9.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary

In Exercises 1–7, fill in the blanks.

1. An _____ is any happening for which the result is uncertain, and the possible results are called _____.
2. The set of all possible outcomes of an experiment is the _____ _____.
3. The formula for the _____ of an event is $P(E) = \dfrac{n(E)}{n(S)}$, where $n(E)$ is the number of equally likely outcomes in the event and $n(S)$ is the number of equally likely outcomes in the sample space.
4. If $P(E) = 0$, then E is an _____ event, and if $P(E) = 1$, then E is a _____ event.
5. Two events A and B (from the same sample space) are _____ _____ when A and B have no outcomes in common.
6. Two events are _____ when the occurrence of one has no effect on the occurrence of the other.
7. The _____ of an event A is the collection of all outcomes in the sample space that are not in A.
8. Match the probability formula with the correct probability name.

(a) Probability of the union of two events	(i) $P(A \cup B) = P(A) + P(B)$
(b) Probability of mutually exclusive events	(ii) $P(A') = 1 - P(A)$
(c) Probability of independent events	(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(d) Probability of a complement	(iv) $P(A \text{ and } B) = P(A) \cdot P(B)$

Skills and Applications

Finding a Sample Space **In Exercises 9–14, find the sample space for the experiment.**

9. You toss a coin and a six-sided die.
10. You toss a six-sided die twice and record the sum.
11. A taste tester ranks three varieties of yogurt, A, B, and C, according to preference.
12. You select two marbles (without replacement) from a bag containing two red marbles, two blue marbles, and one yellow marble. You record the color of each marble.
13. Two county supervisors are selected from five supervisors, A, B, C, D, and E, to study a recycling plan.
14. A sales representative visits three homes per day. In each home, there may be a sale (denote by S) or there may be no sale (denote by F).

Tossing a Coin **In Exercises 15–20, find the probability for the experiment of tossing a coin three times.**

15. The probability of getting exactly one tail
16. The probability of getting exactly two tails
17. The probability of getting a head on the first toss
18. The probability of getting a tail on the last toss
19. The probability of getting at least one head
20. The probability of getting at least two heads

Drawing a Card **In Exercises 21–24, find the probability for the experiment of drawing a card at random from a standard deck of 52 playing cards.**

21. The card is a face card.
22. The card is not a face card.
23. The card is a red face card.
24. The card is a 9 or lower. (Aces are low.)

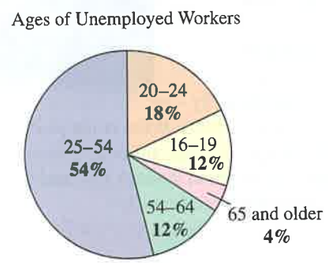
Tossing a Die **In Exercises 25–30, find the probability for the experiment of tossing a six-sided die twice.**

25. The sum is 6.
26. The sum is at least 8.
27. The sum is less than 11.
28. The sum is 2, 3, or 12.
29. The sum is odd and no more than 7.
30. The sum is odd or prime.

Drawing Marbles **In Exercises 31–34, find the probability for the experiment of drawing two marbles at random (without replacement) from a bag containing one green, two yellow, and three red marbles.**

31. Both marbles are red.
32. Both marbles are yellow.
33. Neither marble is yellow.
34. The marbles are different colors.

35. **Unemployment** In 2015, there were approximately 8.3 million unemployed workers in the United States. The circle graph shows the age profile of these unemployed workers. *(Source: U.S. Bureau of Labor Statistics)*



(a) Estimate the number of unemployed workers in the 16–19 age group.
(b) What is the probability that a person selected at random from the population of unemployed workers is in the 20–24 age group?
(c) What is the probability that a person selected at random from the population of unemployed workers is in the 25–54 age group?
(d) What is the probability that a person selected at random from the population of unemployed workers is 55 or older?

36. **Political Poll** An independent polling organization interviewed 100 college students to determine their political party affiliations and whether they favor a balanced-budget amendment to the Constitution. The table lists the results of the study. In the table, D represents Democrat and R represents Republican.

	Favor	Not Favor	Unsure	Total
D	23	25	7	55
R	32	9	4	45
Total	55	34	11	100

Find the probability that a person selected at random from the sample is as described.
(a) A person who does not favor the amendment
(b) A Republican
(c) A Democrat who favors the amendment

37. **Education** In a high school graduating class of 128 students, 52 are on the honor roll. Of these, 48 are going on to college. Of the 76 students not on the honor roll, 56 are going on to college. What is the probability that a student selected at random from the class is (a) going to college, (b) not going to college, and (c) not going to college and on the honor roll?

38. **Alumni Association** A college sends a survey to members of the class of 2016. Of the 1254 people who graduated that year, 672 are women, of whom 124 went on to graduate school. Of the 582 male graduates, 198 went on to graduate school. Find the probability that a class of 2016 alumnus selected at random is as described.
(a) Female
(b) Male
(c) Female and did not attend graduate school

39. **Winning an Election** Three people are running for president of a class. The results of a poll show that the first candidate has an estimated 37% chance of winning and the second candidate has an estimated 44% chance of winning. What is the probability that the third candidate will win?

40. **Payroll Error** The employees of a company work in six departments: 31 are in sales, 54 are in research, 42 are in marketing, 20 are in engineering, 47 are in finance, and 58 are in production. The payroll clerk loses one employee's paycheck. What is the probability that the employee works in the research department?

41. **Exam Questions** A class receives a list of 20 study problems, from which 10 will be part of an upcoming exam. A student knows how to solve 15 of the problems. Find the probability that the student will be able to answer (a) all 10 questions on the exam, (b) exactly eight questions on the exam, and (c) at least nine questions on the exam.

42. **Payroll Error** A payroll clerk addresses five paychecks and envelopes to five different people and randomly inserts the paychecks into the envelopes. Find the probability of each event.
(a) Exactly one paycheck is inserted in the correct envelope.
(b) At least one paycheck is inserted in the correct envelope.

43. **Game Show** On a game show, you are given five digits to arrange in the proper order to form the price of a car. If you are correct, you win the car. What is the probability of winning, given the following conditions?
(a) You guess the position of each digit.
(b) You know the first digit and guess the positions of the other digits.

44. **Card Game** The deck for a card game contains 108 cards. Twenty-five each are red, yellow, blue, and green, and eight are wild cards. Each player is randomly dealt a seven-card hand.
(a) What is the probability that a hand will contain exactly two wild cards?
(b) What is the probability that a hand will contain two wild cards, two red cards, and three blue cards?

45. Drawing a Card You draw one card at random from a standard deck of 52 playing cards. Find the probability that (a) the card is an even-numbered card, (b) the card is a heart or a diamond, and (c) the card is a nine or a face card.

46. Drawing Cards You draw five cards at random from a standard deck of 52 playing cards. What is the probability that the hand drawn is a full house? (A full house consists of three of one kind and two of another.)

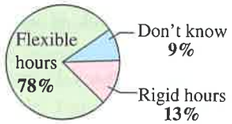
47. Shipment A shipment of 12 microwave ovens contains three defective units. A vending company purchases four units at random. What is the probability that (a) all four units are good, (b) exactly two units are good, and (c) at least two units are good?

48. PIN Code ATM personal identification number (PIN) codes typically consist of four-digit sequences of numbers. Find the probability that if you forget your PIN, you can guess the correct sequence (a) at random and (b) when you recall the first two digits.

49. Random Number Generator A random number generator selects two integers from 1 through 40. What is the probability that (a) both numbers are even, (b) one number is even and one number is odd, (c) both numbers are less than 30, and (d) the same number is selected twice?

50. Flexible Work Hours In a recent survey, people were asked whether they would prefer to work flexible hours—even when it meant slower career advancement—so they could spend more time with their families. The figure shows the results of the survey. What is the probability that three people chosen at random would prefer flexible work hours?

Flexible Work Hours



Probability of a Complement **In Exercises 51–54, you are given the probability that an event *will* happen. Find the probability that the event *will not* happen.**

51. $P(E) = 0.73$
52. $P(E) = 0.28$
53. $P(E) = \frac{1}{5}$
54. $P(E) = \frac{2}{7}$

Probability of a Complement **In Exercises 55–58, you are given the probability that an event *will not* happen. Find the probability that the event *will* happen.**

55. $P(E') = 0.29$
56. $P(E') = 0.89$
57. $P(E') = \frac{14}{25}$
58. $P(E') = \frac{79}{100}$

1971yes/Shutterstock.com

59. Backup System A space vehicle has an independent backup system for one of its communication networks. The probability that either system will function satisfactorily during a flight is 0.985. What is the probability that during a given flight (a) both systems function satisfactorily, (b) both systems fail, and (c) at least one system functions satisfactorily?

60. Backup Vehicle A fire department keeps two rescue vehicles. Due to the demand on the vehicles and the chance of mechanical failure, the probability that a specific vehicle is available when needed is 90%. The availability of one vehicle is independent of the availability of the other. Find the probability that (a) both vehicles are available at a given time, (b) neither vehicle is available at a given time, and (c) at least one vehicle is available at a given time.

61. Roulette American roulette is a game in which a wheel turns on a spindle and is divided into 38 pockets. Thirty-six of the pockets are numbered 1–36, of which half are red and half are black. Two of the pockets are green and are numbered 0 and 00 (see figure). The dealer spins the wheel and a small ball in opposite directions. As the ball slows to a stop, it has an equal probability of landing in any of the numbered pockets.

(a) Find the probability of landing in the number 00 pocket.

(b) Find the probability of landing in a red pocket.

(c) Find the probability of landing in a green pocket or a black pocket.

(d) Find the probability of landing in the number 14 pocket on two consecutive spins.

(e) Find the probability of landing in a red pocket on three consecutive spins.

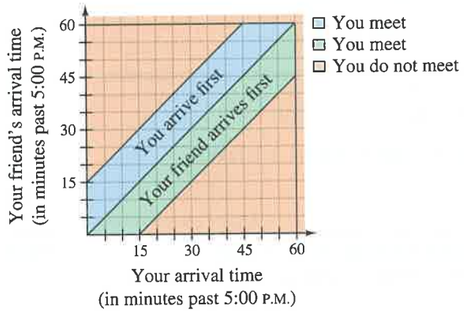
62. A Boy or a Girl? Assume that the probability of the birth of a child of a particular sex is 50%. In a family with four children, find the probability of each event.

(a) All the children are boys.

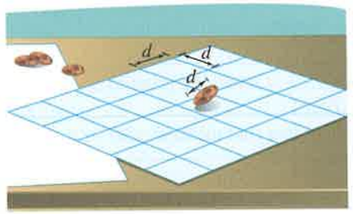
(b) All the children are the same sex.

(c) There is at least one boy.

63. Geometry You and a friend agree to meet at your favorite restaurant between 5:00 P.M. and 6:00 P.M. The one who arrives first will wait 15 minutes for the other, and then will leave (see figure). What is the probability that the two of you will actually meet, assuming that your arrival times are random within the hour?



64. Estimating π You drop a coin of diameter d onto a paper that contains a grid of squares d units on a side (see figure).



(a) Find the probability that the coin covers a vertex of one of the squares on the grid.

(b) Perform the experiment 100 times and use the results to approximate π.

Exploration

True or False? **In Exercises 65 and 66, determine whether the statement is true or false. Justify your answer.**

65. If A and B are independent events with nonzero probabilities, then A can occur when B occurs.

66. Rolling a number less than 3 on a normal six-sided die has a probability of $\frac{1}{3}$. The complement of this event is rolling a number greater than 3, which has a probability of $\frac{1}{2}$.

67. Pattern Recognition Consider a group of n people.

(a) Explain why the pattern below gives the probabilities that the n people have distinct birthdays.

$$n = 2: \frac{365}{365} \cdot \frac{364}{365} = \frac{365 \cdot 364}{365^2}$$

$$n = 3: \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} = \frac{365 \cdot 364 \cdot 363}{365^3}$$

(b) Use the pattern in part (a) to write an expression for the probability that $n = 4$ people have distinct birthdays.

(c) Let P_n be the probability that the n people have distinct birthdays. Verify that this probability can be obtained recursively by

$$P_1 = 1 \text{ and } P_n = \frac{365 - (n-1)}{365} P_{n-1}.$$

(d) Explain why $Q_n = 1 - P_n$ gives the probability that at least two people in a group of n people have the same birthday.

(e) Use the results of parts (c) and (d) to complete the table.

n	10	15	20	23	30	40	50
P_n							
Q_n							

(f) How many people must be in a group so that the probability of at least two of them having the same birthday is greater than $\frac{1}{2}$? Explain.

68. HOW DO YOU SEE IT? The circle graphs show the percents of undergraduate students by class level at two colleges. A student is chosen at random from the combined undergraduate population of the two colleges. The probability that the student is a freshman, sophomore, or junior is 81%. Which college has a greater number of undergraduate students? Explain.