 100\left(1 + \frac{0.02}{12}\right)^{60}.$$

Use the result of Exercise 85 to find $A$.

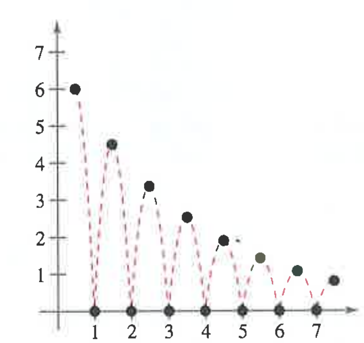
**Multiplier Effect** **In Exercises 87 and 88, use the following information. A state government gives property owners a tax rebate with the anticipation that each property owner will spend approximately $p\%$ of the rebate, and in turn each recipient of this amount will spend $p\%$ of what he or she receives, and so on. Economists refer to this exchange of money and its circulation within the economy as the "multiplier effect." The multiplier effect operates on the idea that the expenditures of one individual become the income of another individual. For the given tax rebate, find the total amount of spending that results, assuming that this effect continues without end.**

| | Tax rebate | $p\%$ |
|---|---|---|
| 87. | $400 | 75% |
| 88. | $600 | 72.5% |

89. **Geometry** The sides of a square are 27 inches in length. New squares are formed by dividing the original square into nine squares. The center square is then shaded (see figure). This process is repeated three more times. Determine the total area of the shaded region.

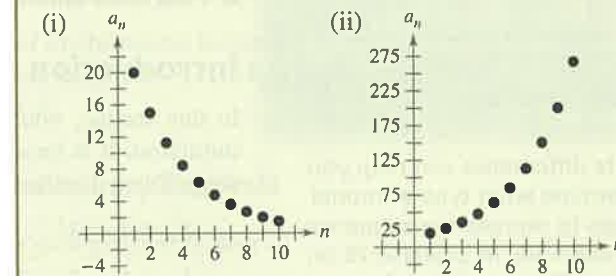


90. **Distance** A ball is dropped from a height of 6 feet and begins bouncing as shown in the figure. The height of each bounce is three-fourths the height of the previous bounce. Find the total vertical distance the ball travels before coming to rest.




91. **Salary** An investment firm has a job opening with a salary of $45,000 for the first year. During the next 39 years, there is a 5% raise each year. Find the total compensation over the 40-year period.

92. **HOW DO YOU SEE IT?** Use the figures shown below.




(a) Without performing any calculations, determine which figure shows terms of a sequence given by $a_n = 20\left(\frac{4}{3}\right)^{n-1}$ and which shows terms of a sequence given by $a_n = 20\left(\frac{3}{4}\right)^{n-1}$. Explain your reasoning.

(b) Which infinite sequence has terms that can be summed? Explain your reasoning.

## Exploration

**True or False?** **In Exercises 93 and 94, determine whether the statement is true or false. Justify your answer.**

93. A sequence is geometric when the ratios of consecutive differences of consecutive terms are the same.

94. To find the $n$th term of a geometric sequence, multiply its common ratio by the first term of the sequence raised to the $(n - 1)$th power.

95. **Graphical Reasoning** Consider the graph of

$$y = \frac{1 - r^x}{1 - r}.$$

(a) Use a graphing utility to graph $y$ for $r = \frac{1}{2}, \frac{2}{3}$, and $\frac{4}{5}$. What happens as $x \to \infty$?

(b) Use the graphing utility to graph $y$ for $r = 1.5, 2$, and 3. What happens as $x \to \infty$?

96. **Writing** Write a brief paragraph explaining why the terms of a geometric sequence decrease in magnitude when $-1 < r < 1$.

**Project: Population** To work an extended application analyzing the population of Delaware, visit this text's website at *LarsonPrecalculus.com*. *(Source: U.S. Census Bureau)*